#32: Graph Vocabulary + Implementation

November 9, 2020 · *G Carl Evans*

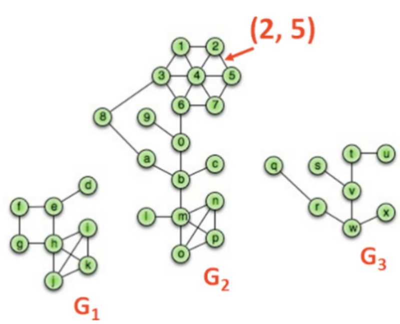
Motivation:

Graphs are awesome data structures that allow us to represent an enormous range of problems. To study these problems, we need:

1. A common vocabulary to talk about graphs
2. Implementation(s) of a graph
3. Traversals on graphs
4. Algorithms on graphs

Graph Vocabulary

Consider a graph **G** with vertices **V** and edges **E**, **G=(V,E)**.



Incident Edges:
  I(v) = { (x, v) in E }

Degree(v): **|I|**

Adjacent Vertices:
  A(v) = { x : (x, v) in E }

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

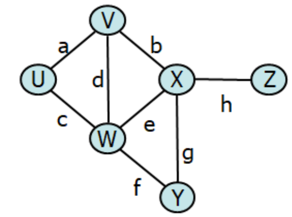
Subgraph(G): **G' = (V', E')**:
  V' ∈ V, E' ∈ E, and (u, v) ∈ E → u ∈ V', v ∈ V'

Graphs that we will study this semester include:
- Complete subgraph(G)
- Connected subgraph(G)
- Connected component(G)
- Acyclic subgraph(G)
- Spanning tree(G)

Size and Running Times

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

For arbitrary graphs, the <u>**minimum**</u> number of edges given a graph that is:



Not Connected:

Minimally Connected:*

The <u>**maximum**</u> number of edges given a graph that is:

Simple:

Not Simple:

The relationship between the degree of the graph and the edges:

Proving the Size of a Minimally Connected Graph

Theorem: Every connected graph **G=(V, E)** has at least **|V|-1** edges.

<u>**Proof of Theorem**</u>

Consider an arbitrary, connected graph **G=(V, E)**.

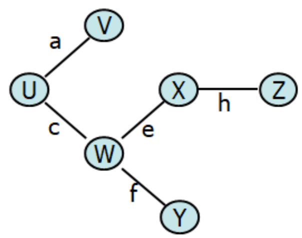
<u>Suppose |V| = 1:</u>

Definition:

Theorem:

<u>Inductive Hypothesis:</u> For any **j** < **|V|**, any connected graph of **j** vertices has at lest **j-1** edges.

<u>Suppose |V| > 1:</u>



1. Choose any vertex:
   -
   -

2. Partitions:
   -
   -

   - **C_0 :=**

   - **C_k, k=[1…d] :=**

3. Count the edges:

|**E_G**| =

*…by application of our IH and Lemma #1, every component **C_k** is a minimally connected subgraph of **G**…*

|**E_G**| =

Graph ADT

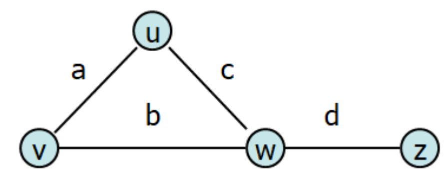
Data	Functions

1. Vertices
 	
2. Edges
 	
3. Some data structure maintaining the structure between vertices and edges.
	`insertVertex(K key);`
	`insertEdge(Vertex v1, Vertex v2, K key);`
	
	`removeVertex(Vertex v);`
	`removeEdge(Vertex v1, Vertex v2);`
	
	`incidentEdges(Vertex v);`
	`areAdjacent(Vertex v1, Vertex v2);`
	
	`origin(Edge e);`
	`destination(Edge e);`

Graph Implementation #1: Edge List

Vert.
u
v
w
z

Edges		
		a
		b
		c
		d



Operations:

insertVertex(K key):

removeVertex(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

incidentEdges(Vertex v):

CS 225 – Things To Be Doing:

1. Exam 4 this Friday!
2. mp_maze Extra Credit due today.
3. Daily POTDs are ongoing!