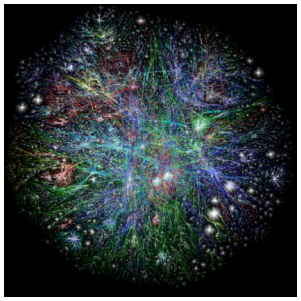
CS 225

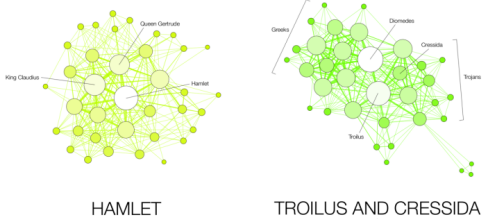
Data Structures

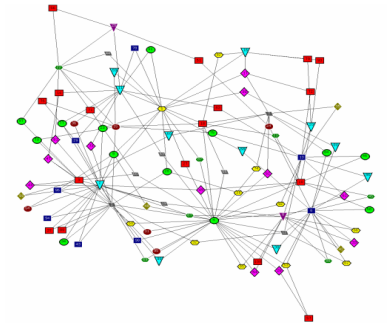
November 8 – Graphs

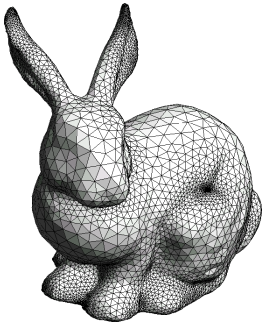
G Carl Evans

Graphs



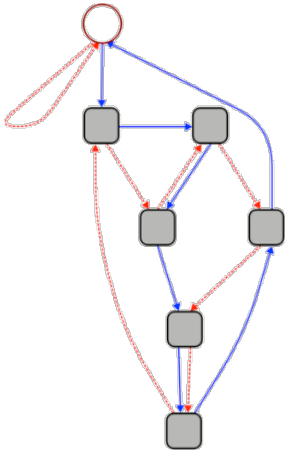


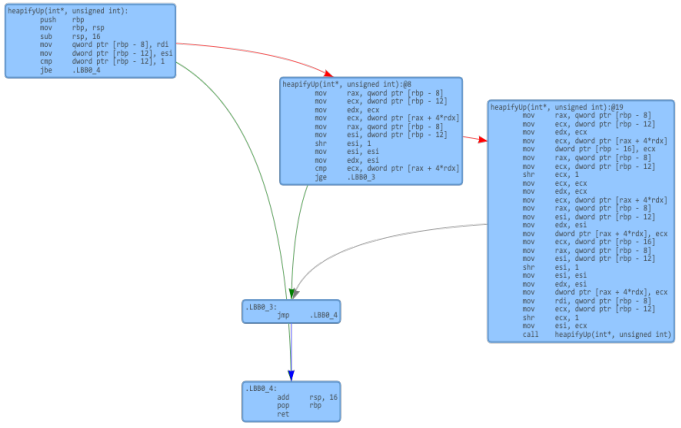




To study all of these structures:

1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms



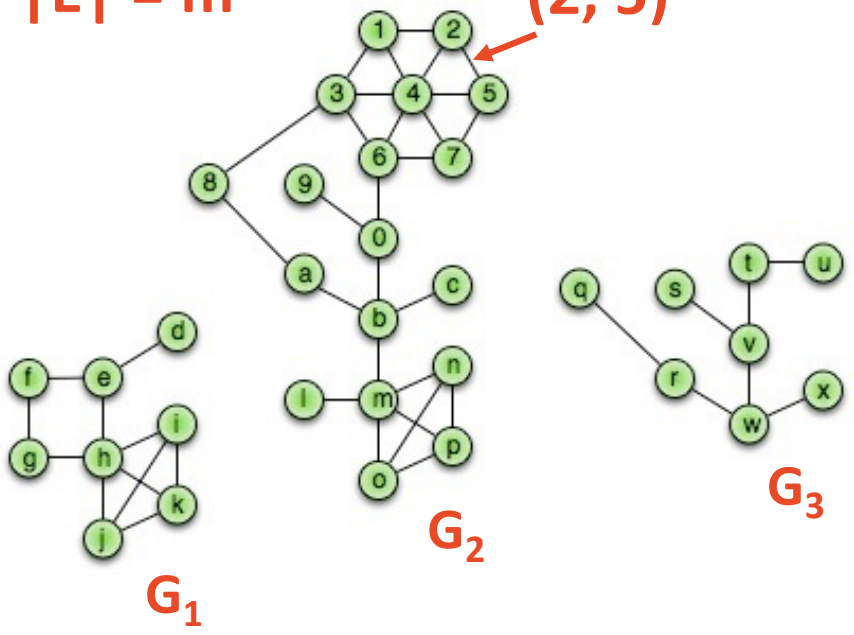


Graph Vocabulary

G = (V, E)

|V| = n

|E| = m



Incident Edges:
I(v) = { {x, v} in E }

Degree(v): |I|

Adjacent Vertices:
A(v) = { x : {x, v} in E }

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

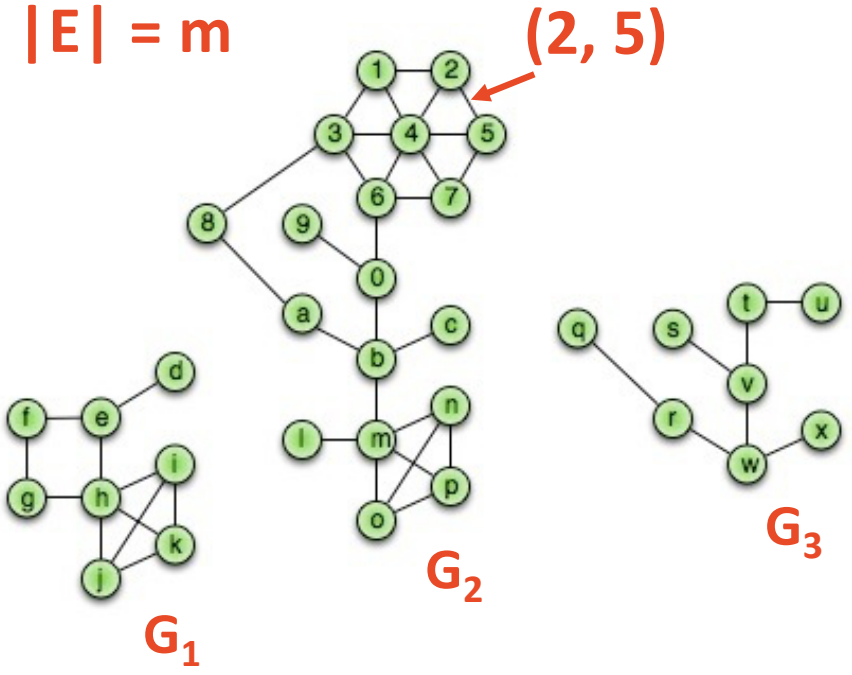
Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

G = (V, E)

|V| = n

|E| = m



Subgraph(G):

G' = (V', E'):

V' ∈ V, E' ∈ E, and

(u, v) ∈ E' → u ∈ V', v ∈ V'

Complete subgraph(G)

Connected subgraph(G)

Connected component(G)

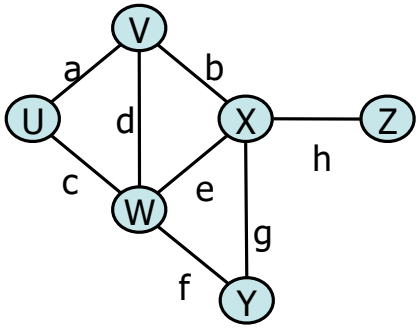
Acyclic subgraph(G)

Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges? **Minimum edges:**

Not Connected:



Connected*:

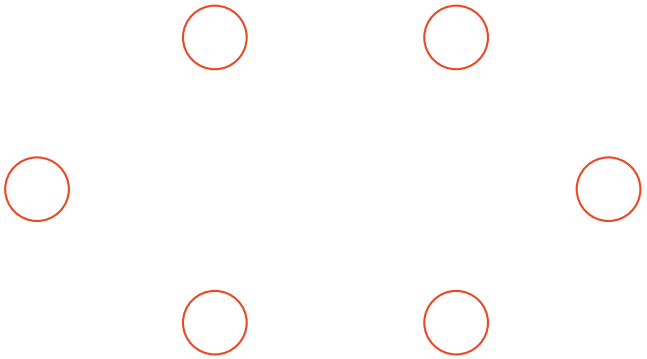
Maximum edges:

Simple:

Not simple:

$$\sum_{v \in V} \deg(v) =$$

Connected Graphs



Proving the size of a minimally connected graph

Theorem:
Every connected graph **G=(V, E)** has at least **|V|-1** edges.

Thm: Every connected graph **G=(V, E)** has at least **|V|-1** edges.

Proof: Consider an arbitrary, connected graph **G=(V, E)**.

Suppose |V| = 1:

Definition: A connected graph of 1 vertex has 0 edges.

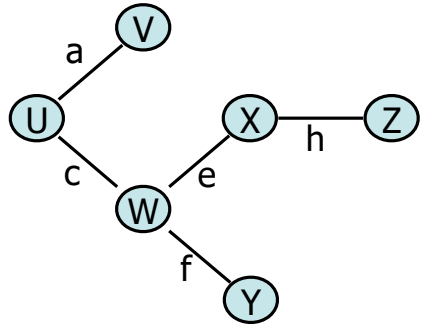
Theorem: |V|-1 edges ➔ 1-1 = 0.

Inductive Hypothesis: For any **j < |V|**, any connected graph of **j** vertices has at least **j-1** edges.

Suppose |V| > 1:

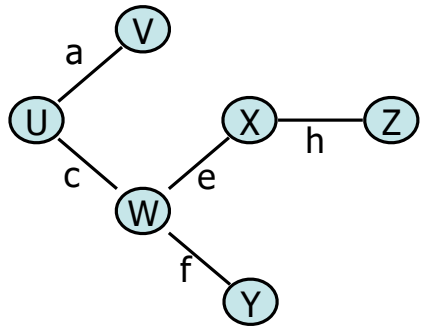
1. Choose any vertex:

2. Partition:



Suppose |V| > 1:

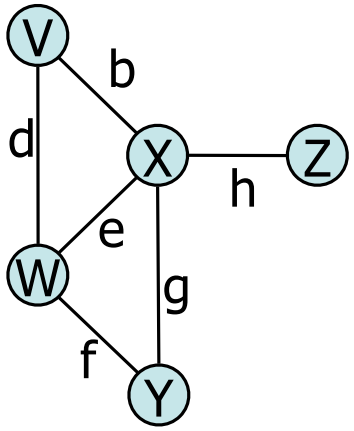
3. Count the edges



Graph ADT

Data:

- **Vertices**
- **Edges**
- **Some data structure maintaining the structure between vertices and edges.**



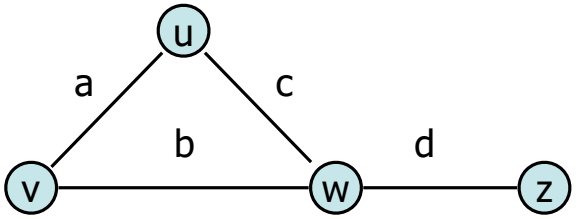
Functions:

- **insertVertex(K key);**
- **insertEdge(Vertex v1, Vertex v2, K key);**

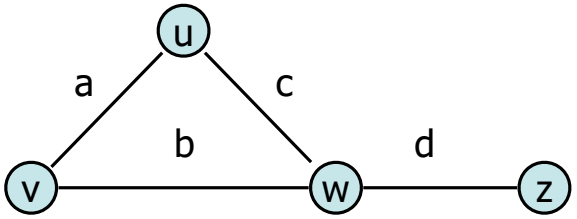
- **removeVertex(Vertex v);**
- **removeEdge(Vertex v1, Vertex v2);**

- **incidentEdges(Vertex v);**
- **areAdjacent(Vertex v1, Vertex v2);**

Graph Implementation Idea



Graph Implementation: Edge List



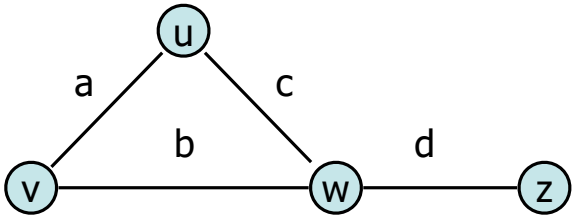
Vertex Collection:

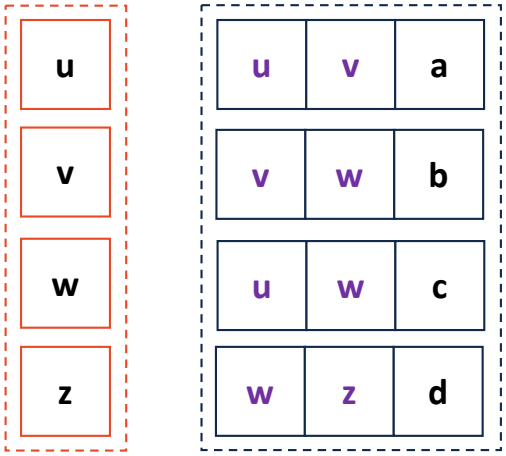
u
v
w
z

Edge Collection:

u	v	a
v	w	b
u	w	c
w	z	d

Graph Implementation: Edge List

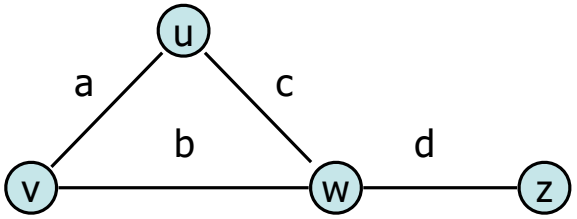


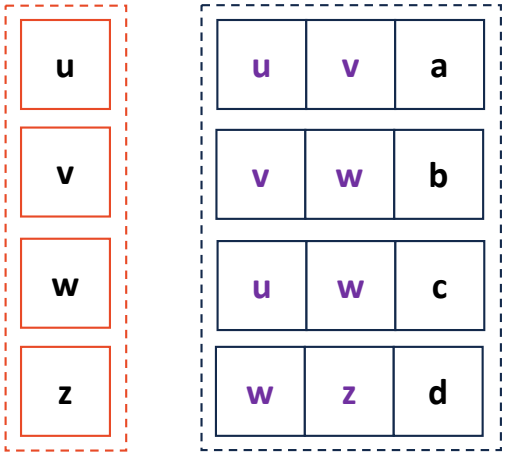


insertVertex(K key):

removeVertex(Vertex v):

Graph Implementation: Edge List



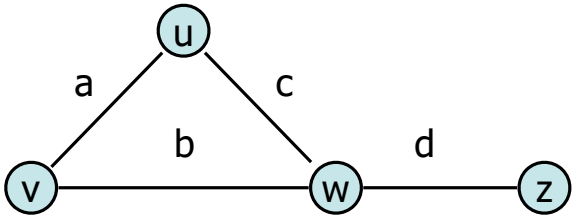


incidentEdges(Vertex v):

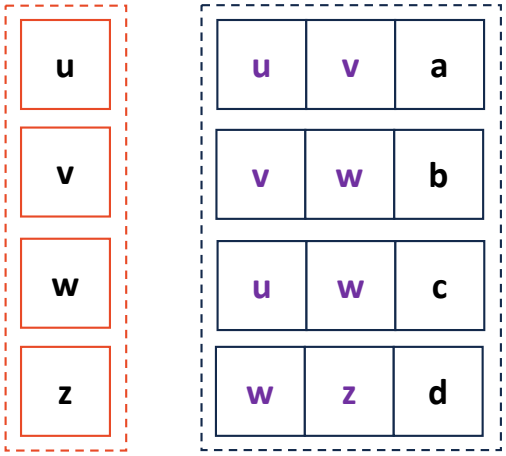
areAdjacent(Vertex v1, Vertex v2):

```
G.incidentEdges(v1).contains(v2)
```

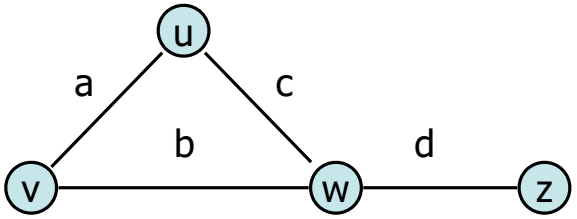
Graph Implementation: Edge List

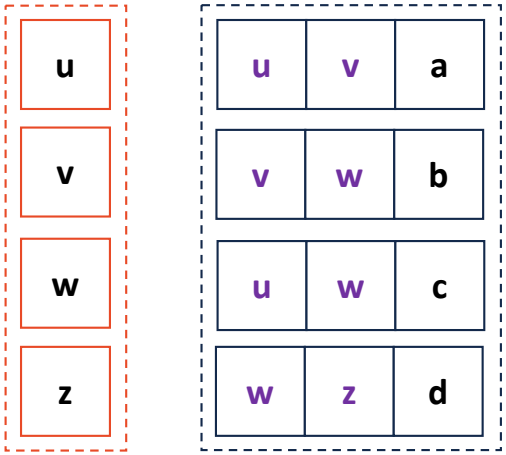


insertEdge(Vertex v1, Vertex v2, K key):



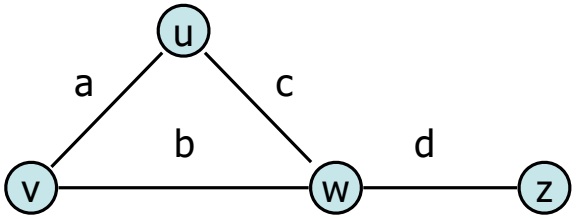
Graph Implementation: Adjacency Matrix

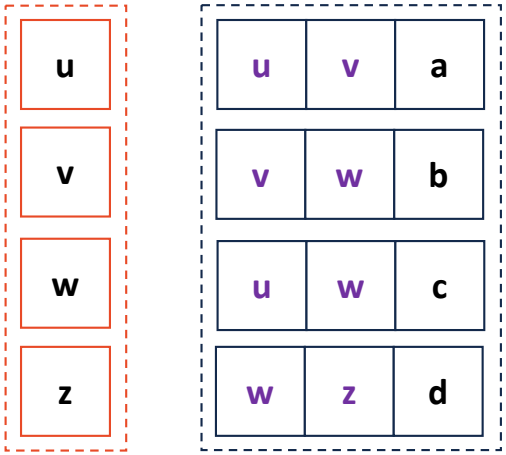




	u	v	w	z
u				
v				
w				
z				

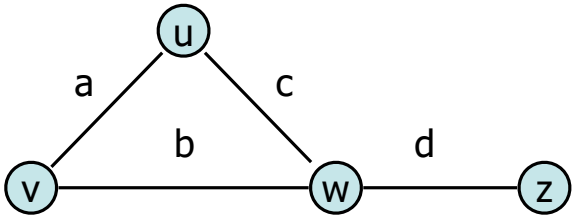
Graph Implementation: Adjacency Matrix

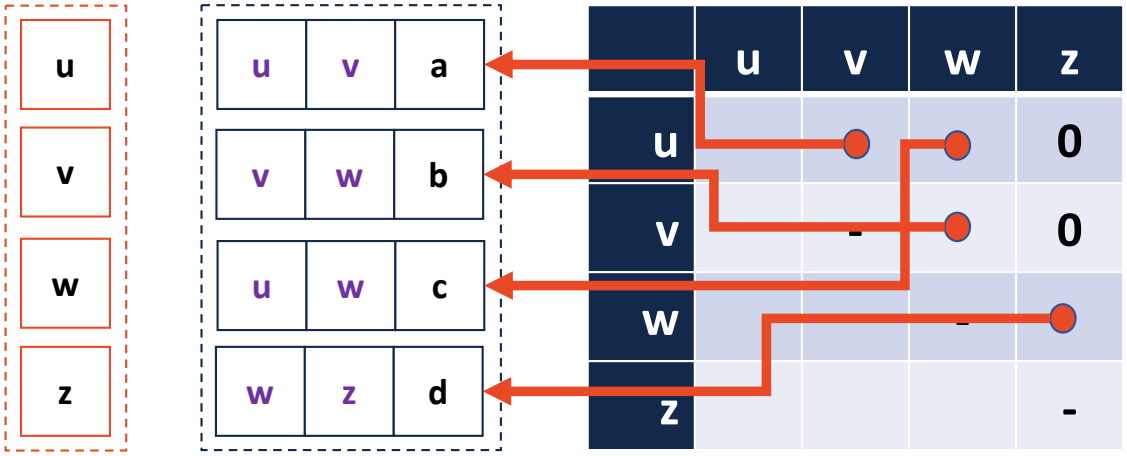




	u	v	w	z
u	-	1	1	0
v		-	1	0
w			-	1
z				-

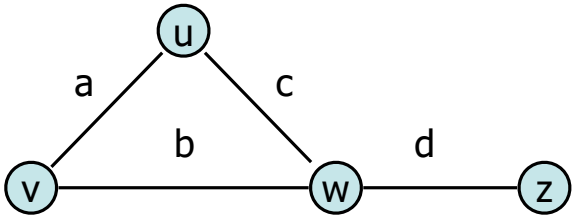
Graph Implementation: Adjacency Matrix

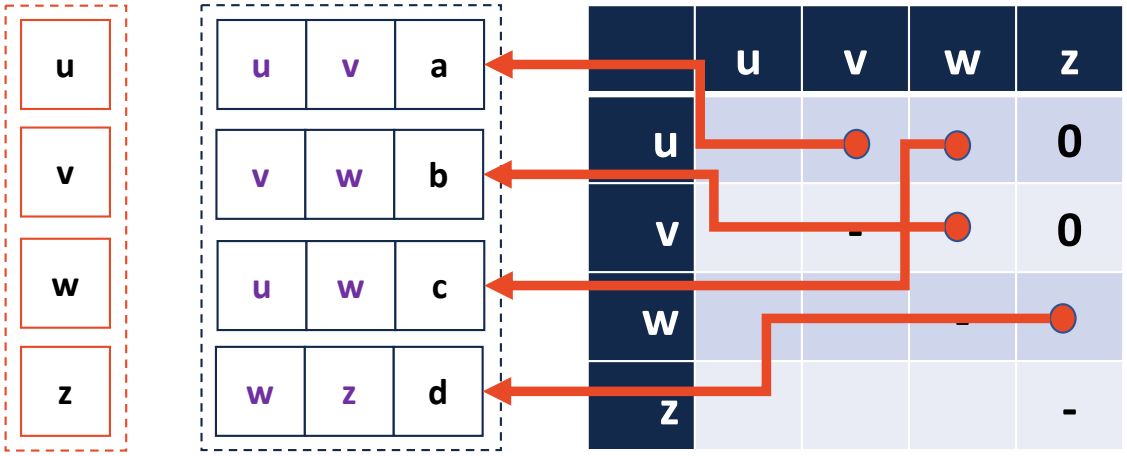




Graph Implementation: Adjacency Matrix

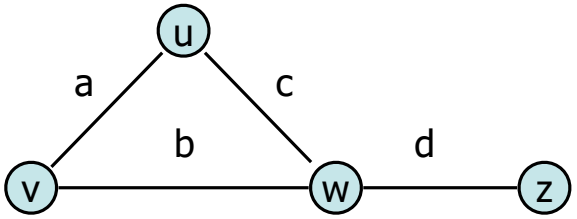
insertVertex(K key):

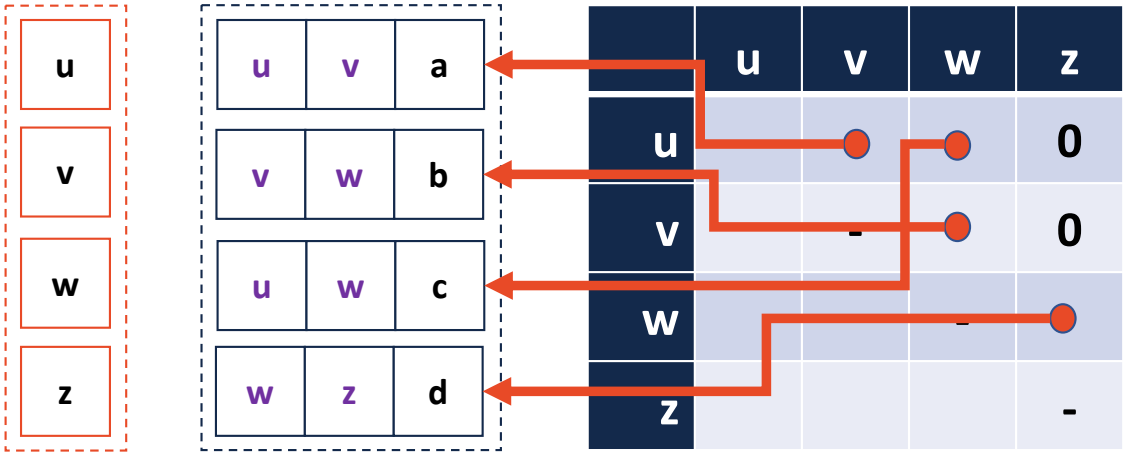




Graph Implementation: Adjacency Matrix

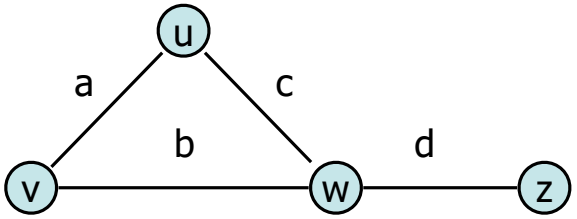
removeVertex(Vertex v):

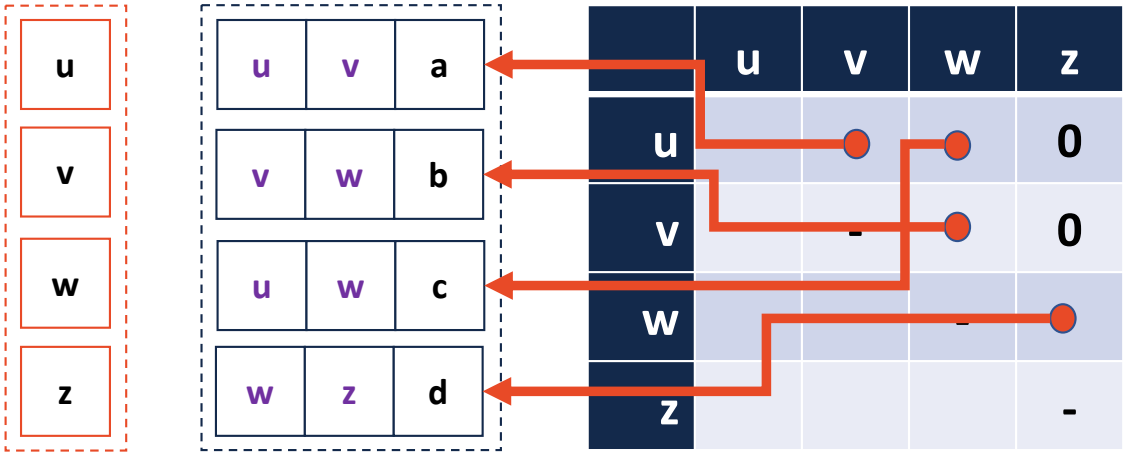




Graph Implementation: Adjacency Matrix

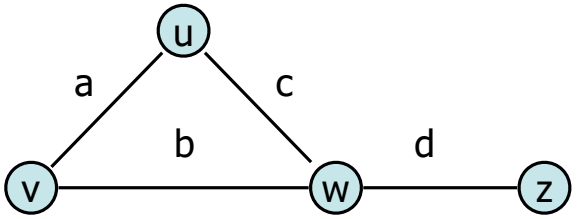
incidentEdges(Vertex v):

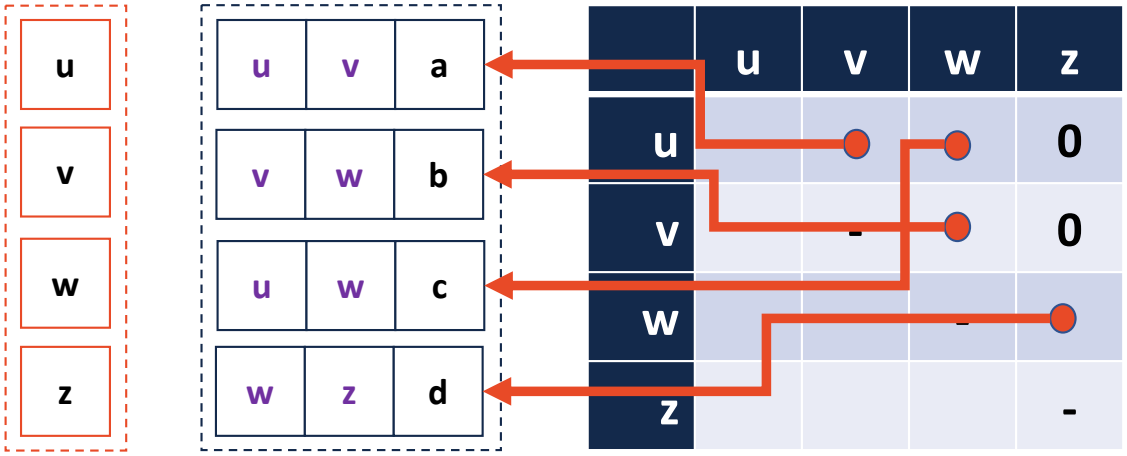




Graph Implementation: Adjacency Matrix

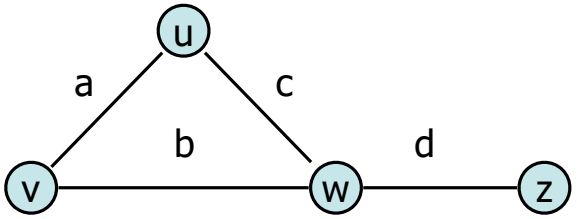
areAdjacent(Vertex v1, Vertex v2):

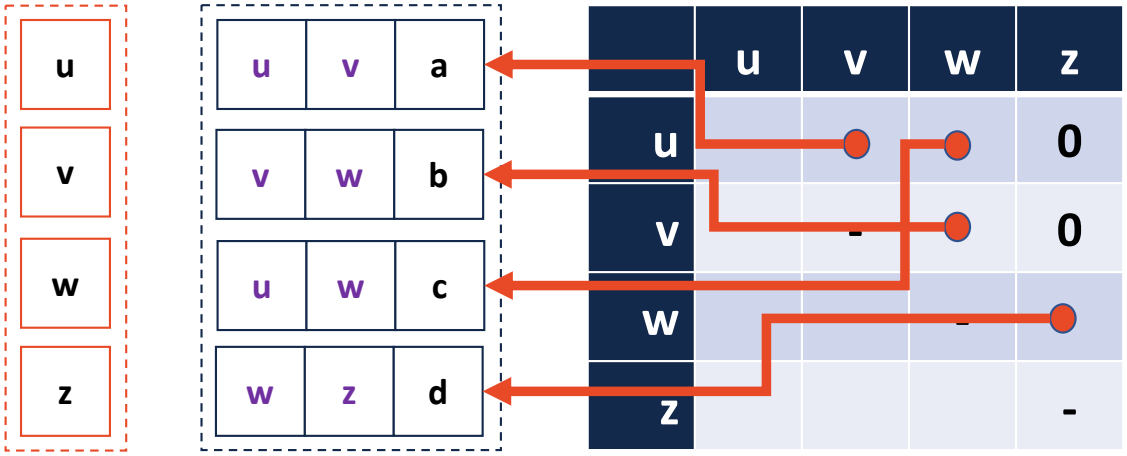




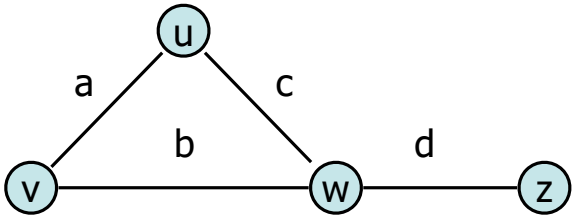
Graph Implementation: Adjacency Matrix

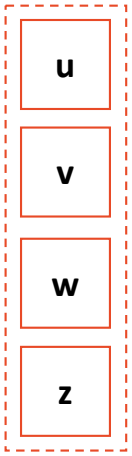
insertEdge(Vertex v1, Vertex v2, K key):

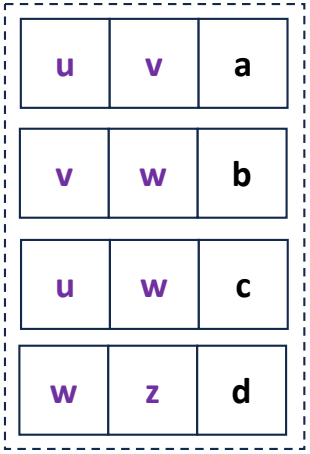




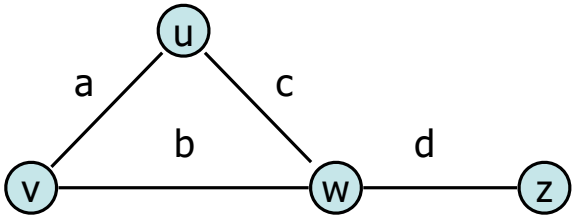
Graph Implementation: Edge List

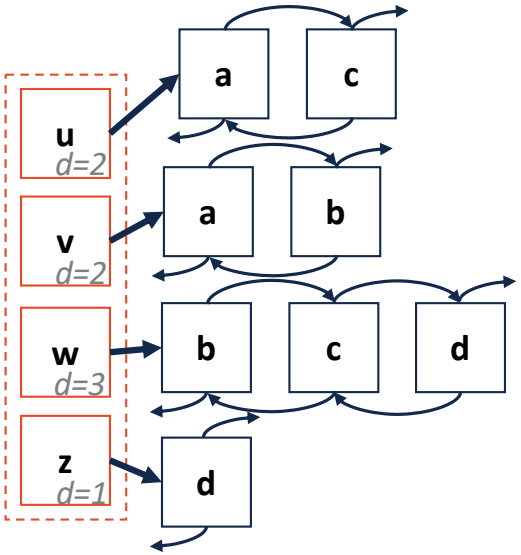


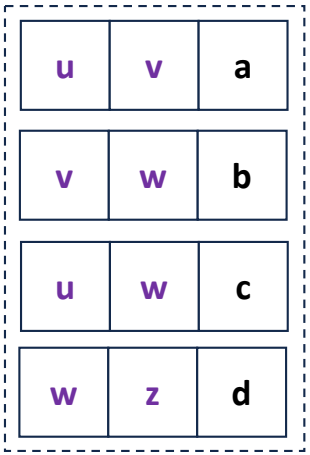




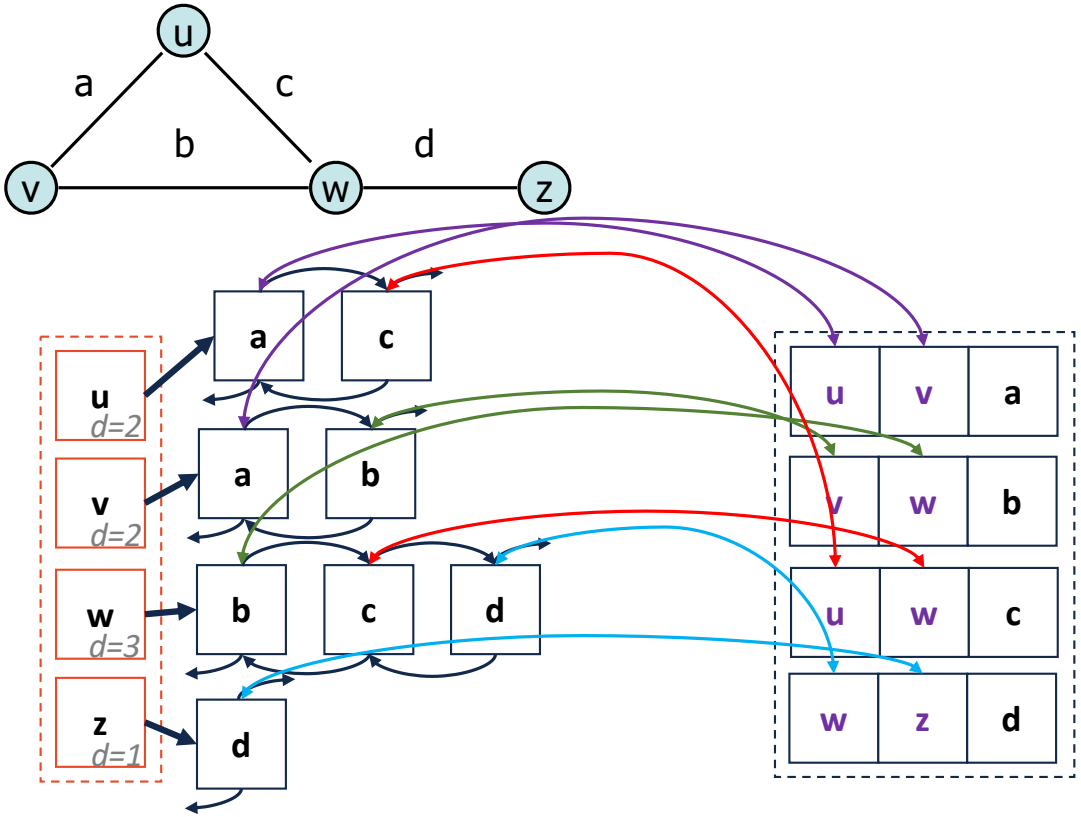
Adjacency List





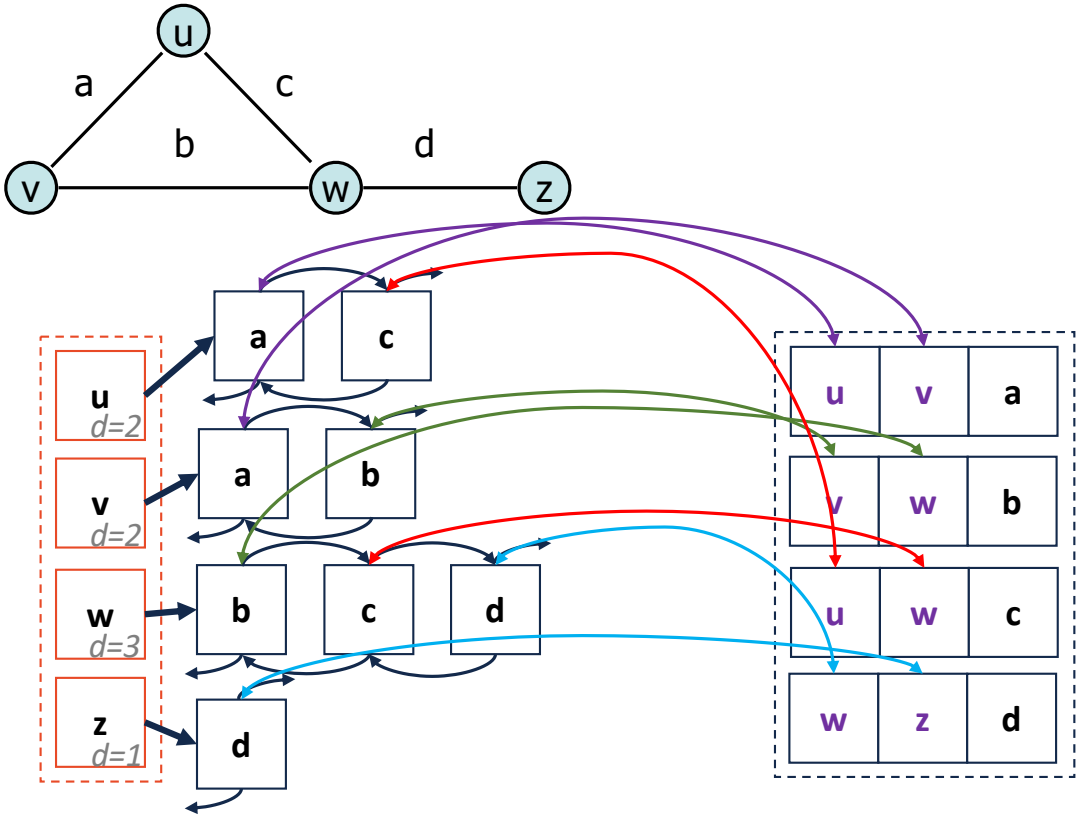


Adjacency List



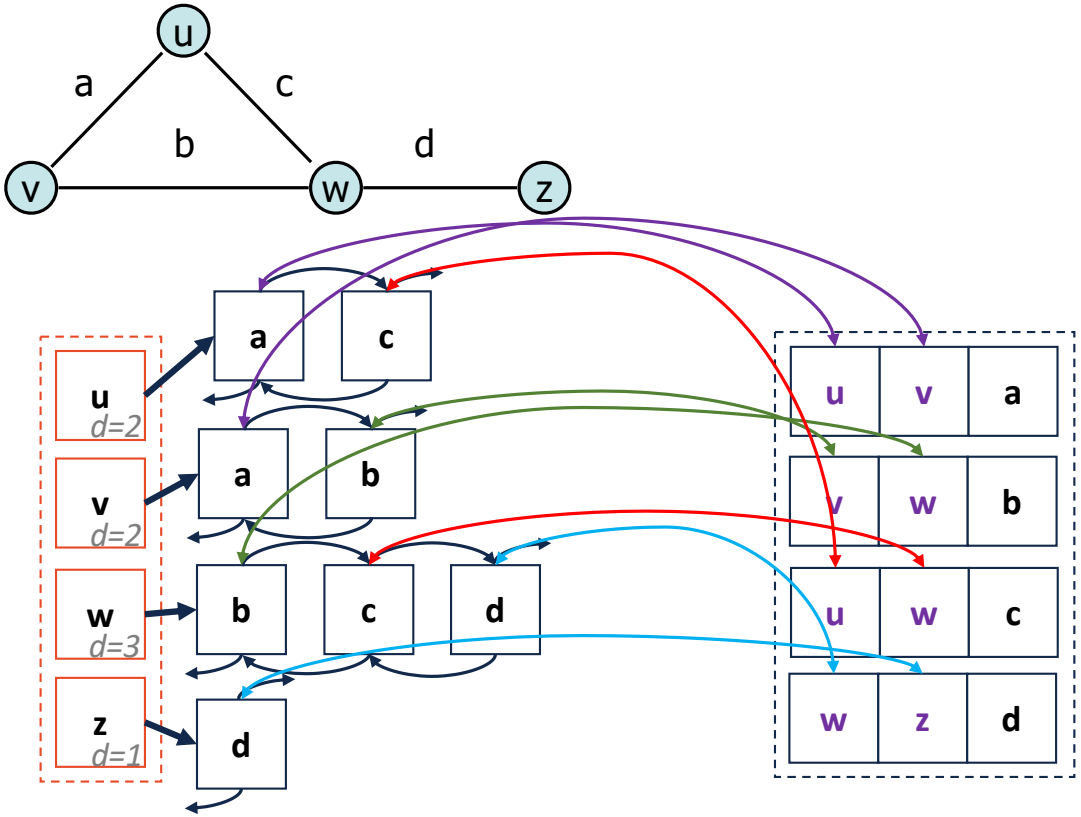
Adjacency List

insertVertex(K key):



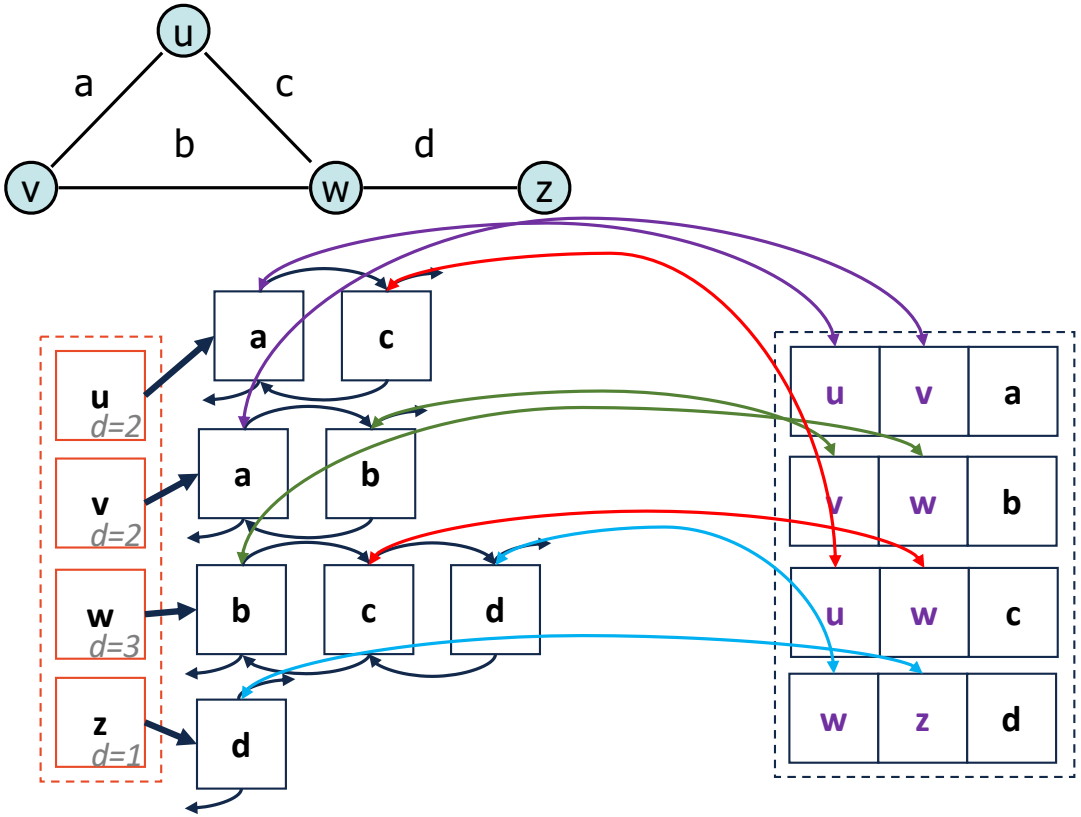
Adjacency List

removeVertex(Vertex v):



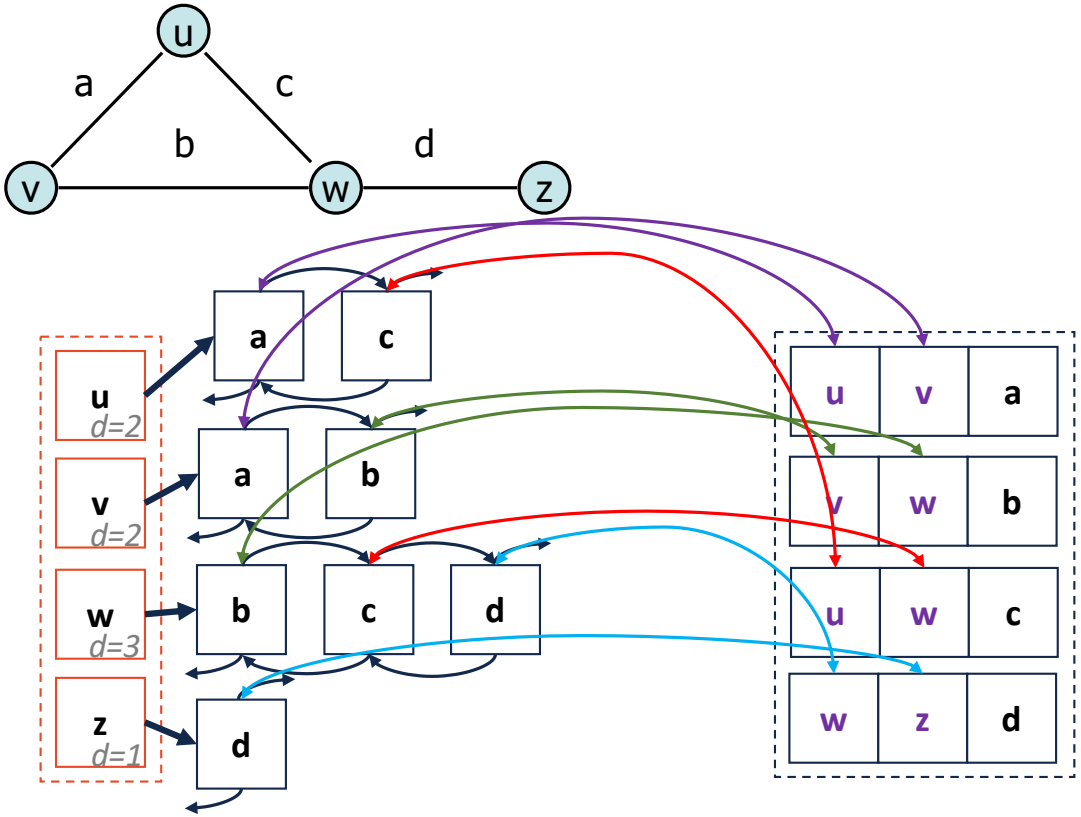
Adjacency List

incidentEdges(Vertex v):



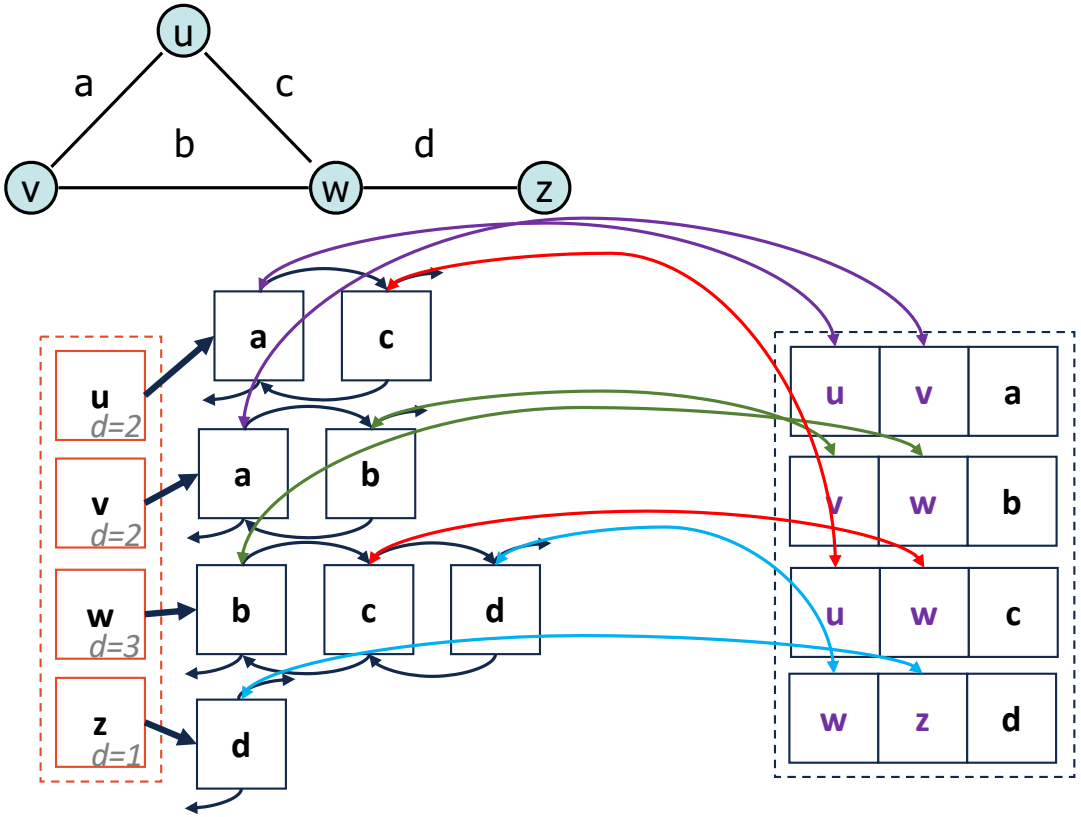
Adjacency List

areAdjacent(Vertex v1, Vertex v2):



Adjacency List

insertEdge(Vertex v1, Vertex v2, K key):



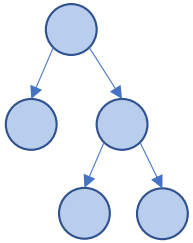
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1	n	1
removeVertex(v)	m	n	deg(v)
insertEdge(v, w, k)	1	1	1
removeEdge(v, w)	1	1	1
incidentEdges(v)	m	n	deg(v)
areAdjacent(v, w)	m	1	min(deg(v), deg(w))

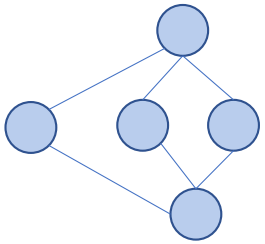
Traversal:

Objective: Visit every vertex and every edge in the graph.

Purpose: Search for interesting sub-structures in the graph.

We've seen traversal before ….but it's different:

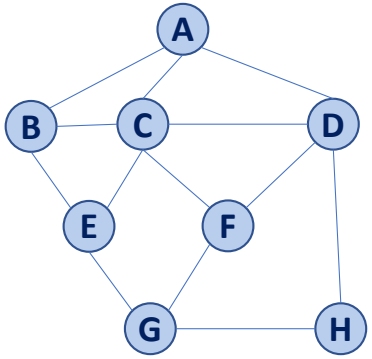




- Ordered
- Obvious Start
-

-
-
-

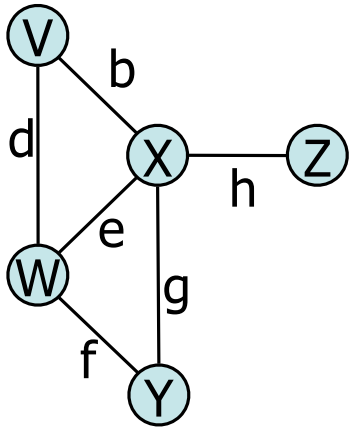
Traversal: BFS



Graph ADT

Data:

- **Vertices**
- **Edges**
- **Some data structure maintaining the structure between vertices and edges.**



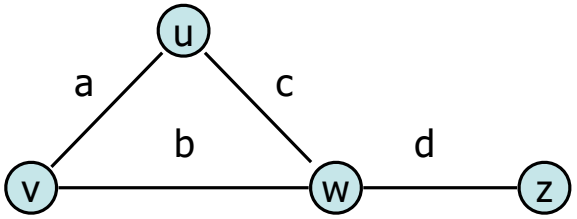
Functions:

- **insertVertex(K key);**
- **insertEdge(Vertex v1, Vertex v2, K key);**

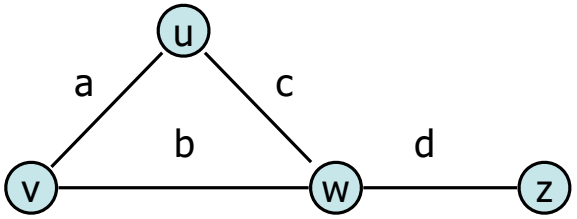
- **removeVertex(Vertex v);**
- **removeEdge(Vertex v1, Vertex v2);**

- **incidentEdges(Vertex v);**
- **areAdjacent(Vertex v1, Vertex v2);**

Graph Implementation Idea



Graph Implementation: Edge List



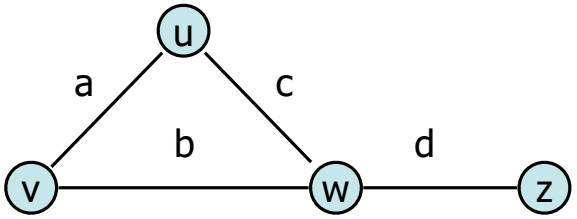
Vertex Collection:

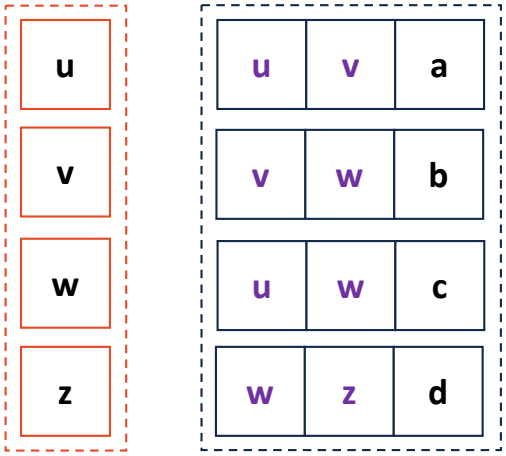
u
v
w
z

Edge Collection:

u	v	a
v	w	b
u	w	c
w	z	d

Graph Implementation: Edge List

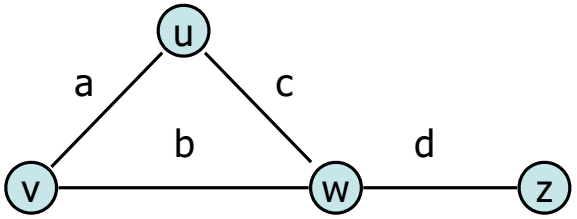


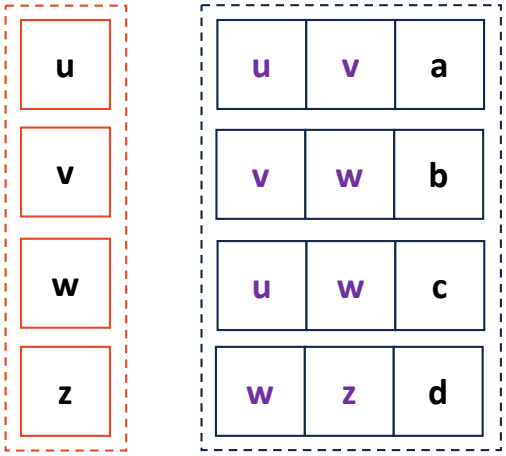


insertVertex(K key):

removeVertex(Vertex v):

Graph Implementation: Edge List



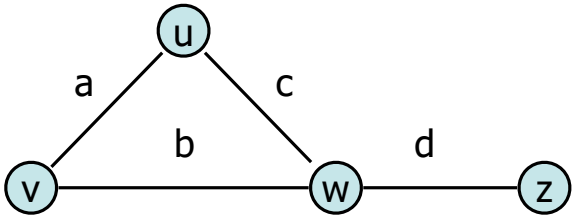


incidentEdges(Vertex v):

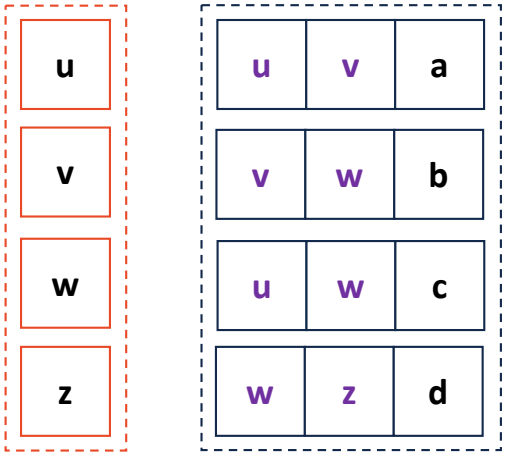
areAdjacent(Vertex v1, Vertex v2):

```
G.incidentEdges(v1).contains(v2)
```

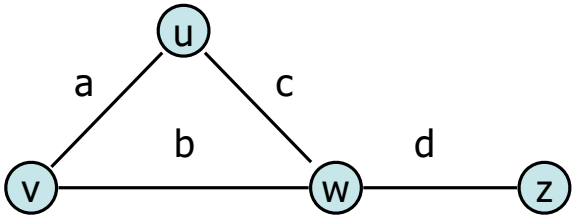
Graph Implementation: Edge List

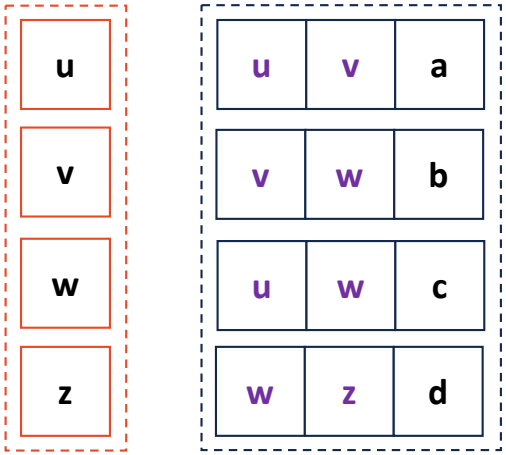


insertEdge(Vertex v1, Vertex v2, K key):



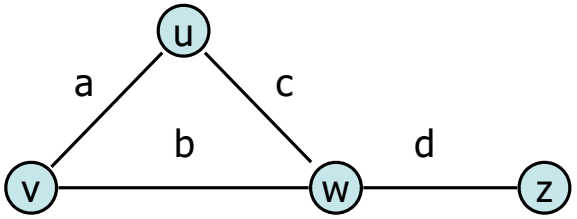
Graph Implementation: Adjacency Matrix

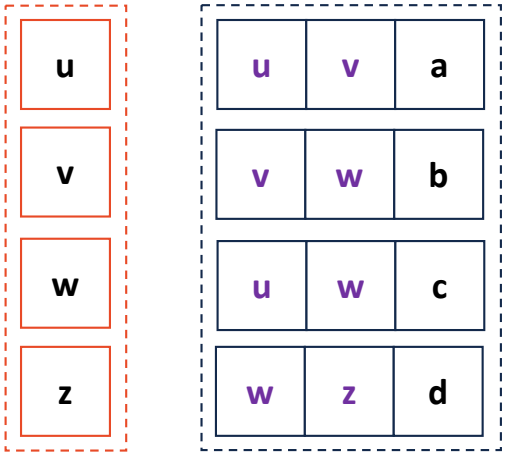




	u	v	w	z
u				
v				
w				
z				

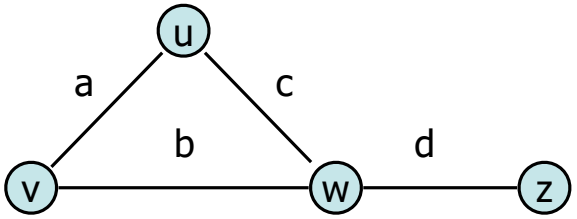
Graph Implementation: Adjacency Matrix

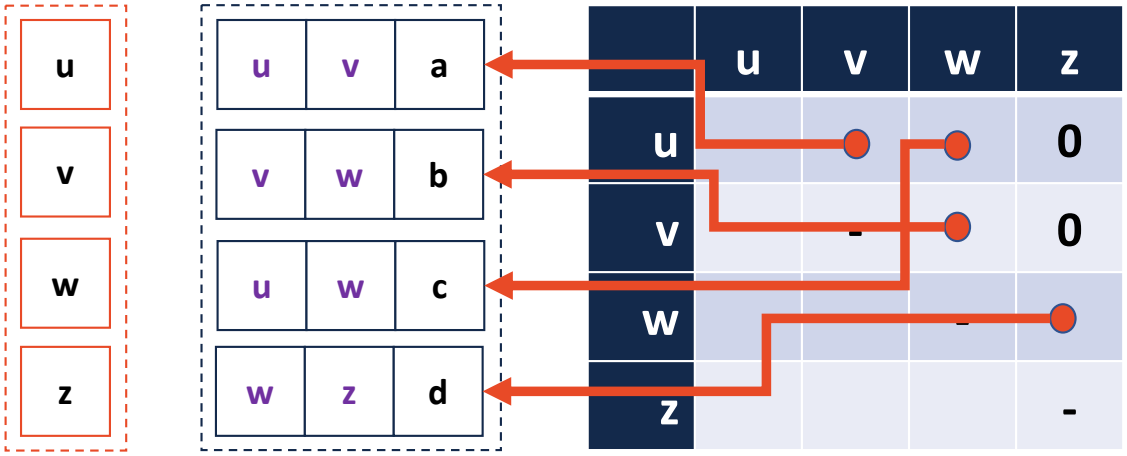




	u	v	w	z
u	-	1	1	0
v		-	1	0
w			-	1
z				-

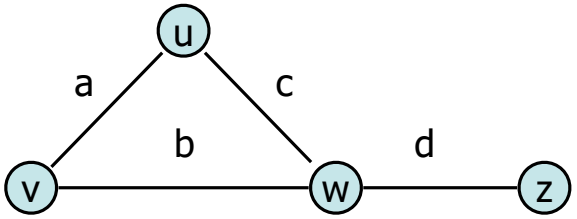
Graph Implementation: Adjacency Matrix

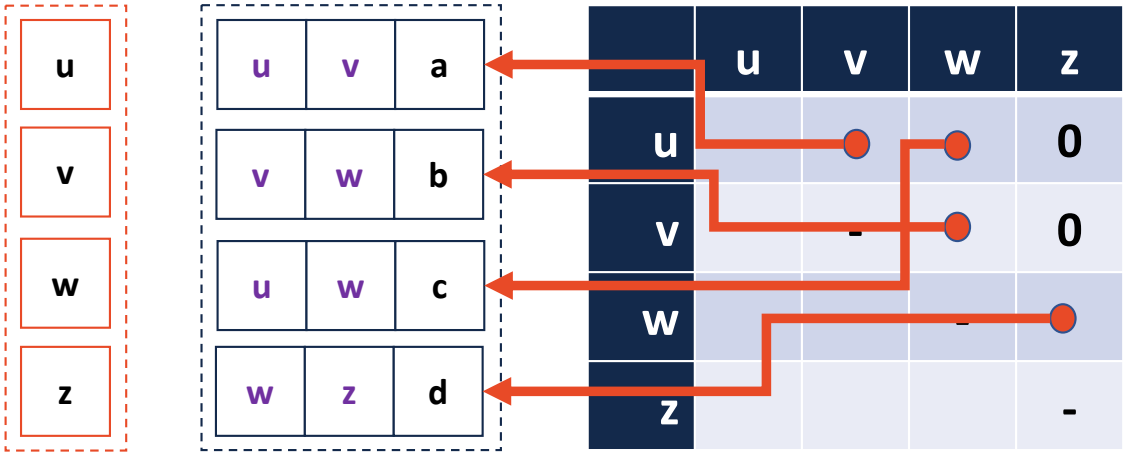




Graph Implementation: Adjacency Matrix

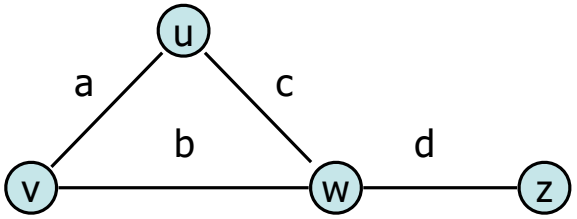
insertVertex(K key):

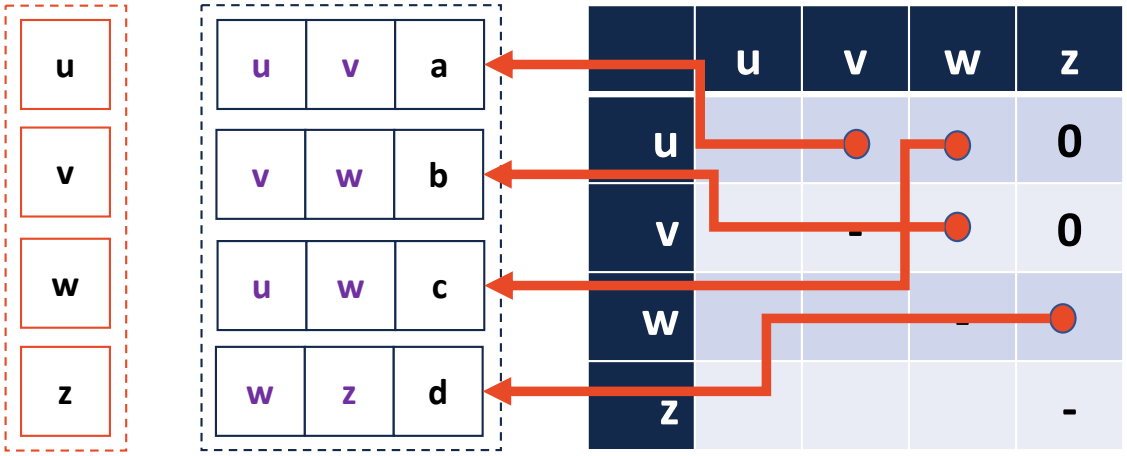




Graph Implementation: Adjacency Matrix

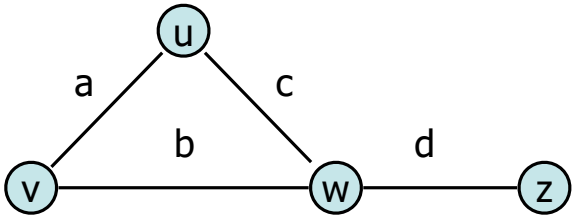
removeVertex(Vertex v):

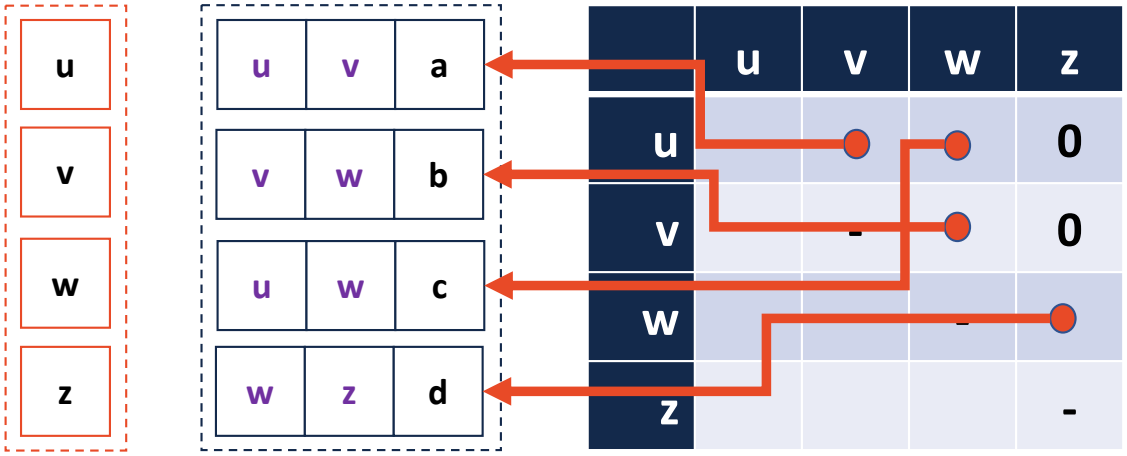




Graph Implementation: Adjacency Matrix

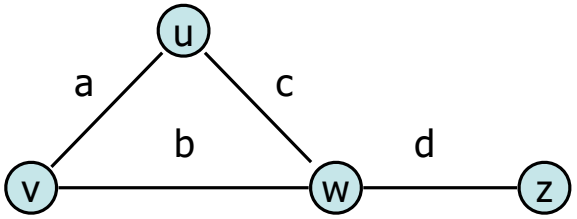
incidentEdges(Vertex v):

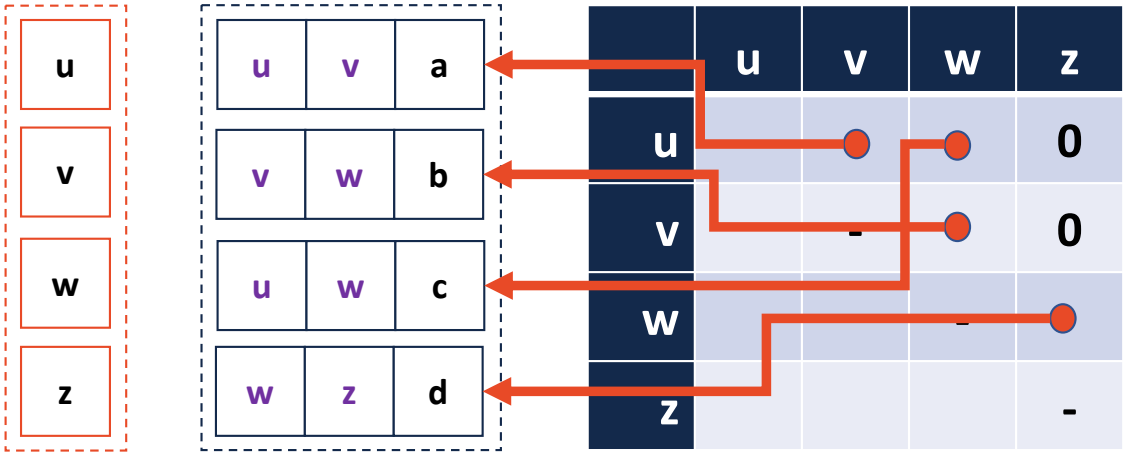




Graph Implementation: Adjacency Matrix

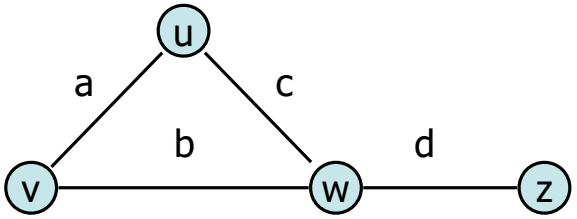
areAdjacent(Vertex v1, Vertex v2):

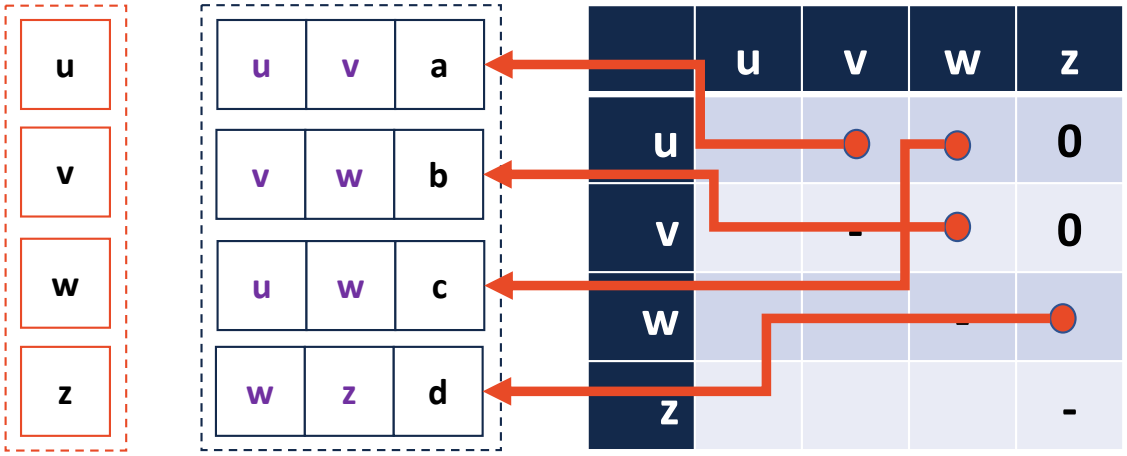




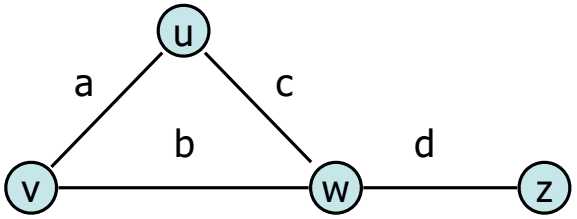
Graph Implementation: Adjacency Matrix

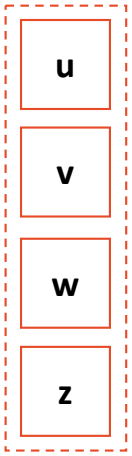
insertEdge(Vertex v1, Vertex v2, K key):

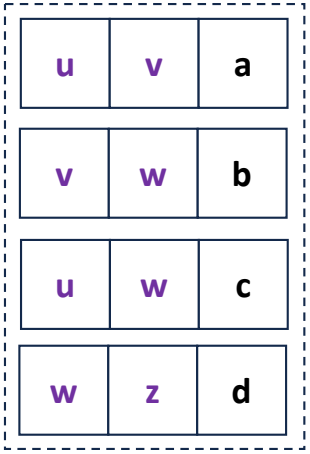




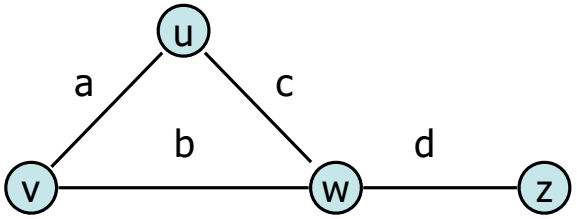
Graph Implementation: Edge List

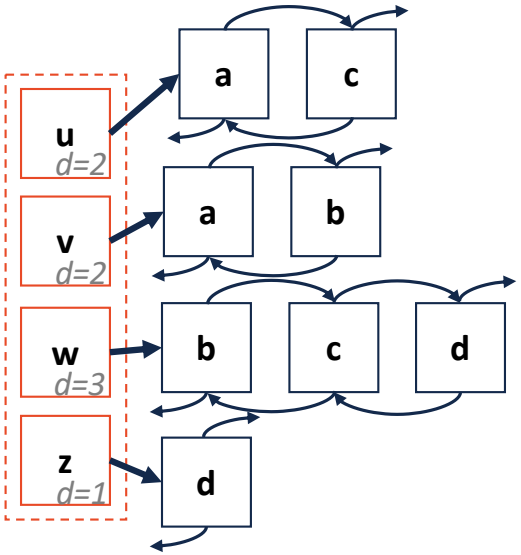


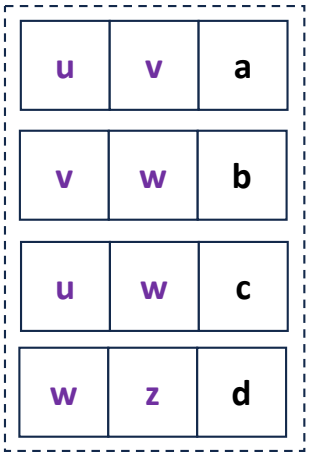




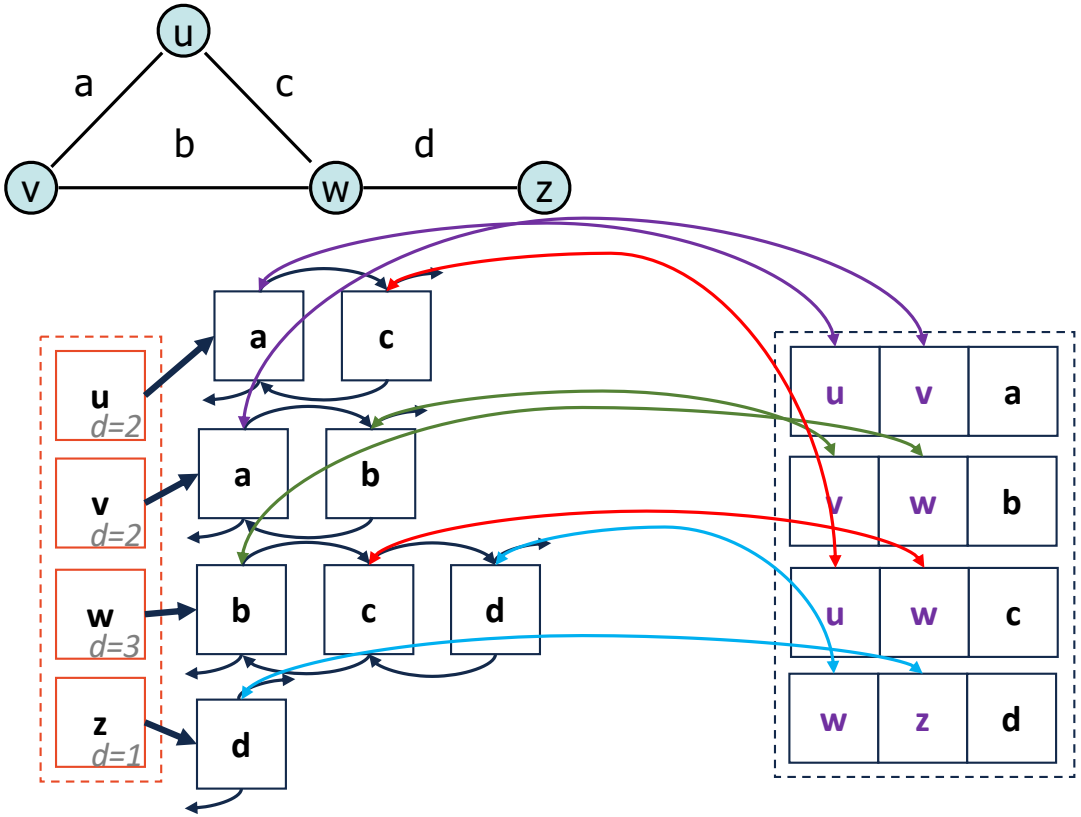
Adjacency List





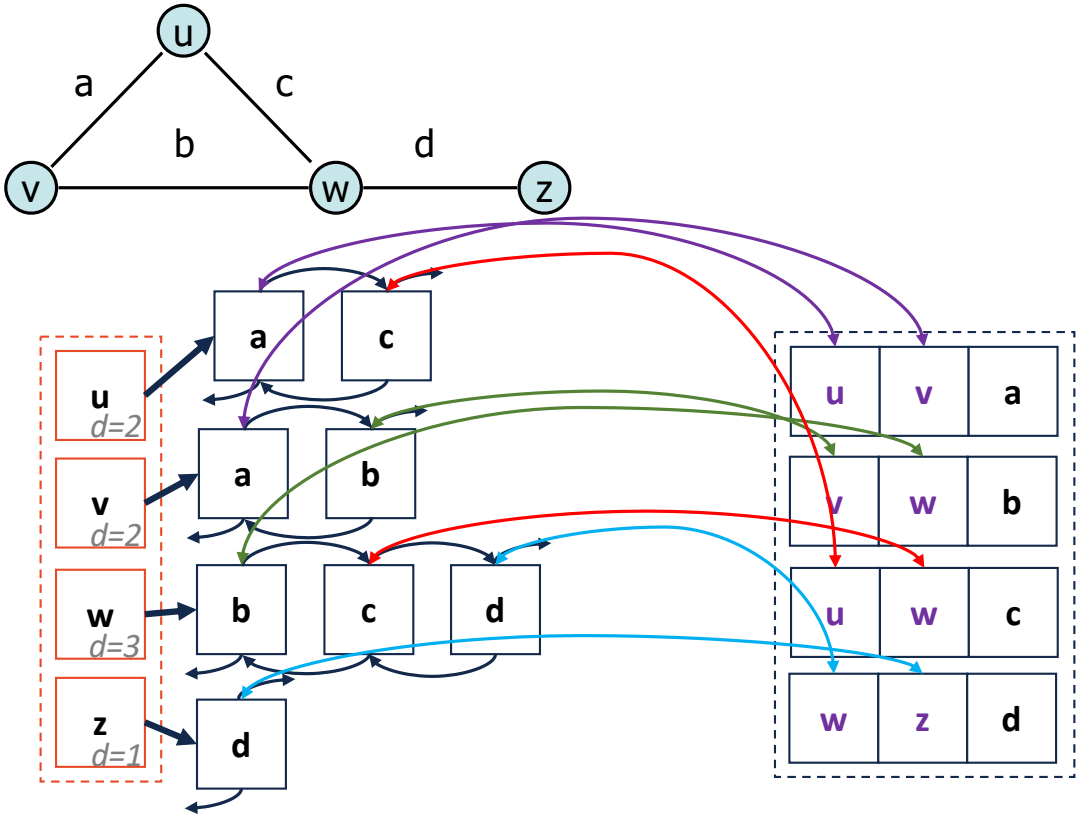


Adjacency List



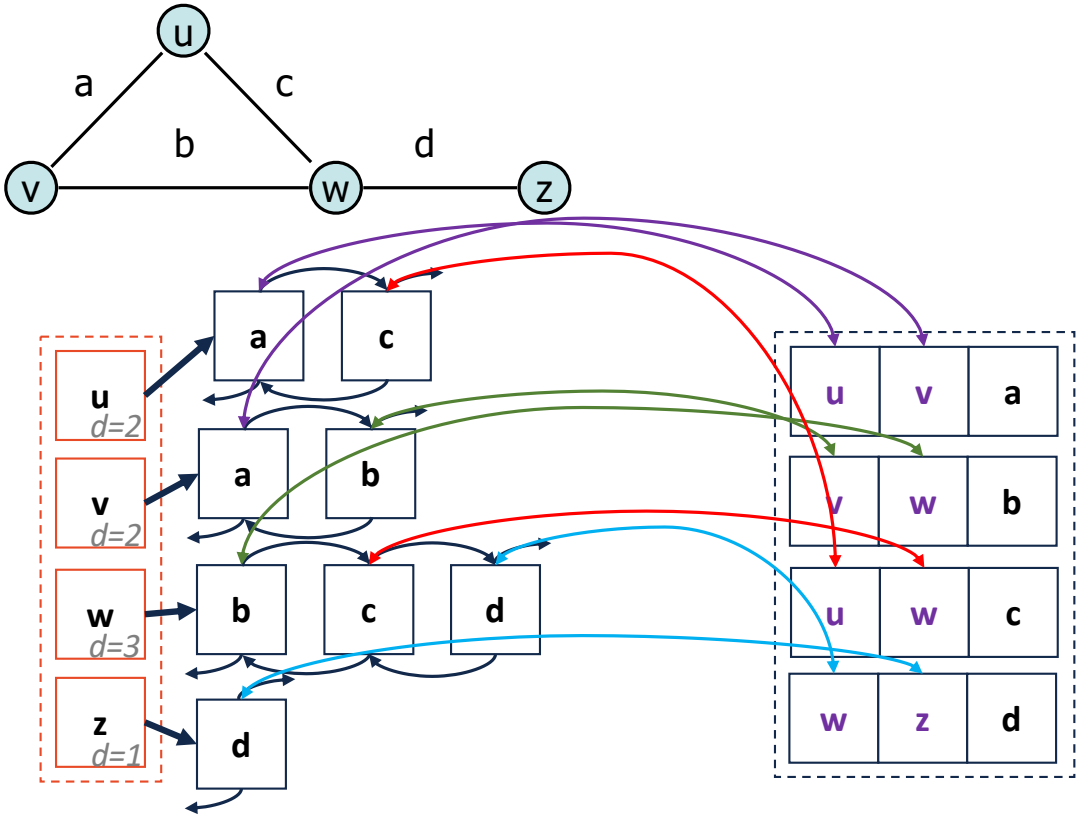
Adjacency List

insertVertex(K key):



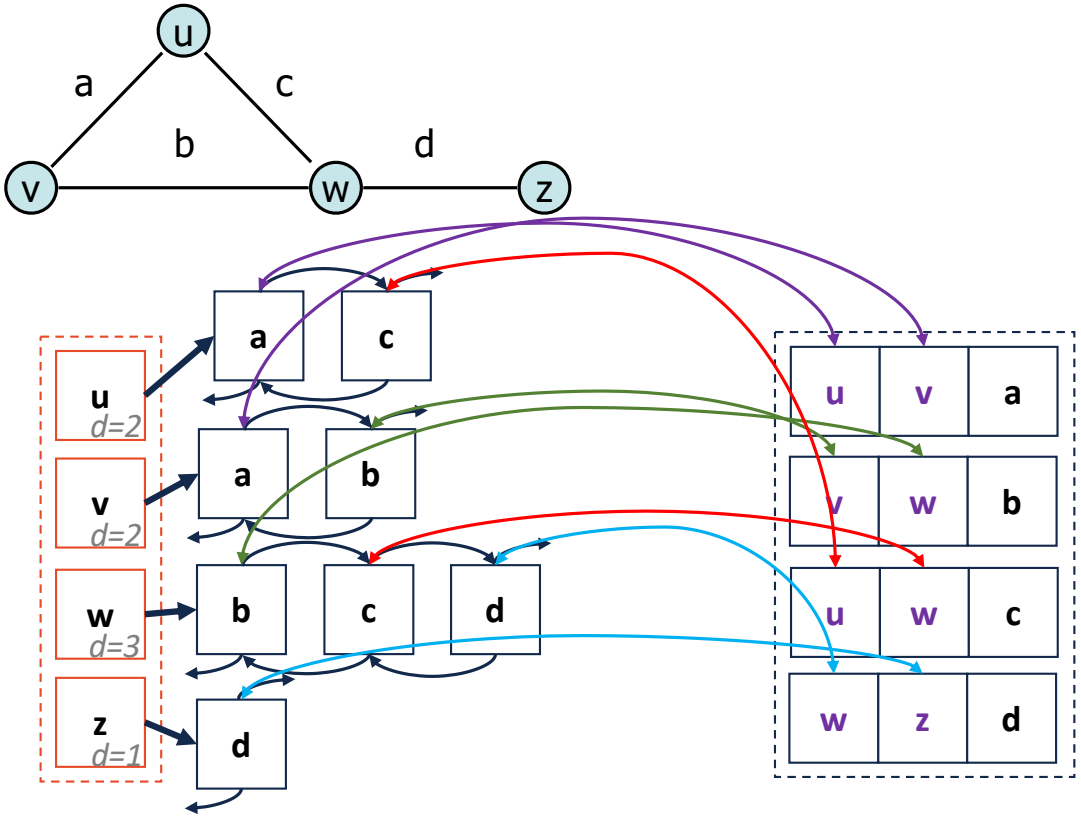
Adjacency List

removeVertex(Vertex v):



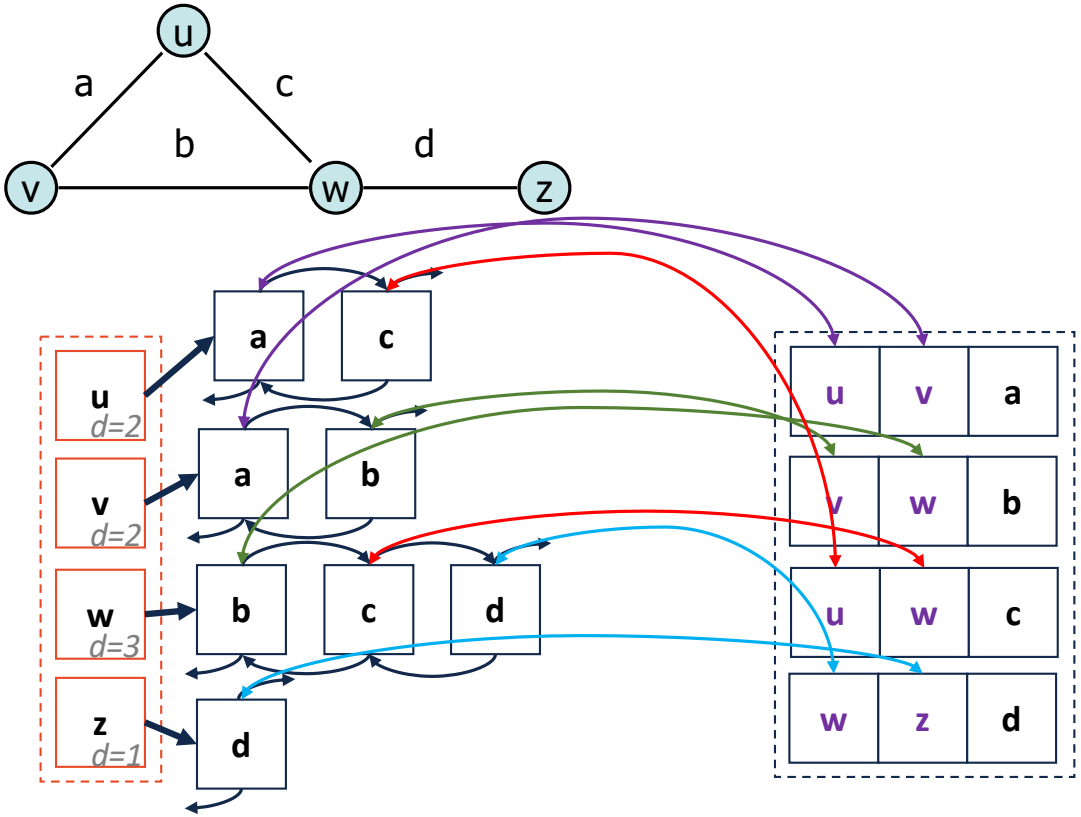
Adjacency List

incidentEdges(Vertex v):



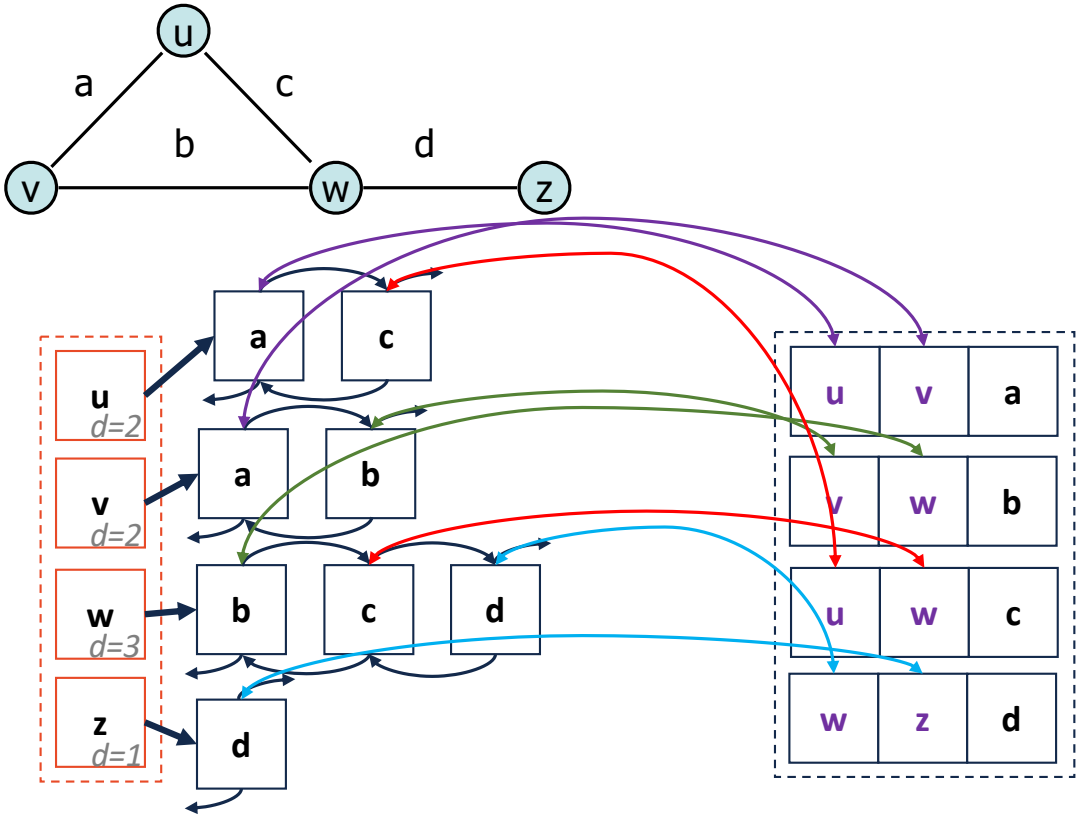
Adjacency List

areAdjacent(Vertex v1, Vertex v2):



Adjacency List

insertEdge(Vertex v1, Vertex v2, K key):



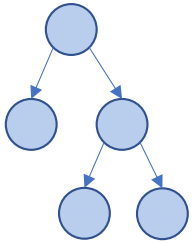
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1	n	1
removeVertex(v)	m	n	deg(v)
insertEdge(v, w, k)	1	1	1
removeEdge(v, w)	1	1	1
incidentEdges(v)	m	n	deg(v)
areAdjacent(v, w)	m	1	min(deg(v), deg(w))

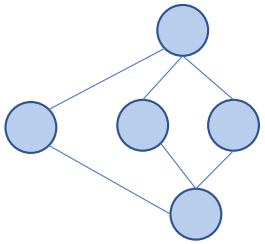
Traversal:

Objective: Visit every vertex and every edge in the graph.

Purpose: Search for interesting sub-structures in the graph.

We've seen traversal before ….but it's different:





- Ordered
- Obvious Start
-

-
-
-

Traversal: BFS