CS 225

Data Structures

Oct. 18 – BTree Analysis

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BTree Analysis

The height of the BTree determines maximum number of _____________ possible in search data.

…and the height of the structure is: _______________.

Therefore: The number of seeks is no more than ___________.

…suppose we want to prove this!

BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given **n**) is the same as finding a lower bound on the nodes (given **h**).

We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

BTree Analysis

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.

BTree Analysis

The minimum number of **nodes** for a BTree of order m **at each level**:

root:

level 1:

level 2:

level 3:

…

level h:

BTree Analysis

The **total number of nodes** is the sum of all of the levels:

BTree Analysis

The **total number of keys**:

BTree Analysis

The **smallest total number of keys** is:

So an inequality about **n**, the total number of keys:

Solving for **h**, since **h** is the number of seek operations:

BTree Analysis

Given **m=101**, a tree of height **h=4** has:

Minimum Keys:

Maximum Keys:

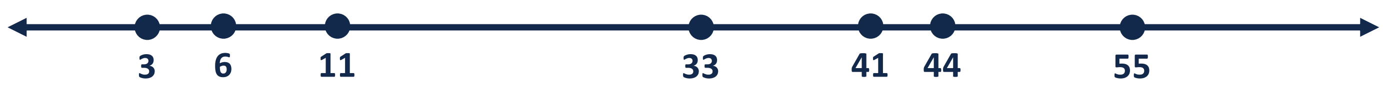
Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

…what points fall in [11, 42]?

Ex:



Range-based Searches

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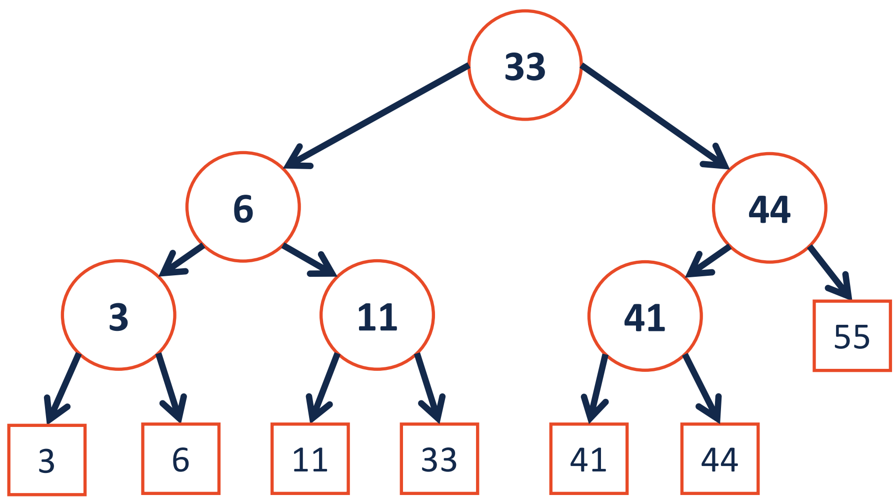
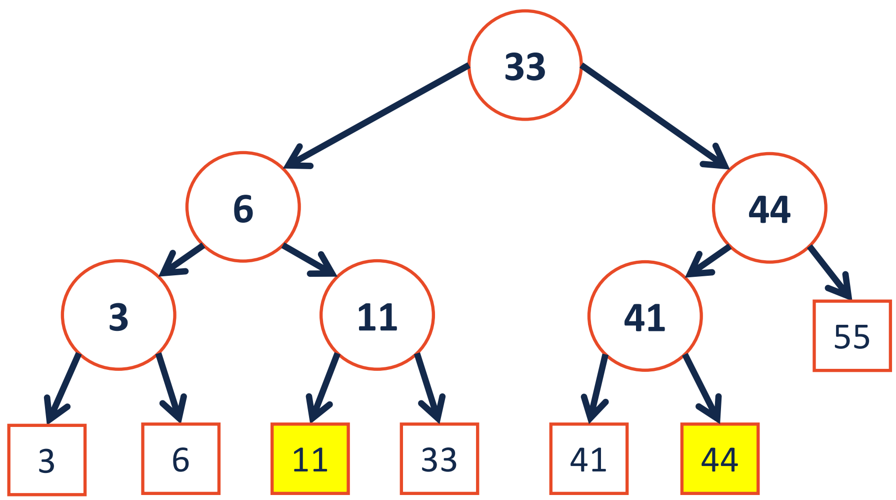
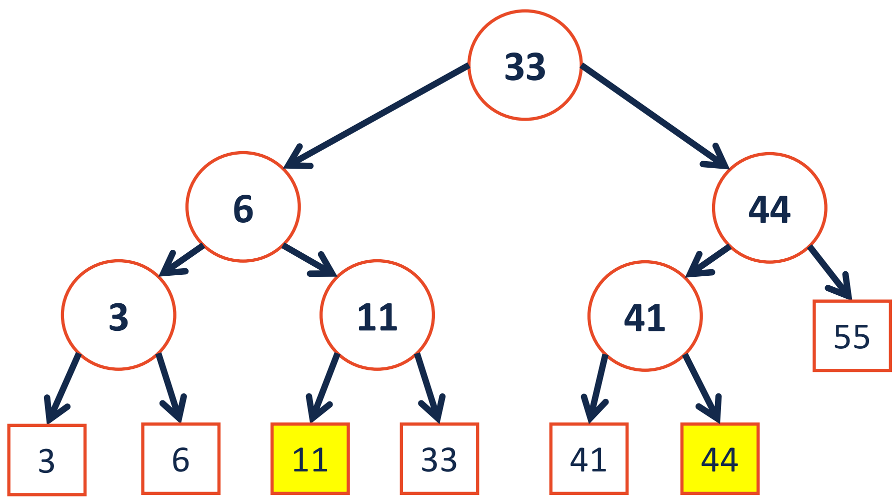


Range-based Searches



Running Time

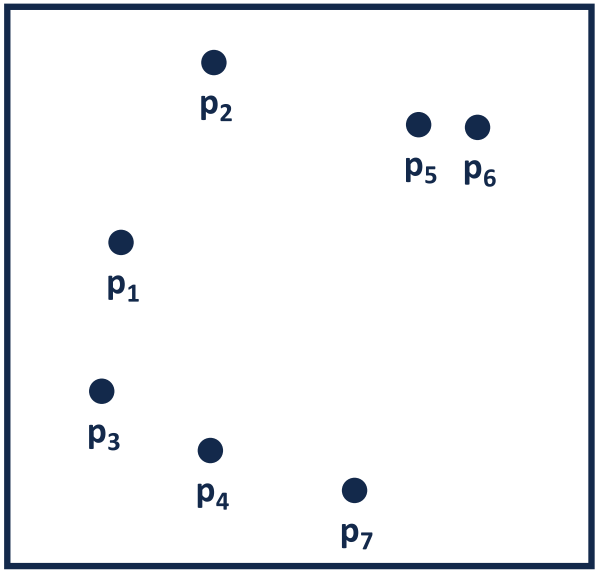


Range-based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

Q: What points are in the rectangle:
[(x_1, y_1), (x_2, y_2)]?

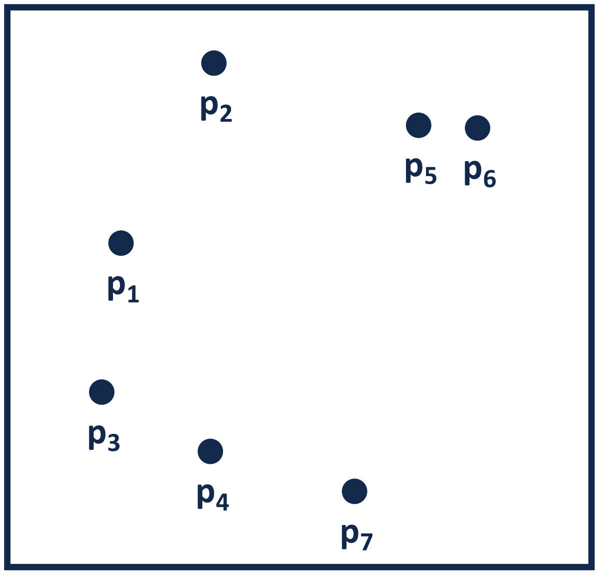
Q: What is the nearest point to $(\mathbf{x_1, y_1})$?



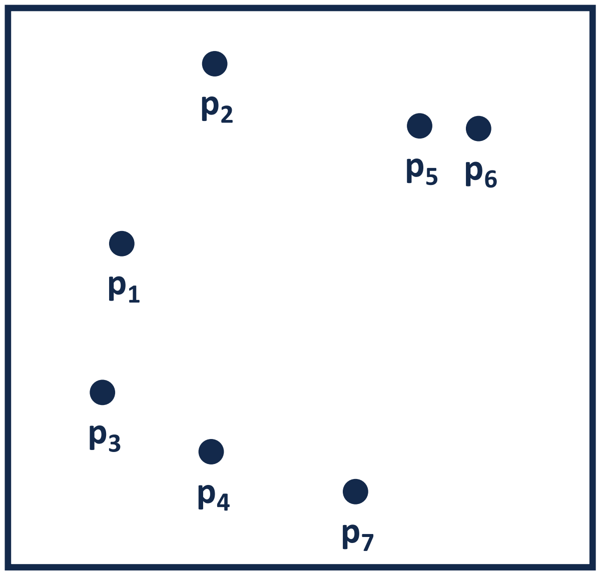
Range-based Searches

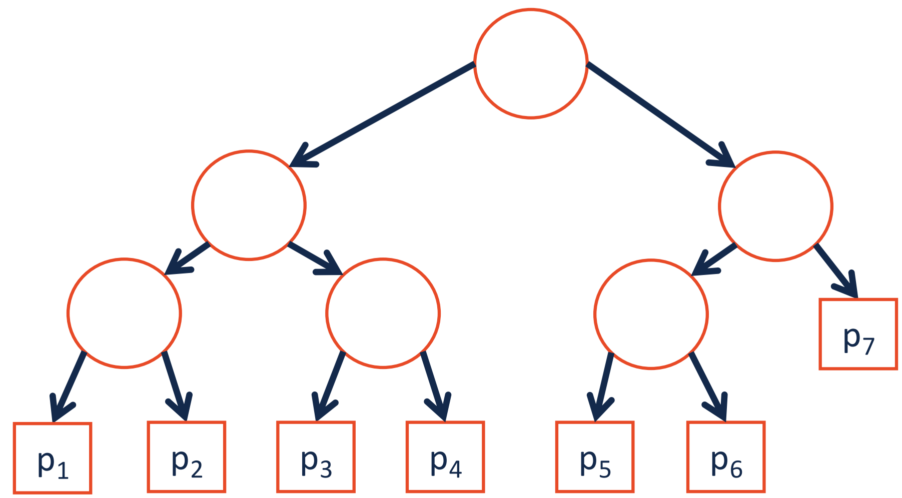
Consider points in 2D: $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$.

Space divisions:

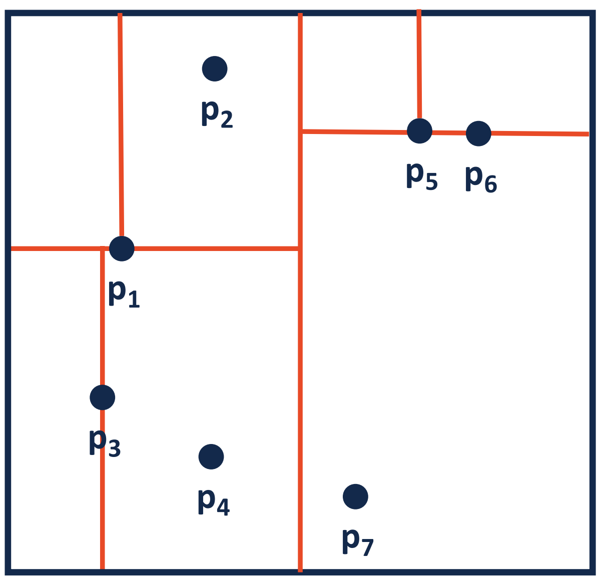


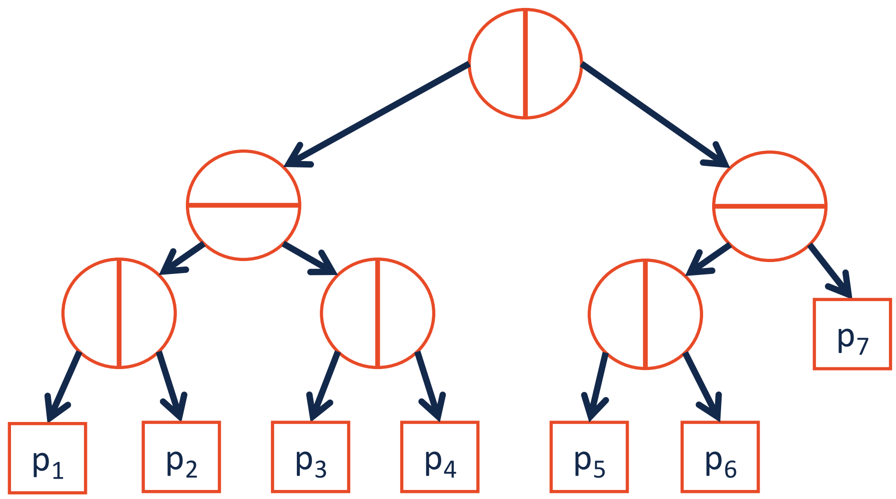
Range-based Searches



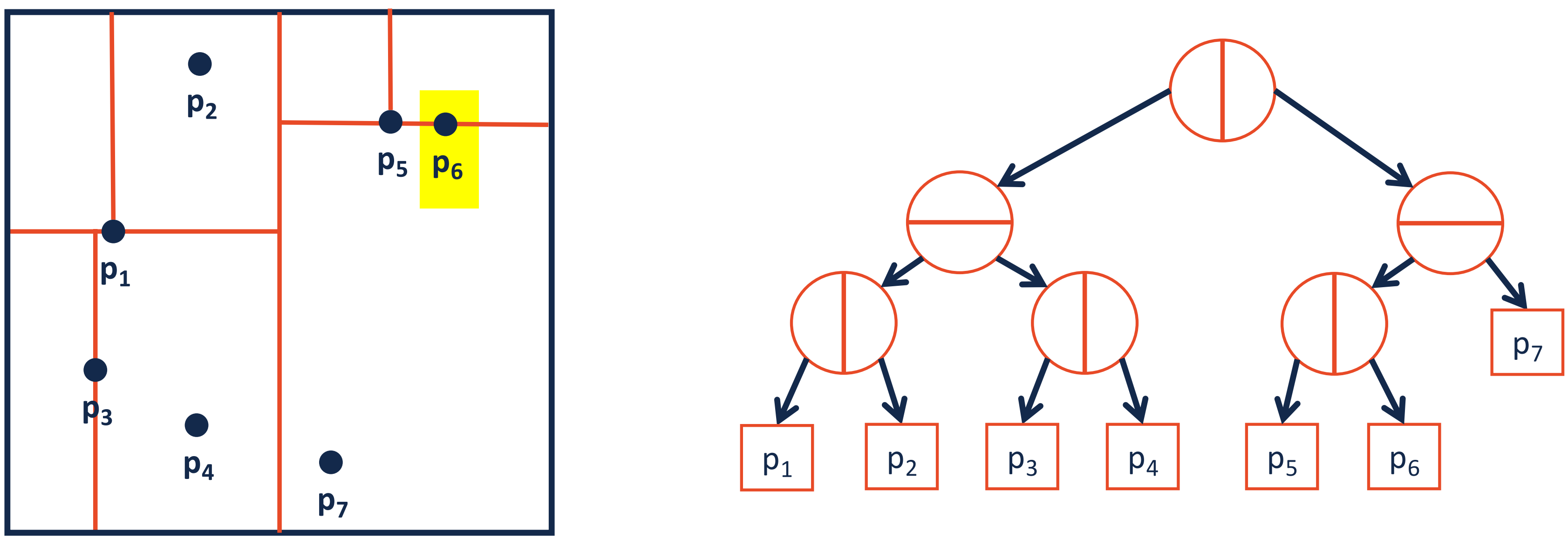


kD-Trees





kD-Trees



Hashing

Hashing

Goals:

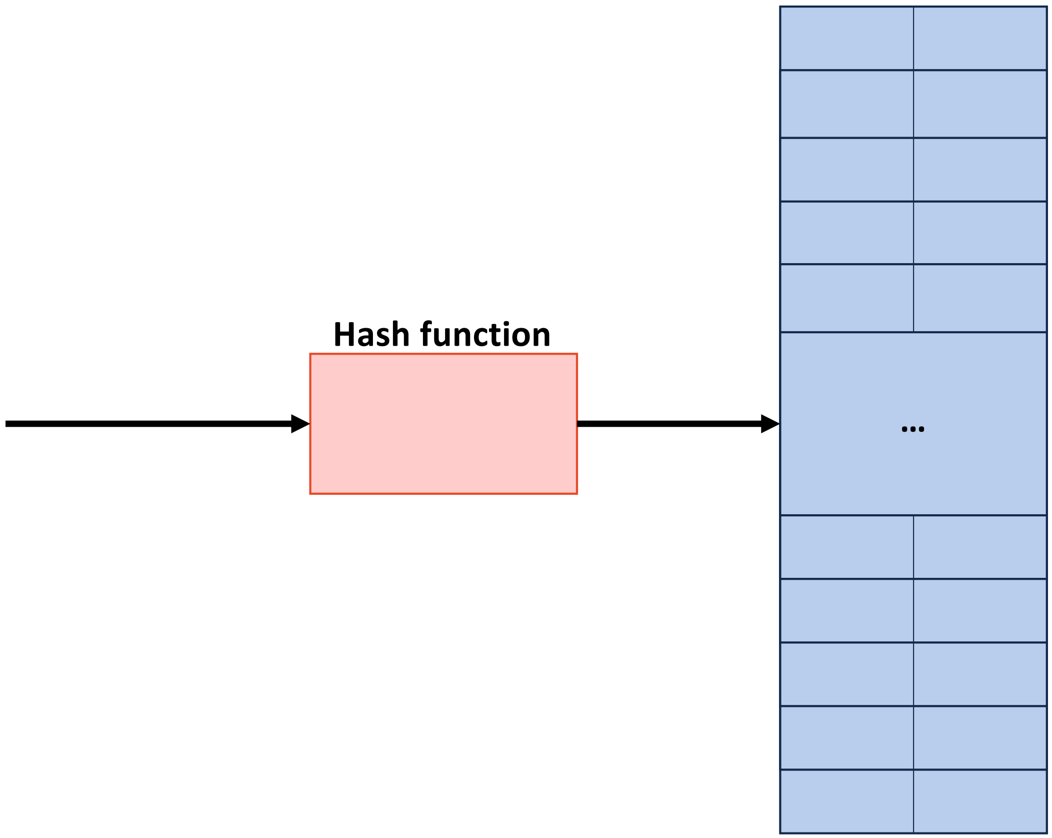
We want to define a **keyspace**, a (mathematical) description of the keys for a set of data.

…use a function to map the **keyspace** into a small set of integers.

Hashing

Locker Number	Name
103	
92	
330	
46	
124	

Hashing



A Hash Table based Dictionary

Client Code:

```
Dictionary<KeyType, ValueType> d;
d[k] = v;
```

A **Hash Table** consists of three things:

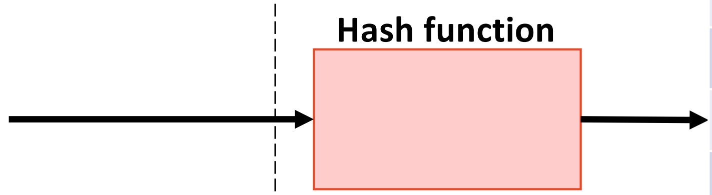
1.

2.

3.

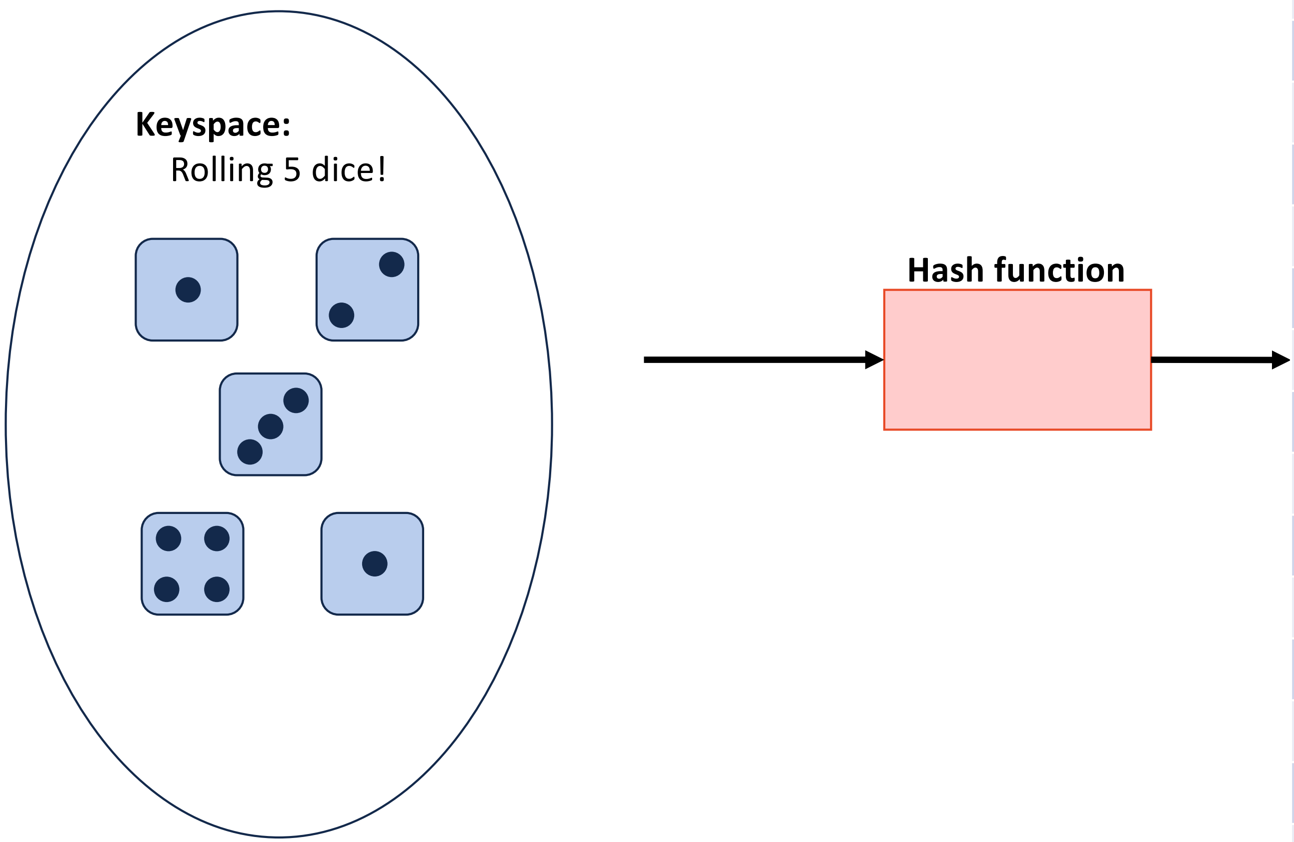
A Perfect Hash Function

(Angrave, CS 241)
(Beckman, CS 421)
(Challon, CS 125)
(Davis, CS 101)
(Evans, CS 225)
(Fagen-Ulmschneider, CS 107)
(Gunter, CS 422)
(Herman, CS 233)



Key	Value

A Perfect Hash Function



Key	Value
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	