CS 225

Data Structures

October 14 – AVL Analysis

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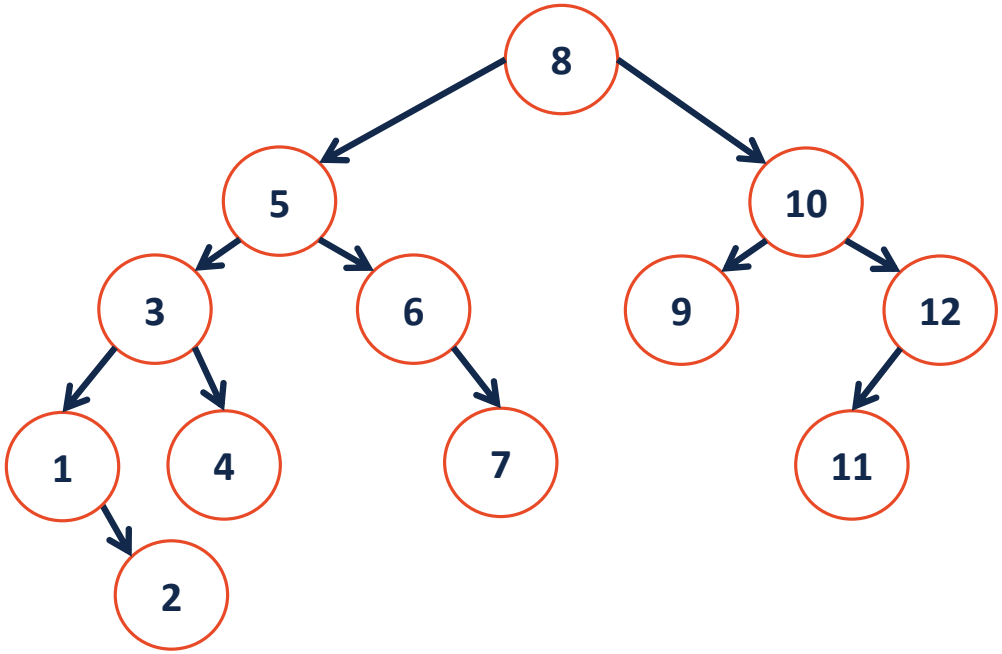
Insertion into an AVL Tree

`_insert(6.5)`

Insert (pseudo code):

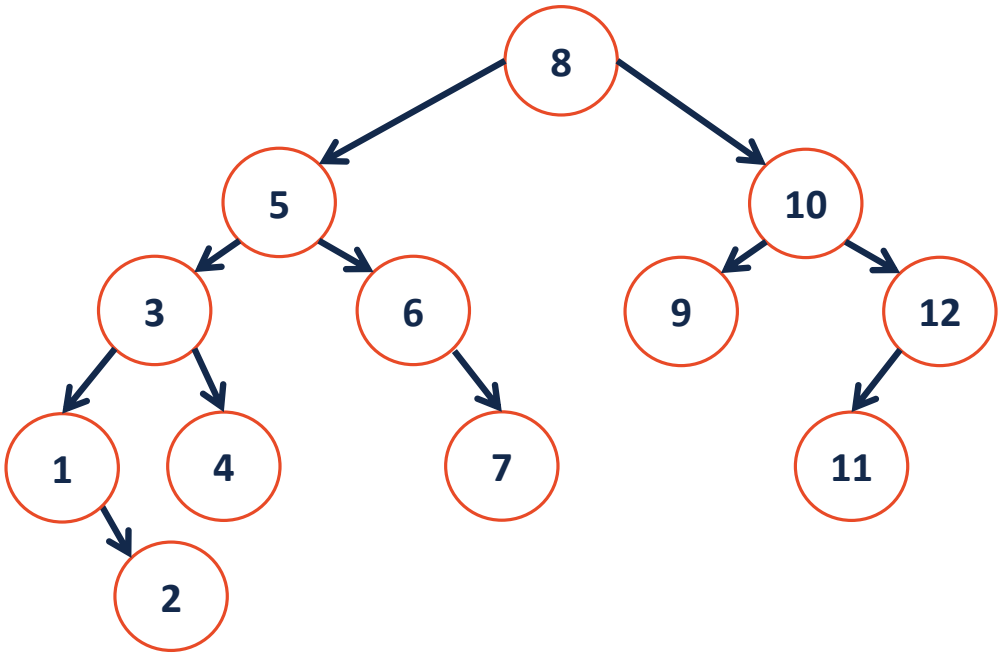
1: Insert at proper place
2: Check for imbalance
3: Rotate, if necessary
4: Update height

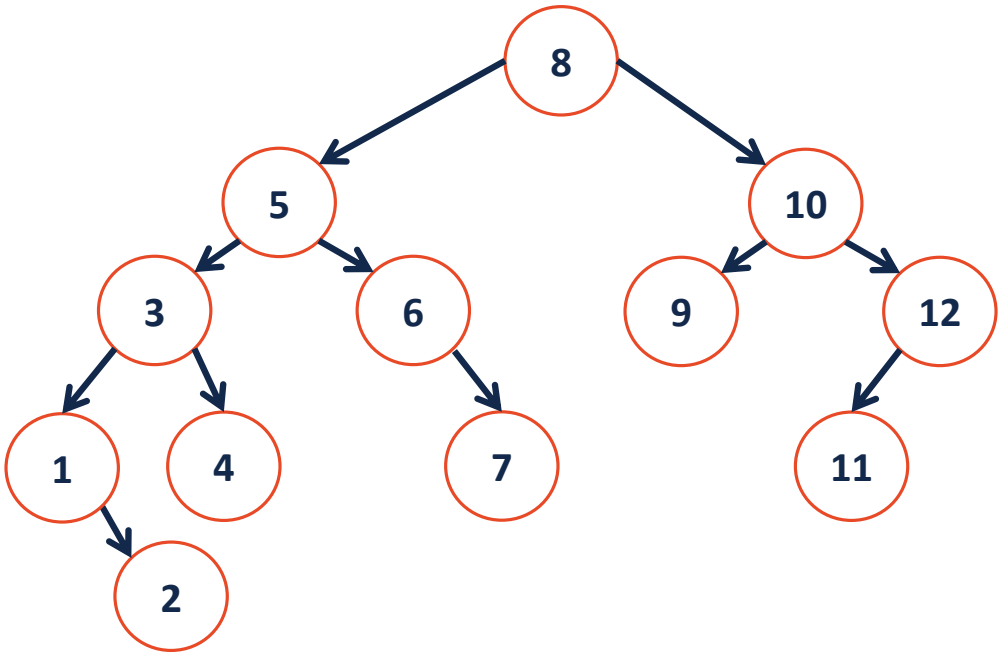
```
1  struct TreeNode {
2    T key;
3    unsigned height;
4    TreeNode *left;
5    TreeNode *right;
6  };
```



```
151 template <typename K, typename V>
152 void AVL<K, D>::_insert(const K & key, const V & data, TreeNode
    *& cur) {
153   if (cur == NULL)          { cur = new TreeNode(key, data);   }
157   else if (key < cur->key) { _insert( key, data, cur->left ); }
160   else if (key > cur->key) { _insert( key, data, cur->right );}
166   _ensureBalance(cur);
167 }
```

```
 1  template <class T> void AVLTree<T>::_insert(const T & x, treeNode<T> * & t ) {
 2    if( t == NULL ) {
 3      t = new TreeNode<T>( x, 0, NULL, NULL);
 4    }
 5
 6    else if( x < t->key ) {
 7      _insert( x, t->left );
 8      int balance = height(t->right) - height(t->left);
 9      int leftBalance = height(t->left->right) - height(t->left->left);
10      if ( balance == -2 ) {
11        if ( leftBalance == -1 ) { rotate_____________( t ); }
12        else                     { rotate_____________( t ); }
13      }
14    }
15
16    else if( x > t->key ) {
17      _insert( x, t->right );
18      int balance = height(t->right) - height(t->left);
19      int rightBalance = height(t->right->right) - height(t->right->left);
20      if( balance == 2 ) {
21        if( rightBalance == 1 ) { rotate_____________( t ); }
22        else                    { rotate_____________( t ); }
23      }
24    }
25
26    t->height = 1 + max(height(t->left), height(t->right));
27  }
```


8
5
10
3
6
9
12
1
4
7
11
2


8
5
10
3
6
9
12
1
4
7
11
2

AVL Tree Analysis

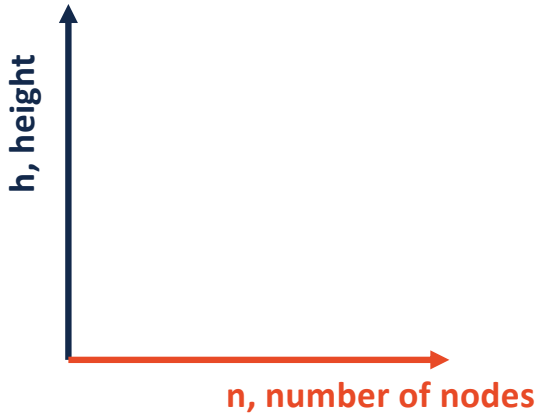
We know: insert, remove and find runs in: _____.

We will argue that: h is _____.

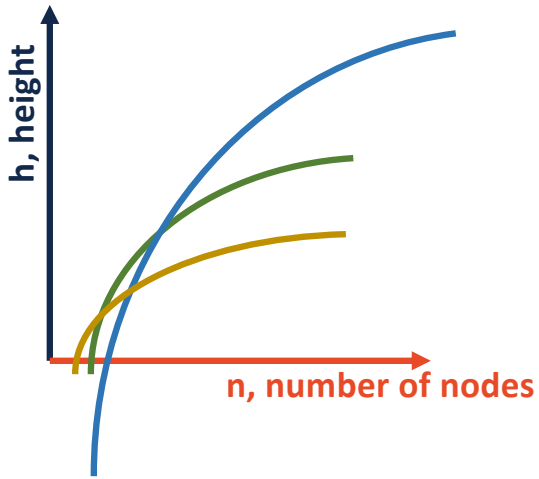
AVL Tree Analysis

Definition of big-O:

…or, with pictures:

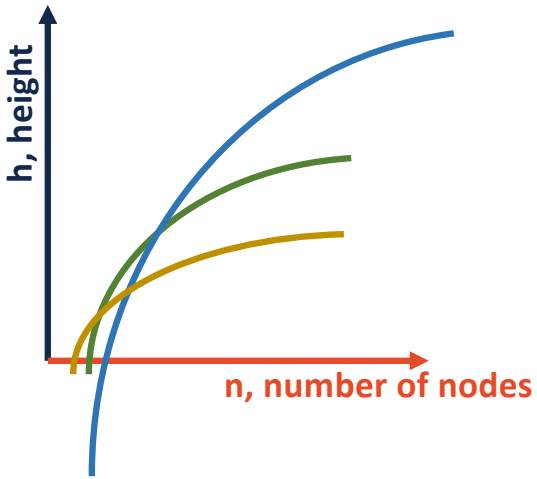


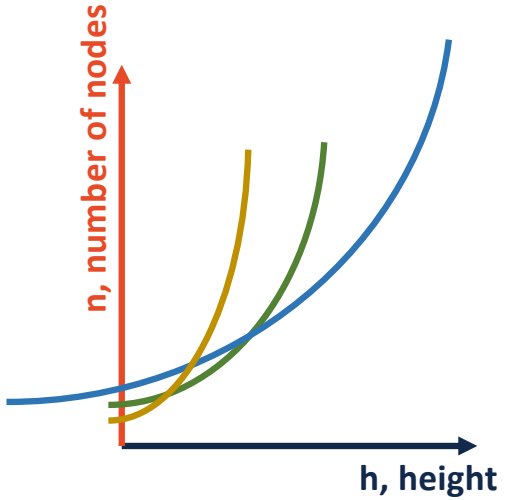
AVL Tree Analysis



- The height of the tree, **f(n)**, will always be <u>less than</u> **c × g(n)** for all values where **n > k**.

AVL Tree Analysis





- The number of nodes in the tree, $\mathbf{f^{-1}(h)}$, will always be <u>greater than</u> $\mathbf{c \times g^{-1}(h)}$ for all values where $\mathbf{n > k}$.

Plan of Action

Since our goal is to find the lower bound on **n** given **h**, we can begin by defining a function given **h** which describes the smallest number of nodes in an AVL tree of height **h**:

Simplify the Recurrence

N(h) = 1 + N(h - 1) + N(h - 2)

State a Theorem

Theorem: An AVL tree of height h has at least _____.

Proof:

I. Consider an AVL tree and let **h** denote its height.

II. Case: _____

An AVL tree of height ____ has at least ____ nodes.

Prove a Theorem

III. Case: _____

An AVL tree of height ____ has at least ____ nodes.

Prove a Theorem

IV. Case: _____

By an Inductive Hypothesis (IH):

We will show that:

An AVL tree of height ____ has at least ____ nodes.

Prove a Theorem

V. Using a proof by induction, we have shown that:

…and inverting:

Summary of Balanced BST

Red-Black Trees

- Max height: 2 * lg(n)
- Constant number of rotations on insert, remove, and find

AVL Trees

- Max height: 1.44 * lg(n)
- Rotations:

Summary of Balanced BST

Pros:

- Running Time:

  - Improvement Over:

- Great for specific applications:

Summary of Balanced BST

Cons:

- Running Time:

- In-memory Requirement: