CS 225

Data Structures

November 12 – Graph Traversals and MST

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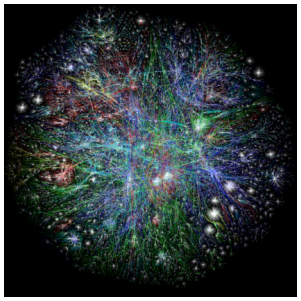
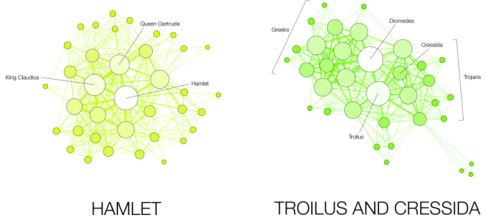
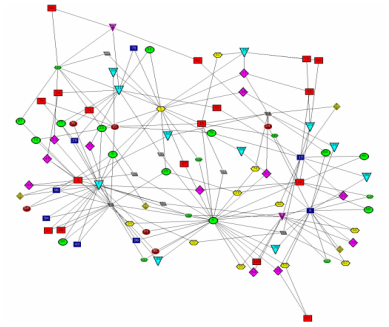
Graphs

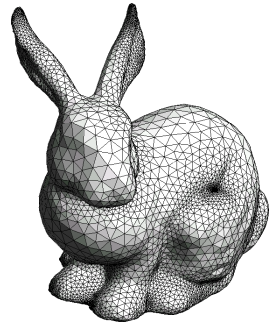


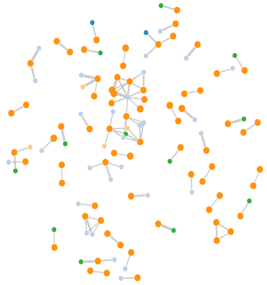


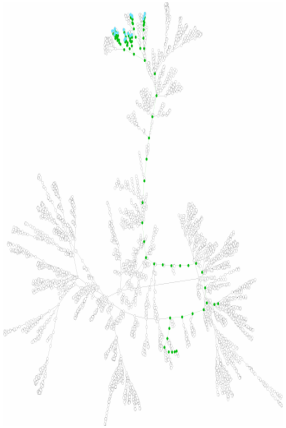


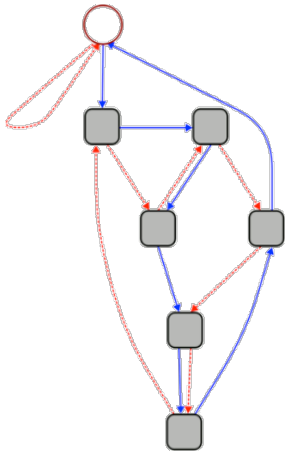
To study all of these structures:

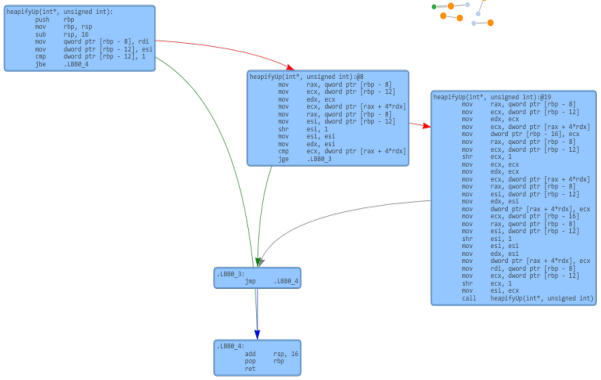
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms











BFS Observations

Obs. 1: Traversals can be used to count components.

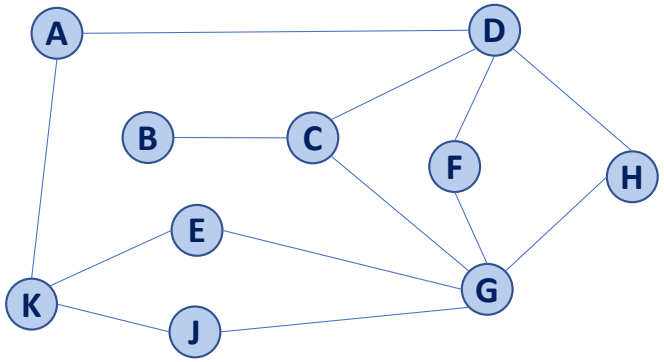
Obs. 2: Traversals can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| = 1$$

Traversal: DFS

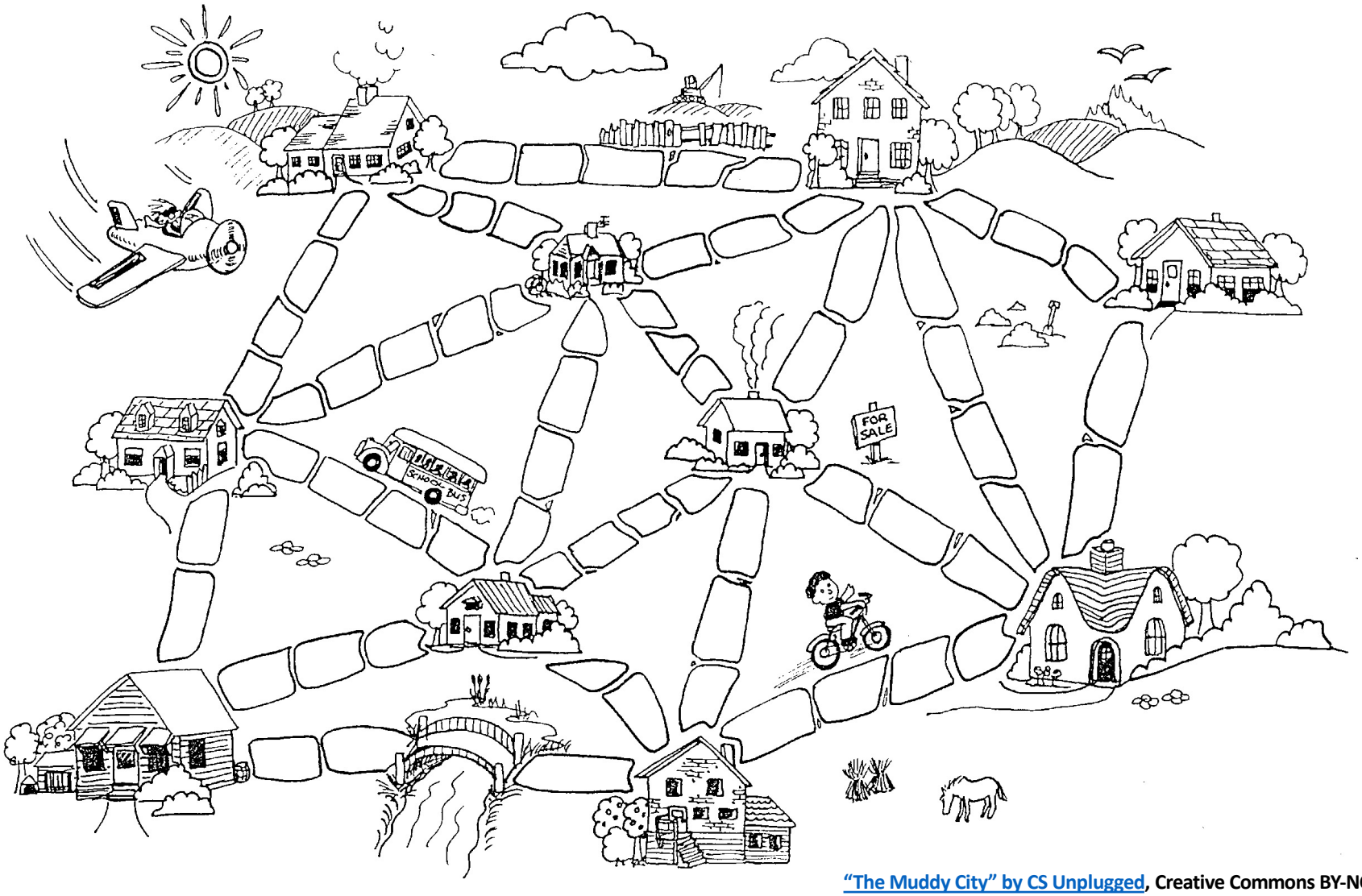


```
 1  BFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and cross edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         BFS(G, v)
```

```
14  BFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23           setLabel(v, w, DISCOVERY)
24           setLabel(w, VISITED)
25           q.enqueue(w)
26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, CROSS)
```

```
 1  DFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and back edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         DFS(G, v)
```

```
14  DFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23           setLabel(v, w, DISCOVERY)
24           setLabel(w, VISITED)
25           DFS(G, w)
26        elseif getLabel(v, w) == UNEXPLORED:
27           setLabel(v, w, BACK)
```

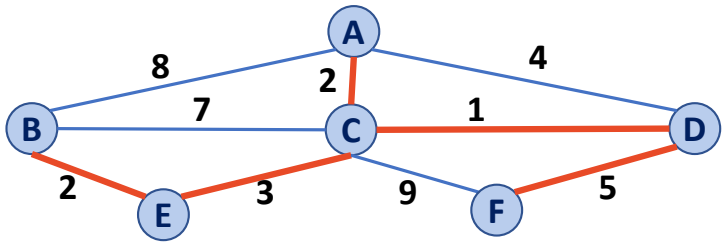


Minimum Spanning Tree Algorithms

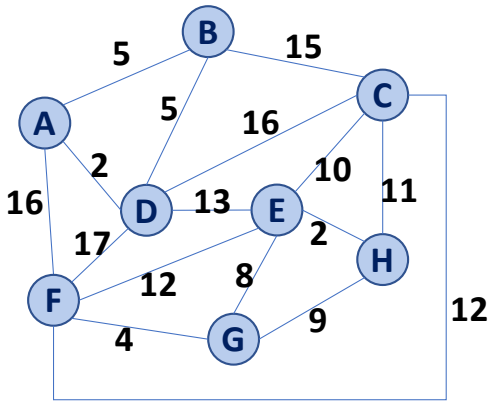
Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

Output: A graph G' with the following properties:

- G' is a spanning graph of G
- G' is a tree (connected, acyclic)
- G' has a minimal total weight among all spanning trees

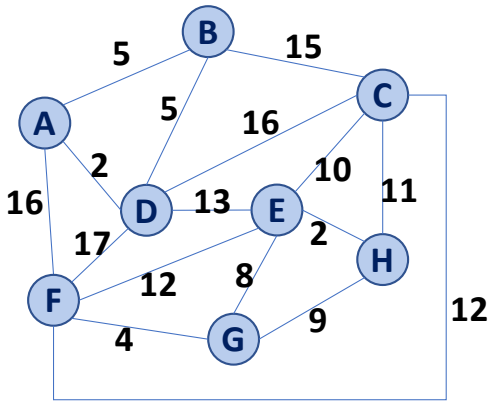


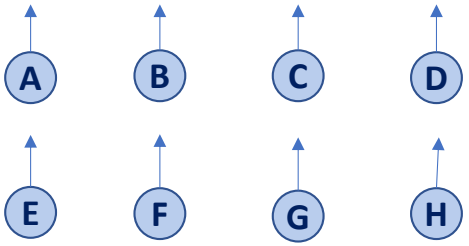
Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

Kruskal's Algorithm





(A, D)
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(G, H)
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(C, H)
(E, F)
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Kruskal's Algorithm

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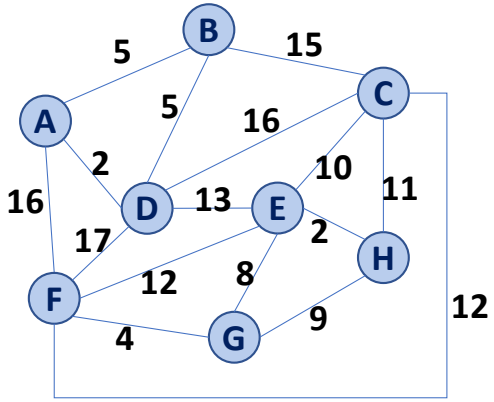

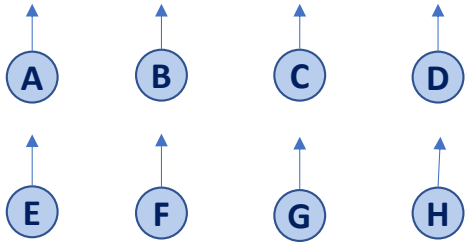


```
1   KruskalMST(G):
2     DisjointSets forest
3     foreach (Vertex v : G):
4       forest.makeSet(v)
5
6     PriorityQueue Q     // min edge weight
7     foreach (Edge e : G):
8       Q.insert(e)
9
10    Graph T = (V, {})
11
12    while |T.edges()| < n-1:
13      Vertex (u, v) = Q.removeMin()
14      if forest.find(u) != forest.find(v):
15         T.addEdge(u, v)
16         forest.union( forest.find(u),
17                       forest.find(v) )
18
19    return T
```

Kruskal's Algorithm

Priority Queue:	Heap	Sorted Array
Building :7-9		
Each removeMin :13		

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Kruskal's Algorithm

Priority Queue:	Total Running Time
Heap	
Sorted Array	

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