



CS 225

Data Structures

December 7 – Floyd-Warshall's Algorithm

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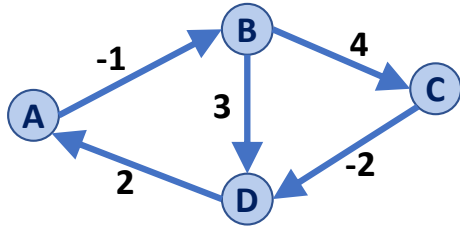
Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alternative to Dijkstra in the presence of **negative-weight edges** (not **negative weight cycles**).

```
FloydWarshall(G):
6   Let d be a adj. matrix initialized to +inf
7   foreach (Vertex v : G):
8       d[v][v] = 0
9   foreach (Edge (u, v) : G):
10      d[u][v] = cost(u, v)
11
12  foreach (Vertex w : G):
13      foreach (Vertex u : G):
14          foreach (Vertex v : G):
15              if (d[u, v] > d[u, w] + d[w, v])
16                  d[u, v] = d[u, w] + d[w, v]
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Floyd-Warshall Algorithm

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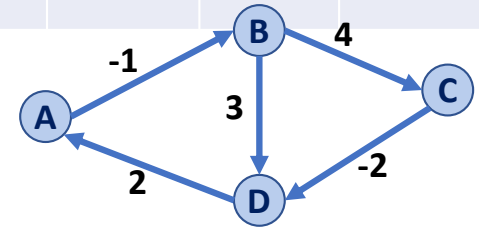


	A	B	C	D
A				
B				
C				
D				

Floyd-Warshall Algorithm

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16                   $d[u, v] = d[u, \mathbf{k}] + d[\mathbf{k}, v]$ 
```

	A	B	C	D
A	0	-1	∞	∞
B	∞	0	4	3
C	∞	∞	0	-2
D	2	∞	∞	0

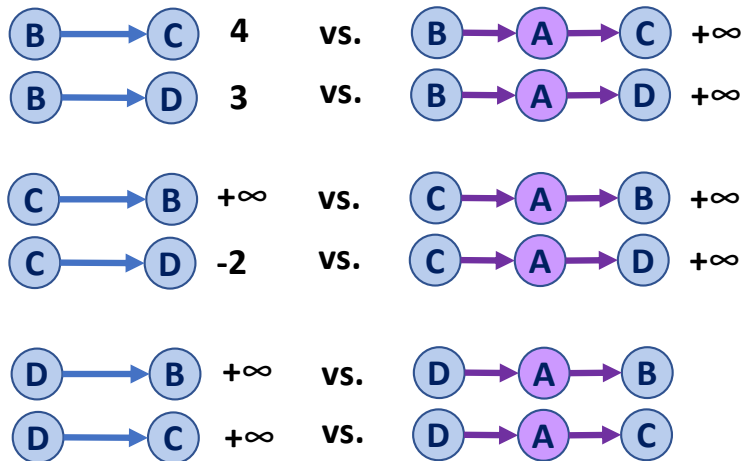


Floyd-Warshall Algorithm

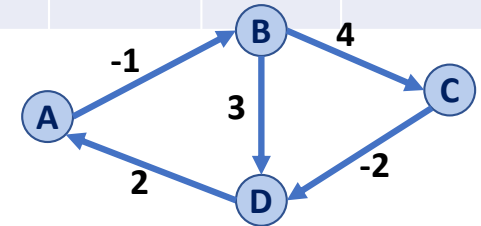
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```

Let us consider k=A:



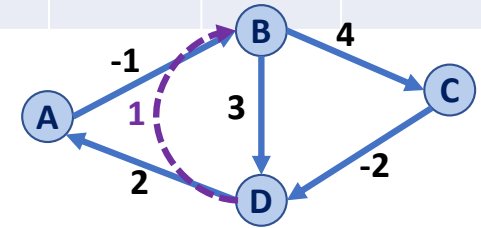
	A	B	C	D
A	0	-1	∞	∞
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Floyd-Warshall Algorithm

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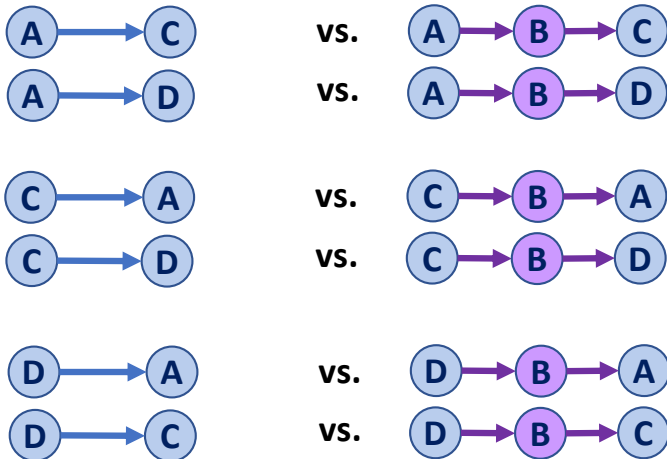
Floyd-Warshall Algorithm

```

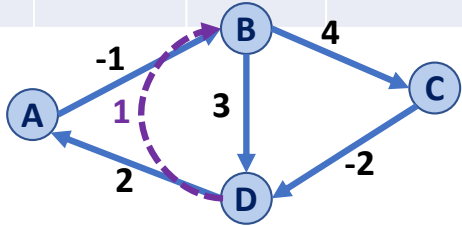
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14          foreach (Vertex v : G) :
15              if  $d[u, v] > d[u, \mathbf{k}] + d[\mathbf{k}, v]$  :
16                   $d[u, v] = d[u, \mathbf{k}] + d[\mathbf{k}, v]$ 

```

Let us consider $k=B$:

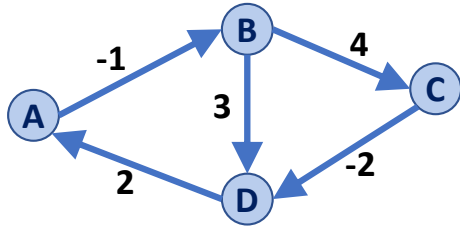


	A	B	C	D
A	0	-1	∞	∞
B	∞	0	4	3
C	∞	∞	0	-2
D	2	1	∞	0



Floyd-Warshall Algorithm

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16          d[u, v] = d[u, w] + d[w, v]
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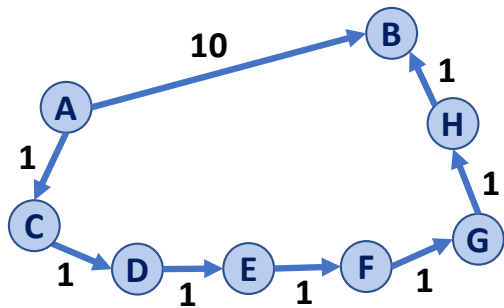


	A	B	C	D
A				
B				
C				
D				

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A				
B				
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D				

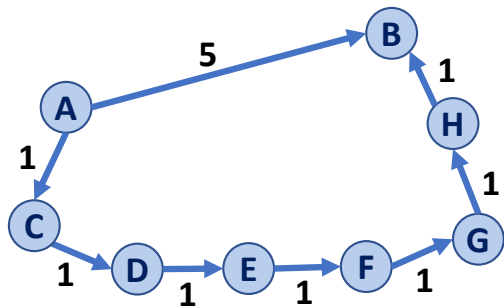
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



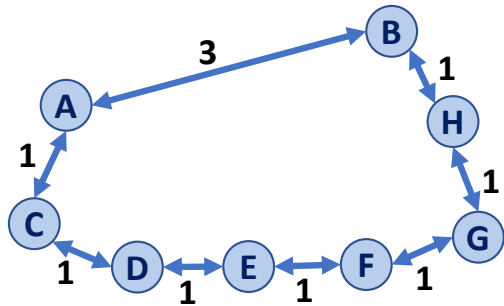
Dijkstra's Algorithm (SSSP)

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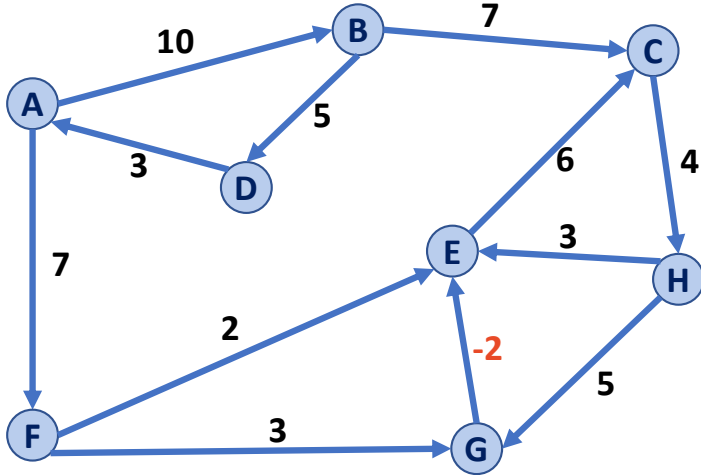
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges, without a negative weight cycle?



Floyd-Warshall Algorithm

Running Time?

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