# String Algorithms and Data Structures

# Approximate Pattern Matching

CS 199-225

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# Learning Objectives

Review approximate pattern matching

Formalize edit distance storage as an ‘edit string’

Discuss strategies for efficient APM with edits

Introduce dynamic programming

# Approximate Pattern Matching

**Input:** A text *T*, a pattern *P*, and a distance *d*

**Output:** All positions in *T* where *P* has at most *d* mismatches or edits

*P:* word

*T:* There would have been a time for such a word

Alignment 1:
```
would
word
```

~~Not a match!~~

Distance 2 match!

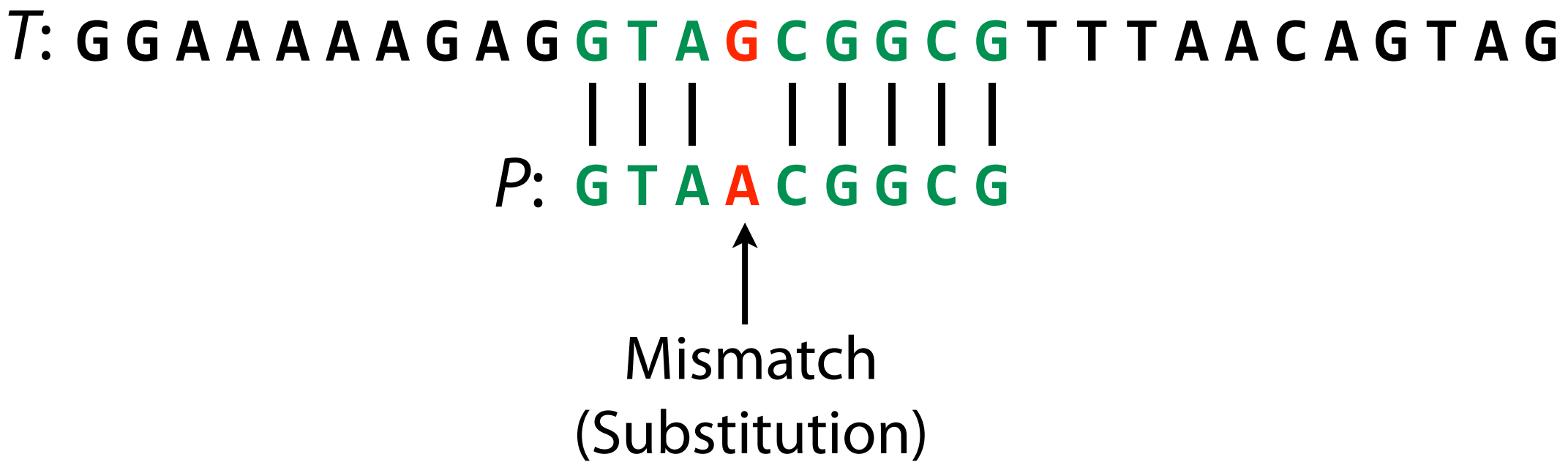
Alignment 2:
```
word
word
```

Match!

Distance 0 match!

# Hamming Distance

The number of **substitutions** required to turn one string into another



# Approximate Pattern Matching

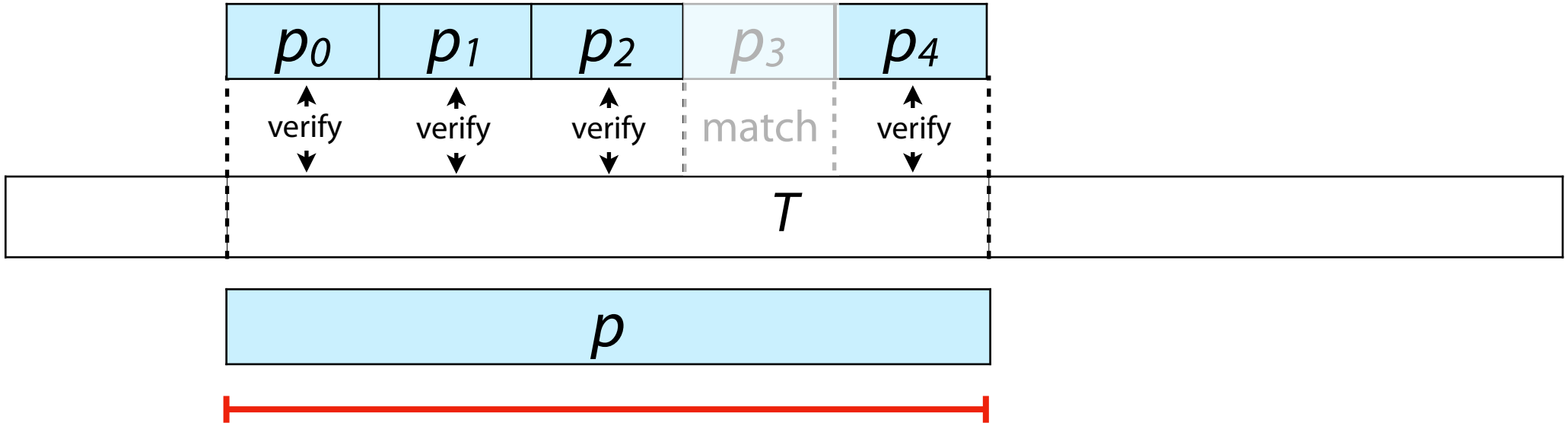
As a ***heuristic***, seed and extend reduces the overall search space

*T:* **There would have been a time for such a word**

**word**

**word**

**word**




Only consider mismatches while verifying a seed hit

# Edit Distance

```
Score =  248 bits (129), Expect = 1e-63
Identities = 213/263 (80%), Gaps = 34/263 (12%)
Strand = Plus / Plus
```

Substitution

```
Query: 161 atatcaccacgtcaaaggtgactccaactcca---ccactccattttgttcagataatgc 217
           ||||||||||||||||||||||||||||  |     | |   || ||||||||||||||
Sbjct: 481 atatcaccacgtcaaaggtgactccaact-tattgatagtgtttttatgttcagataatgc 539

Query: 218 ccgatgatcatgtcatgcagctccaccgattgtgagaacgacagcgacttccgtcccagc 277
           |||||||   ||||||||||||||||||| || |           ||||||||||||
Sbjct: 540 ccgatgactttgtcatgcagctccaccgattttg-g----------ttccgtcccagc 586
```

Deletion

```
Query: 278 c-gtgcc--aggtgctgcctcagattcaggttatgccgctcaattcgctgcgtatatcgc 334
           |  ||  |  | ||||||||||||||||||||||||||||||||||||| |||||||||
Sbjct: 587 caatgacgta-gtgctgcctcagattcaggttatgccgctcaattcgctgggtatatcgc 645

Query: 335 ttgctgattacgtgcagctttcccttcaggcggga----------ccagccatccgtc 382
           ||||||||||||||||||||||||||||||||           ||||||||||||
Sbjct: 646 ttgctgattacgtgcagctttcccttcaggcgggattcatacagcggccagccatccgtc 705
```

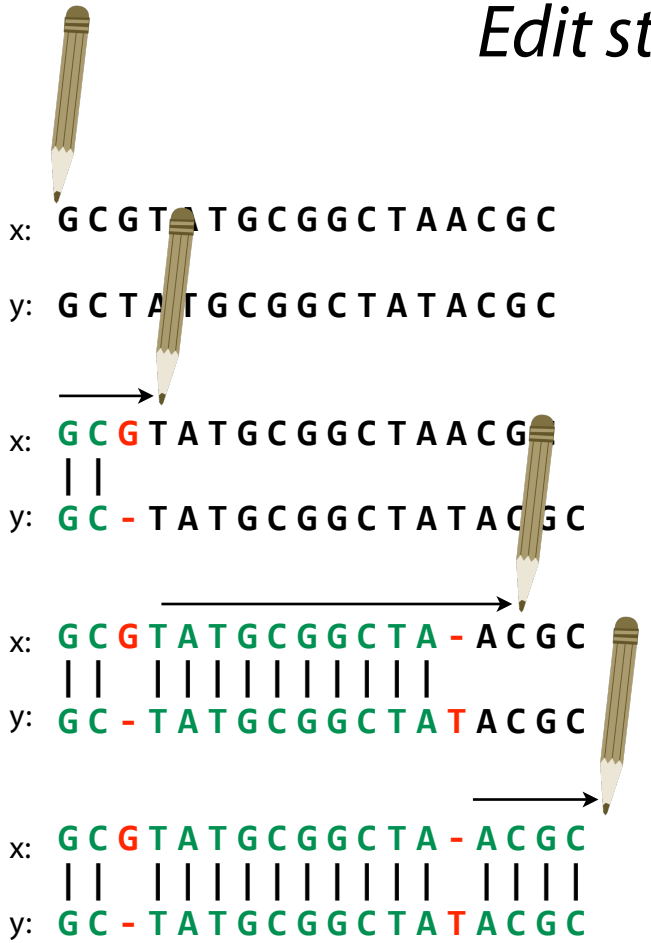
Insertion

```
Query: 383 ctccatatc-accacgtcaaagg 404
           ||||||||  ||||||||||||
Sbjct: 706 atccatatcaaccacgtcaaagg 728
```

# Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

*Edit string* summarizes how editor turns *x* into *y:*




Operations:

**M** = match, **R** = replace (substitute),

**I** = insert into *x*, **D** = delete from *x*

**MMD**

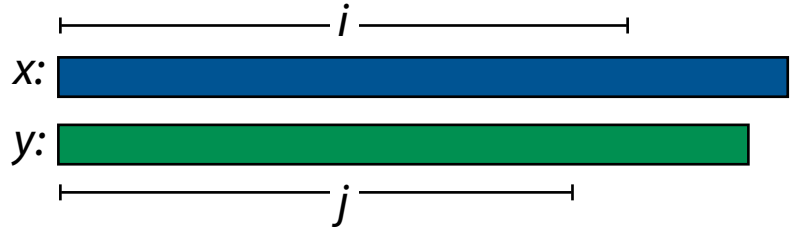
Reminder: this is **D**, not **I**, because we have to delete a character from *x* to make it more like *y*

**MMDMMMMMMMMMI**

**MMDMMMMMMMMMIMMMM**

# Edit Distance

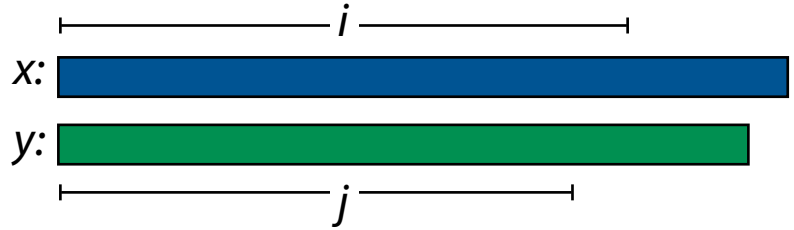
D[*i*, *j*]: edit distance between length-*i* prefix of *x* and length-*j* prefix of *y*




Optimal edit string for D[*i*, *j*] is built by ***extending a shorter optimal string by 1 operation***. 3 options:

# Edit Distance

D[*i*, *j*]: edit distance between length-*i* prefix of *x* and length-*j* prefix of *y*




Optimal edit string for D[*i*, *j*] is built by ***extending a shorter optimal string by 1 operation***. 3 options:

Append **D** to transcript for D[*i*-1, *j*]

Append **I** to transcript for D[*i*, *j*-1]

Append **M** or **R** to transcript for D[*i*-1, *j*-1]

We choose based on whichever option has the fewest edits

# Edit Distance

X: GTTTAA Y: GGTTTA

D[*5*, *6*]

```
GTTTA
GGTTTA
```

D[*5*, *5*]

```
GTTTA
GGTTT
```

D[*6*, *5*]

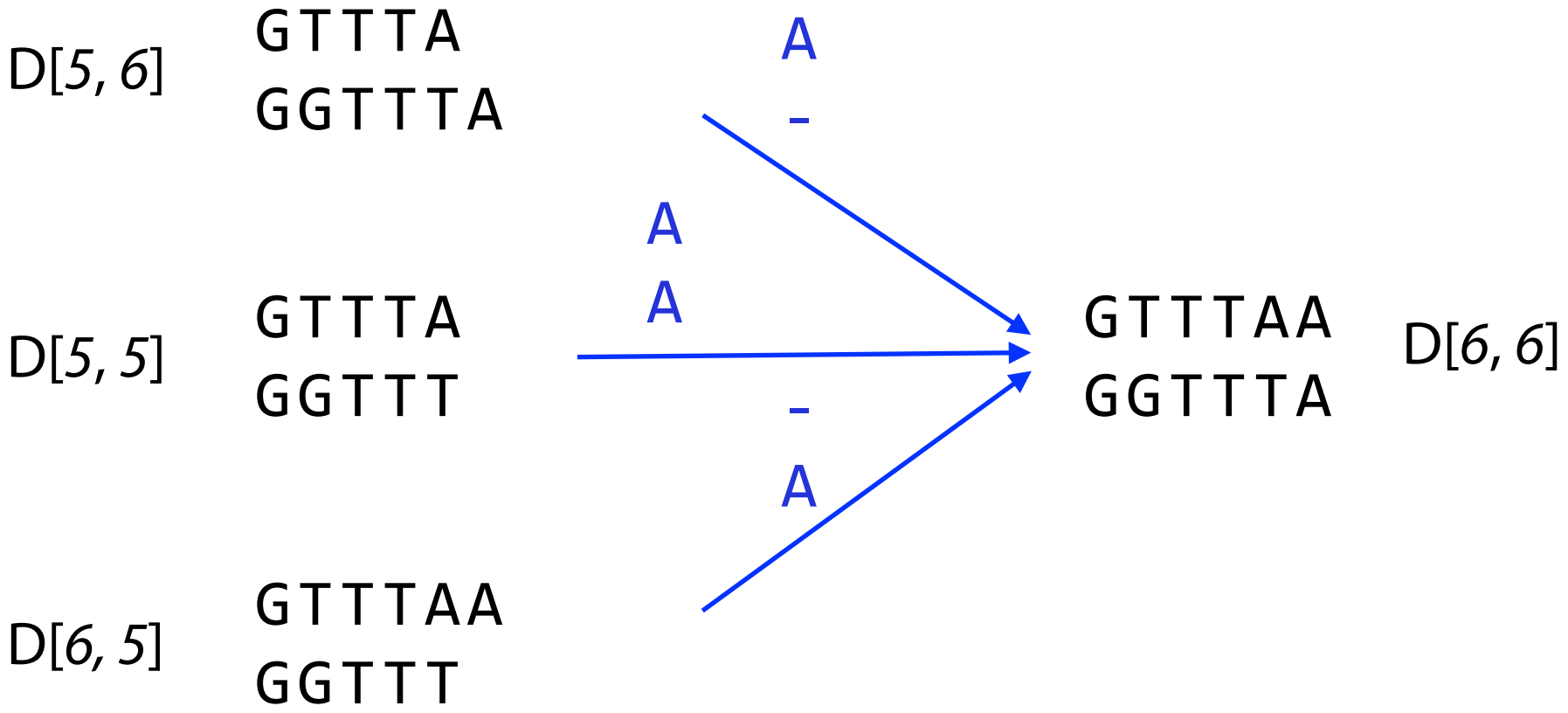
```
GTTTAA
GGTTT
```

```
GTTTAA
GGTTTA
```

D[*6*, *6*]

# Edit Distance

X: GTTTAA Y: GGTTTA



# Edit Distance

X: GTTTAA Y: GGTTTA

D[*5*, *6*]

```
GTTTA
GGTTTA
```

D[*5*, *5*]

```
GTTTA
GGTTT
```

D[*6*, *5*]

```
GTTTAA
GGTTT
```

# Edit Distance

X: GTTTAA  Y: GGTTTA

MIMMMM

D[*5*, *6*]

| | | | | | |
|---|---|---|---|---|---|
| G | - | T | T | T | A |
| G | G | T | T | T | A |

D[*5*, *5*]

GTTTA
GGTTT

D[*6*, *5*]

GTTTAA
GGTTT

# Edit Distance

X: GTTTAA Y: GGTTTA

**MIMMMM**

D[*5*, *6*]

| G | - | T | T | T | A |
|---|---|---|---|---|---|
| G | G | T | T | T | A |

**MRMMR**

D[*5*, *5*]

| G | T | T | T | A |
|---|---|---|---|---|
| G | G | T | T | T |

D[*6*, *5*]

GTTTAA
GGTTT

# Edit Distance

X: GTTTAA Y: GGTTTA

**MIMMMM**

D[*5*, *6*]

| G | - | T | T | T | A |
|---|---|---|---|---|---|
| G | G | T | T | T | A |

**MRMMR**

D[*5*, *5*]

| G | T | T | T | A |
|---|---|---|---|---|
| G | G | T | T | T |

**MRMMRD**

D[*6*, *5*]

| G | T | T | T | A | A |
|---|---|---|---|---|---|
| G | G | T | T | T | - |

# Edit Distance

X: GTTTAA Y: GGTTTA

**MIMMMM**

D[*5*, *6*]

| G | - | T | T | T | A |
|---|---|---|---|---|---|
| G | G | T | T | T | A |

| G | - | T | T | T | A | |
|---|---|---|---|---|---|---|
| G | G | T | T | T | A | |

D[*6*, *6*][1]

**MRMMR**

D[*5*, *5*]

| G | T | T | T | A |
|---|---|---|---|---|
| G | G | T | T | T |

| G | T | T | T | A | |
|---|---|---|---|---|---|
| G | G | T | T | T | |

D[*6*, *6*][2]

**MRMMRD**

D[*6*, *5*]

| G | T | T | T | A | A |
|---|---|---|---|---|---|
| G | G | T | T | T | - |

| G | T | T | T | A | A | |
|---|---|---|---|---|---|---|
| G | G | T | T | T | - | |

D[*6*, *6*][3]

# Edit Distance

X: GTTTAA Y: GGTTTA

D[*5*, *6*]

MIMMMM

| G | - | T | T | T | A |
|---|---|---|---|---|---|
| G | G | T | T | T | A |

MIMMMMD

| G | - | T | T | T | A | A |
|---|---|---|---|---|---|---|
| G | G | T | T | T | A | - |

D[*6*, *6*][1]

D[*5*, *5*]

MRMMR

| G | T | T | T | A |
|---|---|---|---|---|
| G | G | T | T | T |

MRMMRM

| G | T | T | T | A | A |
|---|---|---|---|---|---|
| G | G | T | T | T | A |

D[*6*, *6*][2]

D[*6*, *5*]

MRMMRD

| G | T | T | T | A | A |
|---|---|---|---|---|---|
| G | G | T | T | T | - |

MRMMRDI

| G | T | T | T | A | A | - |
|---|---|---|---|---|---|---|
| G | G | T | T | T | - | A |

D[*6*, *6*][3]

# Edit Distance

X: ACCT Y: AGCC

D[*3*, *4*]

| | | | | |
|---|---|---|---|---|
| A | - | C | C | |
| A | G | C | C | |

MIMM

D[*3*, *3*]

| | | | |
|---|---|---|---|
| A | C | C | |
| A | G | C | |

MRM

D[*4*, *3*]

| | | | | |
|---|---|---|---|---|
| A | C | C | T | |
| A | G | C | - | |

MRMD

D[4,4] extends one of these options

D[*4*, *4*]

# Edit Distance

X: ACCT Y: AGCC

D[*3*, *4*]

| | | | | |
|---|---|---|---|---|
| A | - | C | C | T |
| A | G | C | C | - |

MIMMD

D[*3*, *3*]

| | | | |
|---|---|---|---|
| A | C | C | T |
| A | G | C | C |

MRMR

D[*4*, *3*]

| | | | | |
|---|---|---|---|---|
| A | C | C | T | - |
| A | G | C | - | C |

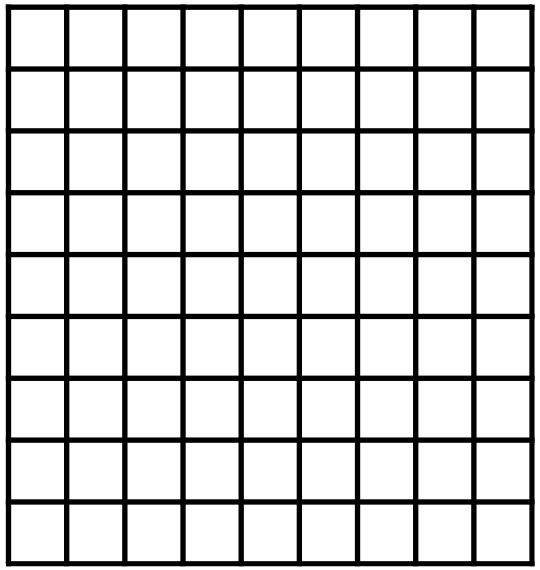
MRMDI

D[4,4] extends one of these options

D[*4*, *4*]

# Edit Distance

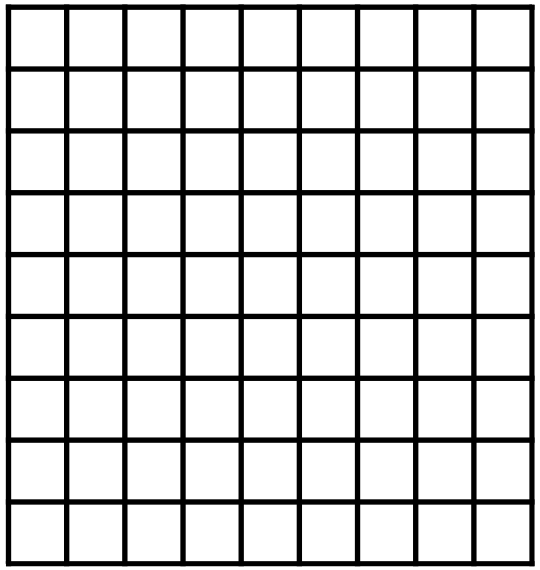
We can store $D$ as a 2D matrix:



Let $D[0, j] = j$, and let $D[i, 0] = i$

# Edit Distance

We can store $D$ as a 2D matrix:



Is at beginning

Ds at beginning

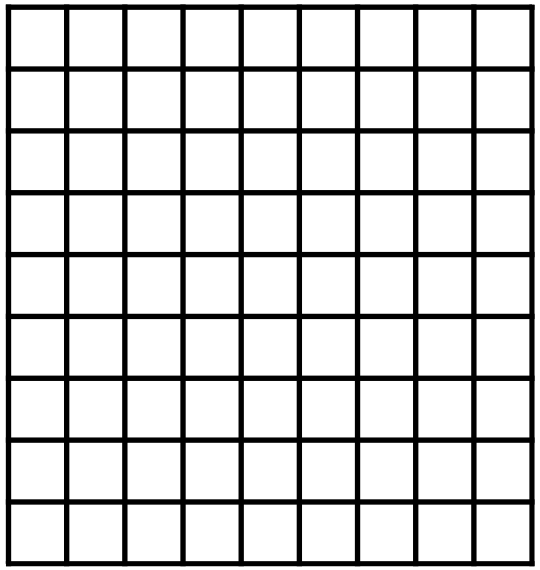
Let $D[0, j] = j$, and let $D[i, 0] = i$

```
----BBBBBBB
    |||||||
AAAABBBBBBB
```

```
the longest----
    |||||||
----longest day
```

# Edit Distance

We can store $D$ as a 2D matrix:



Is at beginning

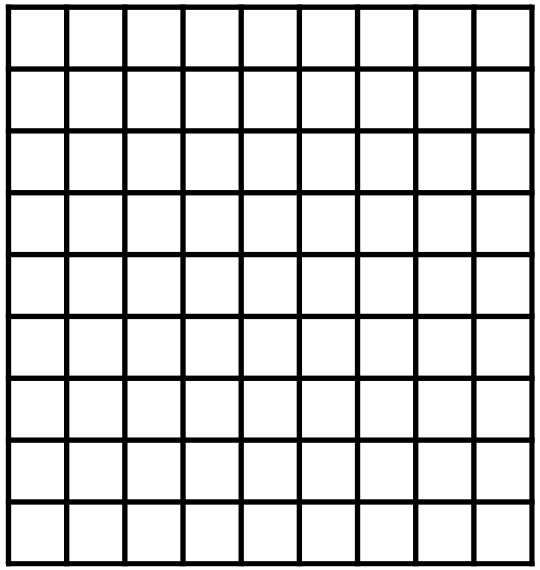
Ds at beginning

Let $D[0, j] = j$, and let $D[i, 0] = i$

Otherwise, let $D[i, j] = \min \begin{cases} D[i-1, j] + 1 \\ D[i, j-1] + 1 \\ D[i-1, j-1] + \delta(x[i-1], y[j-1]) \end{cases}$

$\delta(a, b)$ is 0 if $a = b$, 1 otherwise

# Edit Distance



We can store $D$ as a 2D matrix:

Is at beginning

Ds at beginning

Let $D[0, j] = j$, and let $D[i, 0] = i$

Otherwise, let $D[i, j] = \min \begin{cases} D[i-1, j] + 1 \\ D[i, j-1] + 1 \\ D[i-1, j-1] + \delta(x[i-1], y[j-1]) \end{cases}$

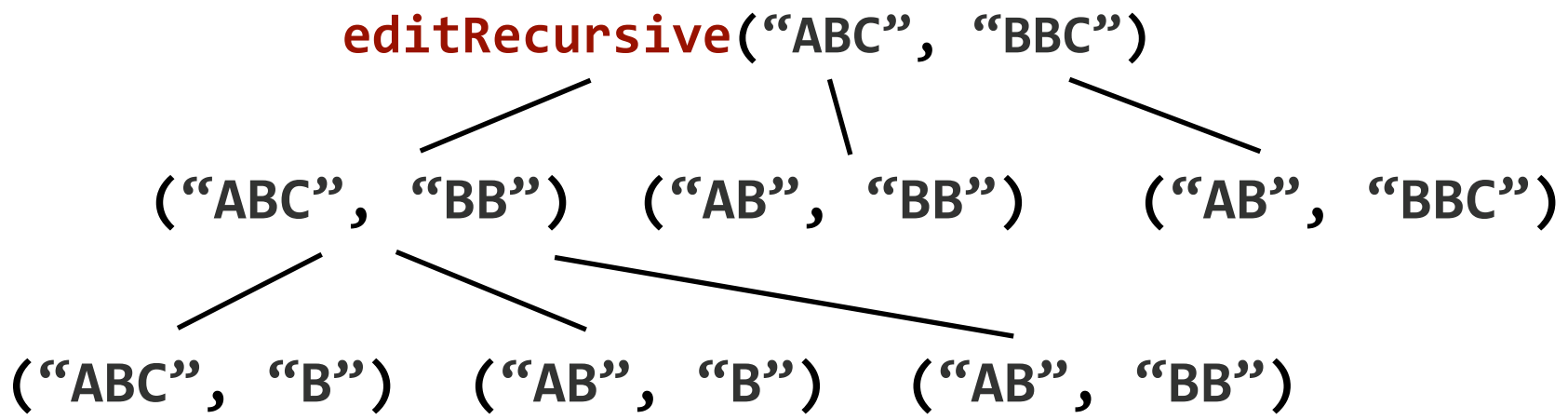
vertical (D)

horizontal (I)

diagonal (M or R)

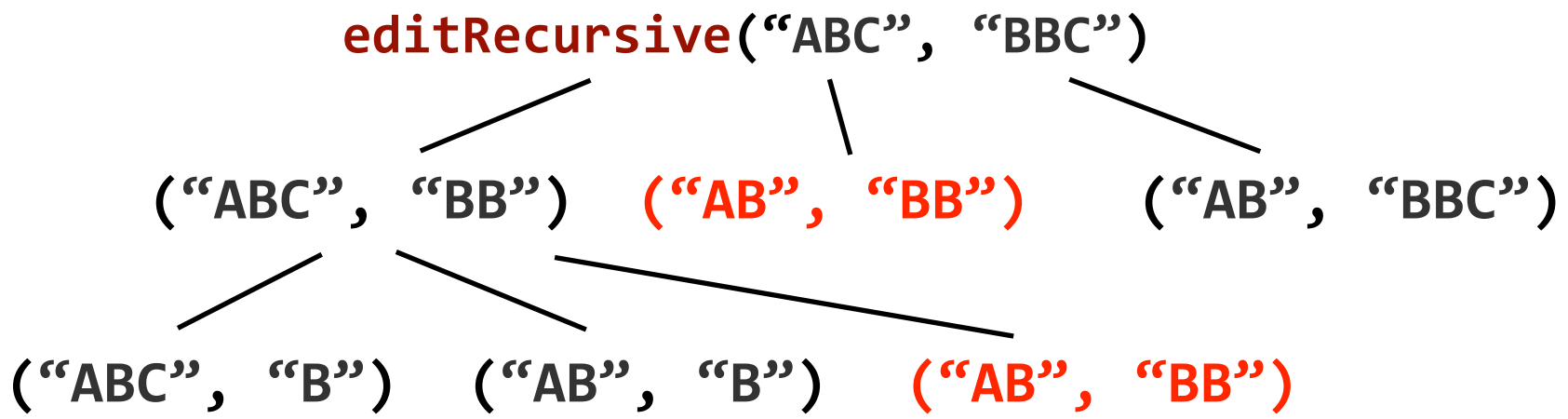
$\delta(a, b)$ is 0 if $a = b$, 1 otherwise

# Edit Distance




*(only part of recursion tree shown)*

# Edit Distance



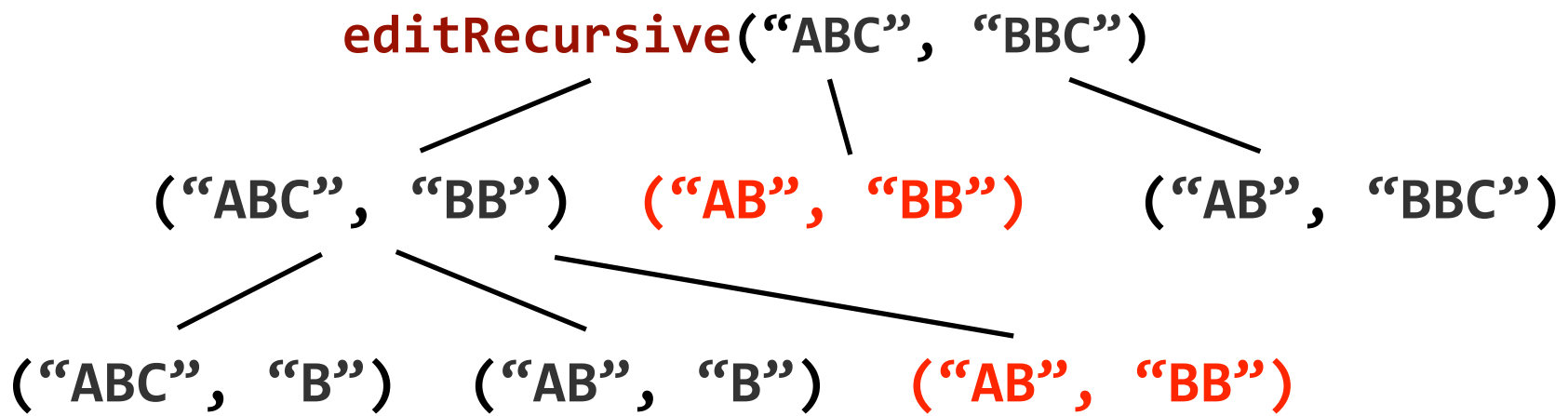

*(only part of recursion tree shown)*

`editRecursive("Shakespeare", "shake spear")`

`Calculate ("Shake", "shake") 8989 times`

How can we address this problem?

# Edit Distance

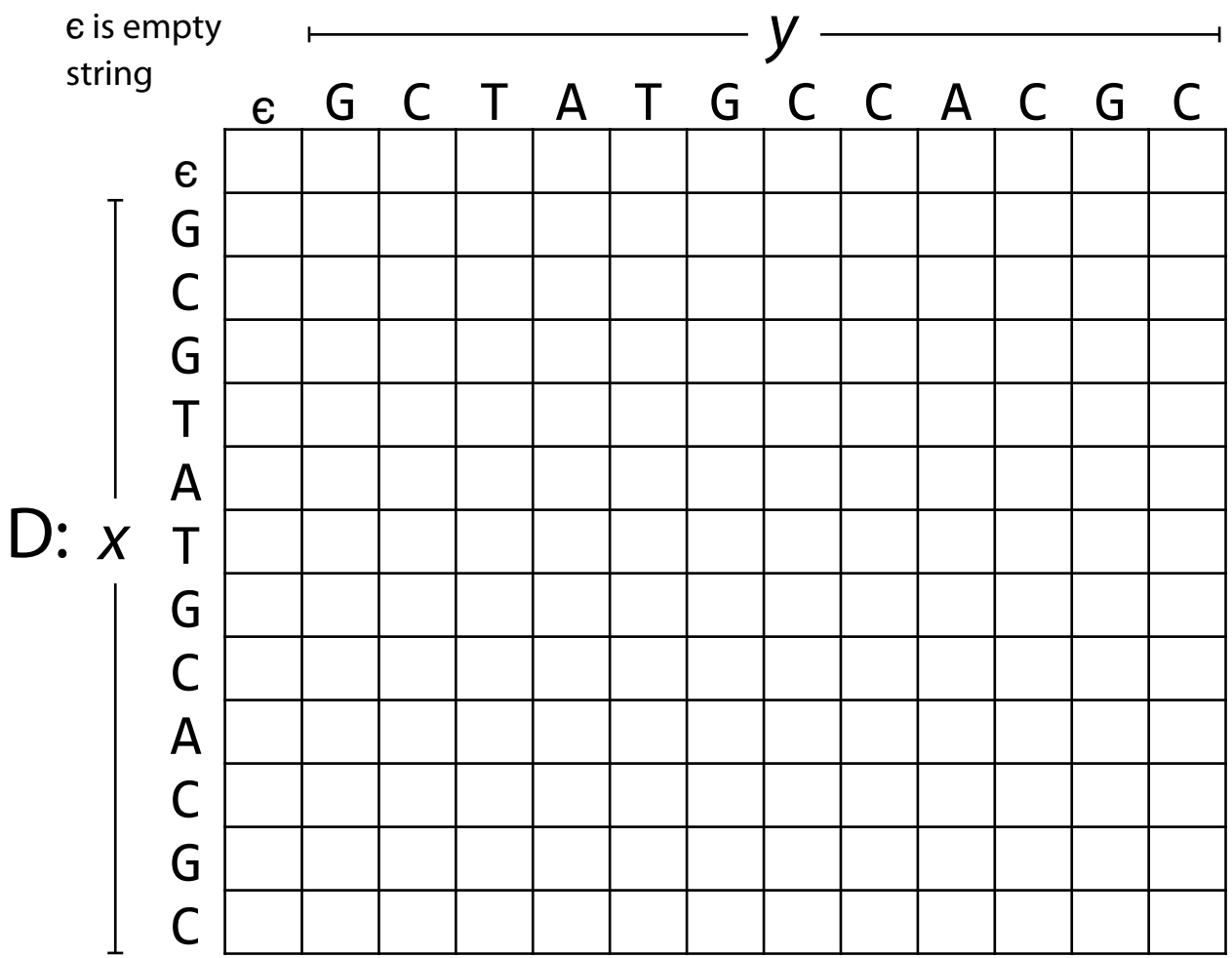



**Memoization:** Top-down

**Dynamic Programming:** Bottom-up

**Both:** Solve individual sub-problems once

# Edit Distance: dynamic programming



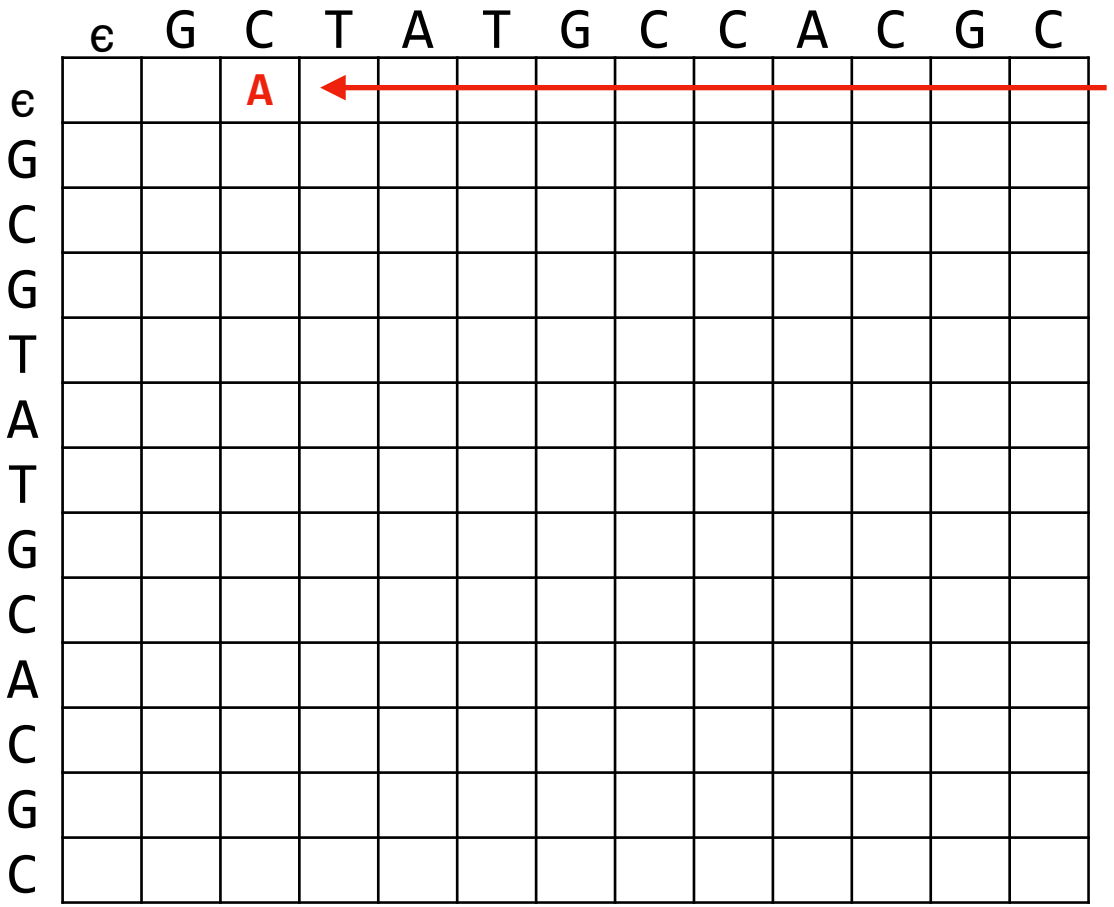

Let $n = |x|$, $m = |y|$

D: $(n+1)$ x $(m+1)$ matrix

D[$i$, $j$] = edit distance b/t length-$i$ prefix of $x$ and length-$j$ prefix of $y$

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

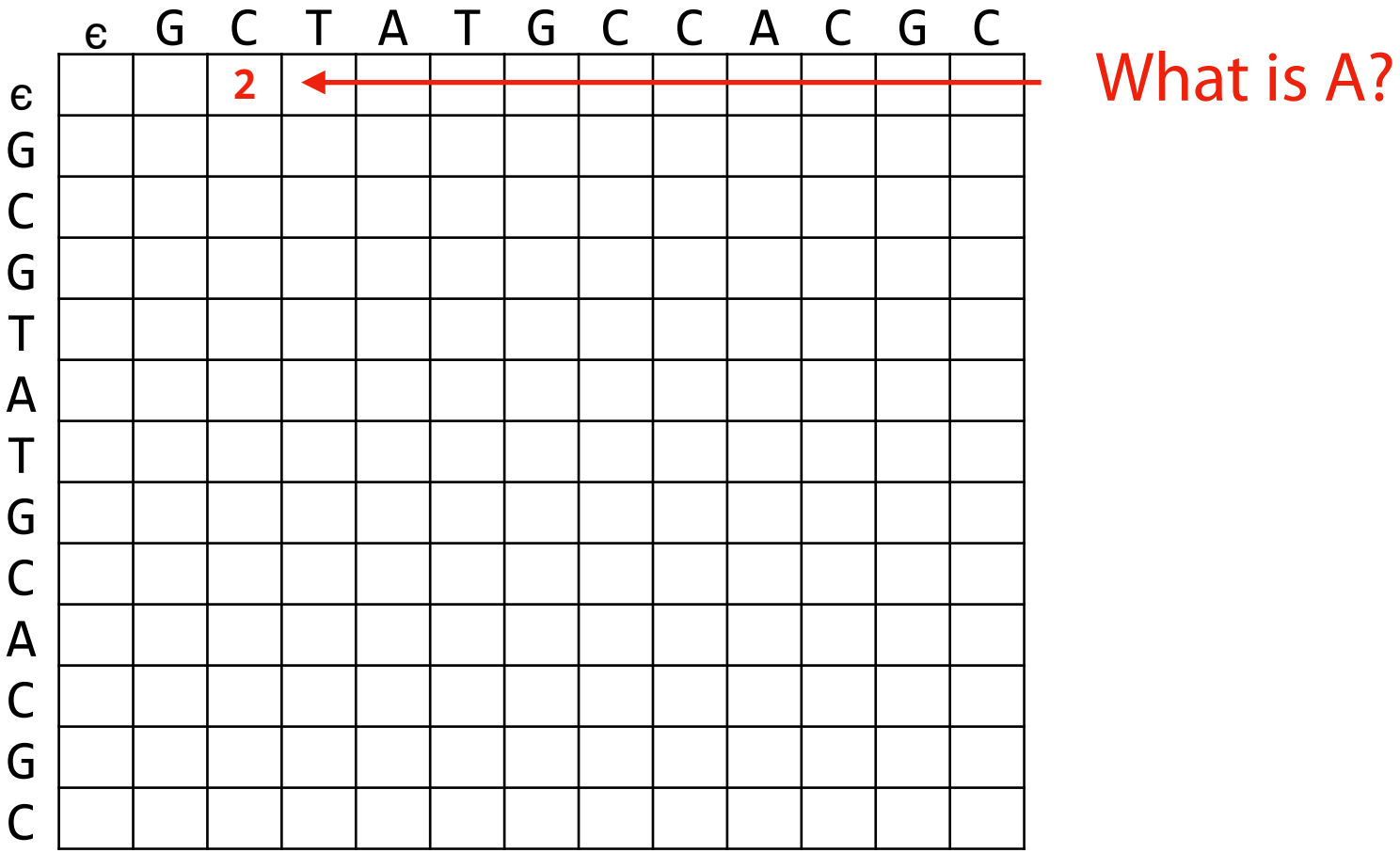



What is A?

D[*i*, *j*] = edit distance b/t length-*i* prefix of *x* and length-*j* prefix of *y*

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$



# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

- - -

G C T

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

G C G T A T

- - - - - -

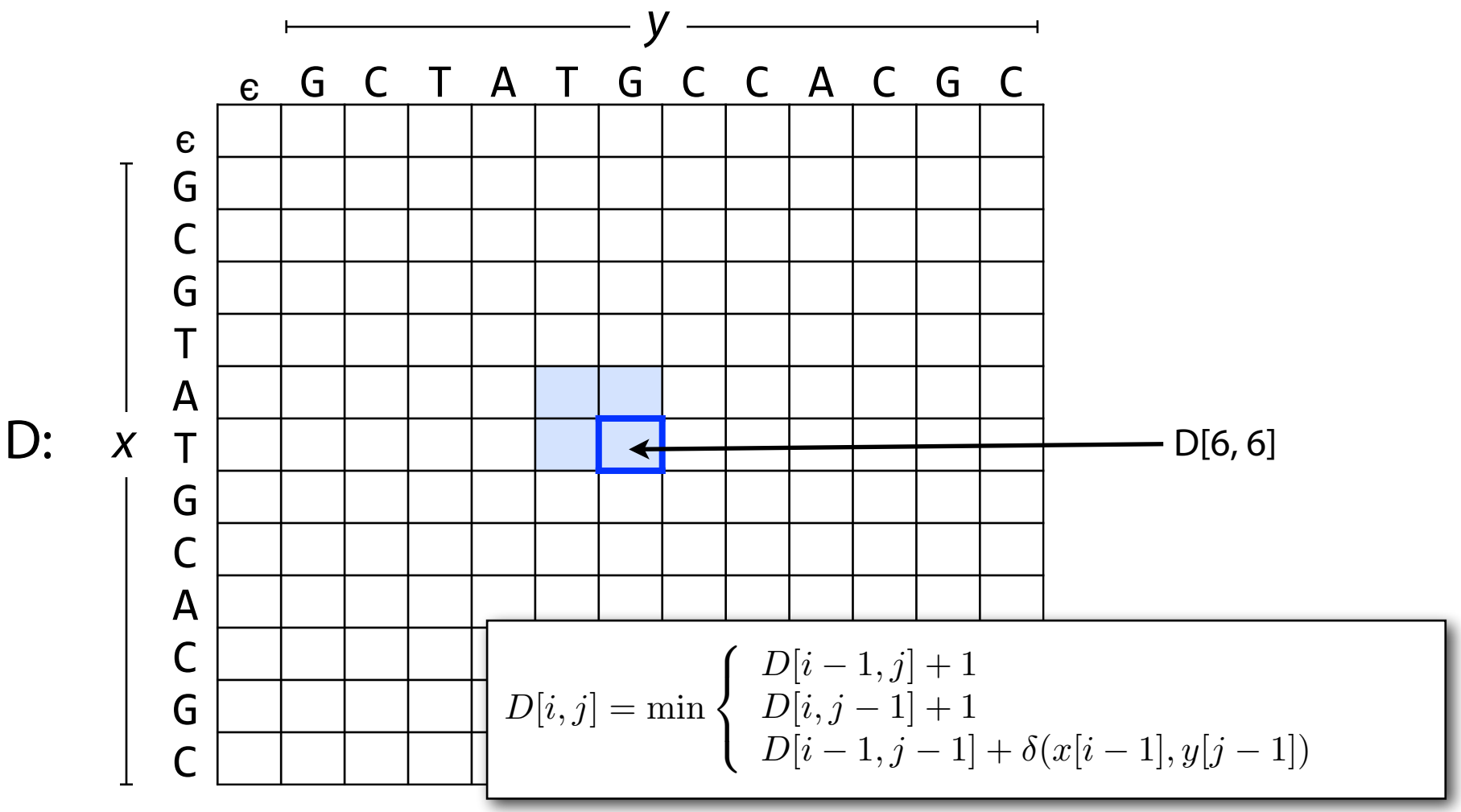
# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ϵ | 0 | | | | | | | | | | | | |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

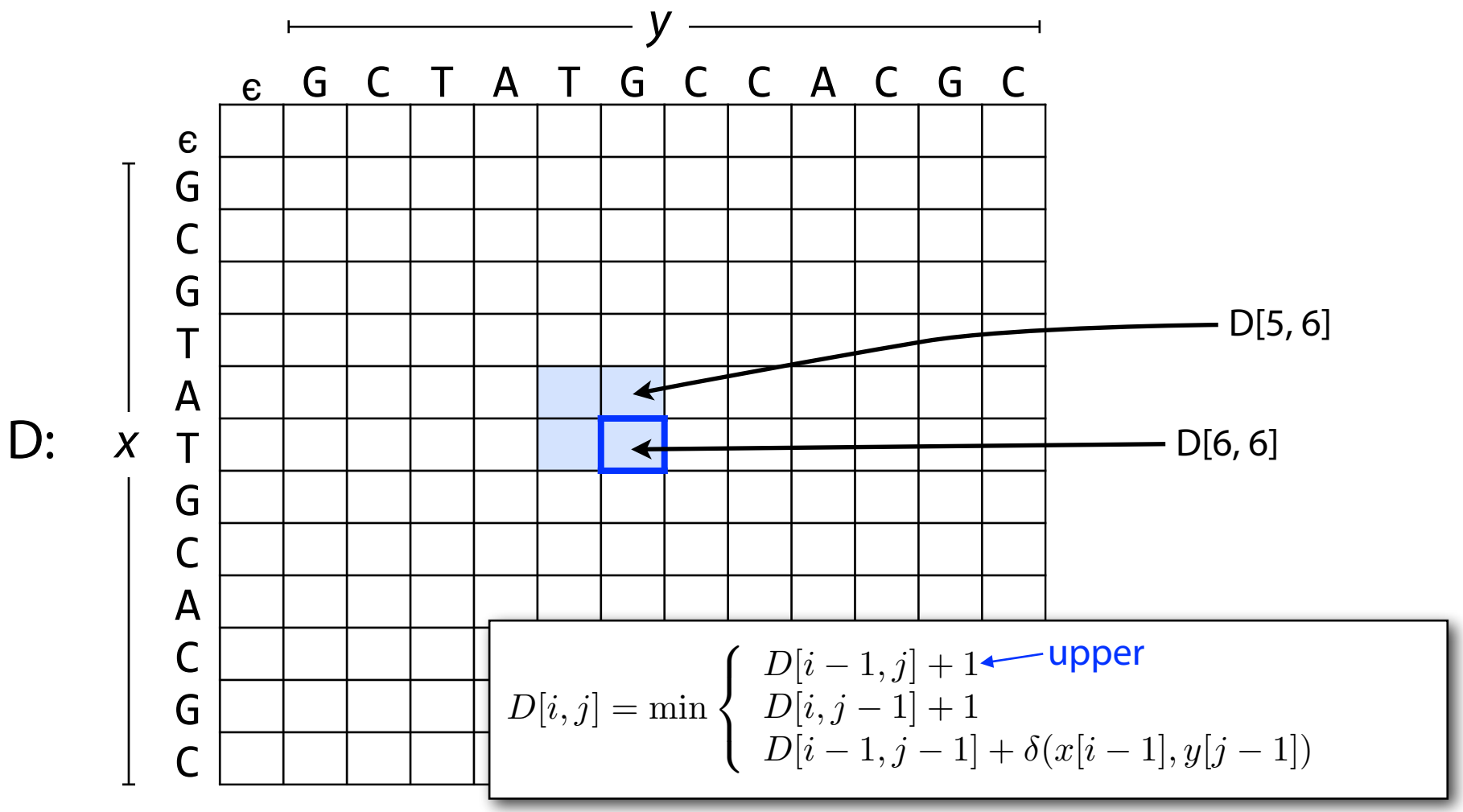
*D* is one row / column larger than x / y!

# Edit Distance: dynamic programming




Cell depends upon its upper, left, and upper-left neighbors

# Edit Distance: dynamic programming

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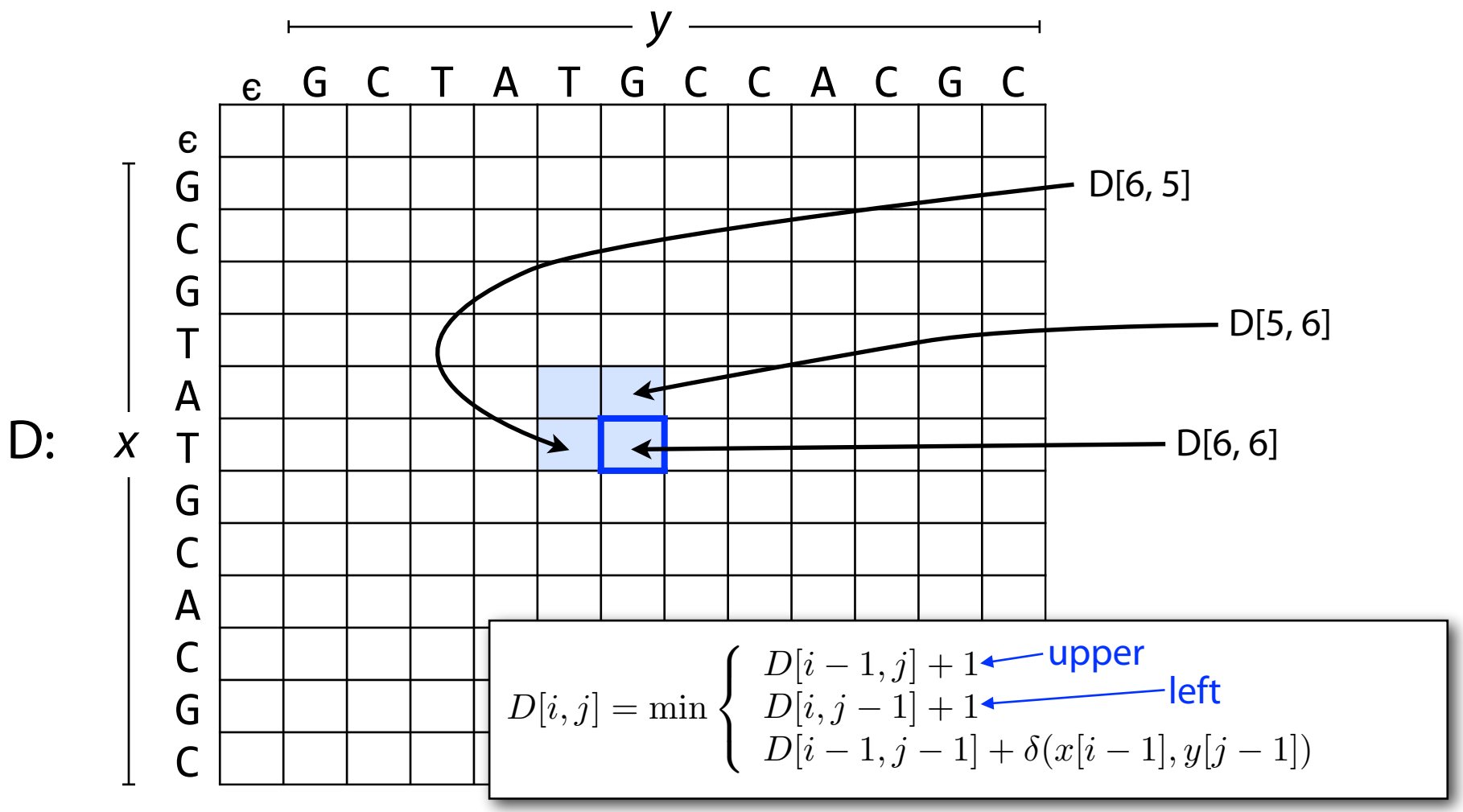
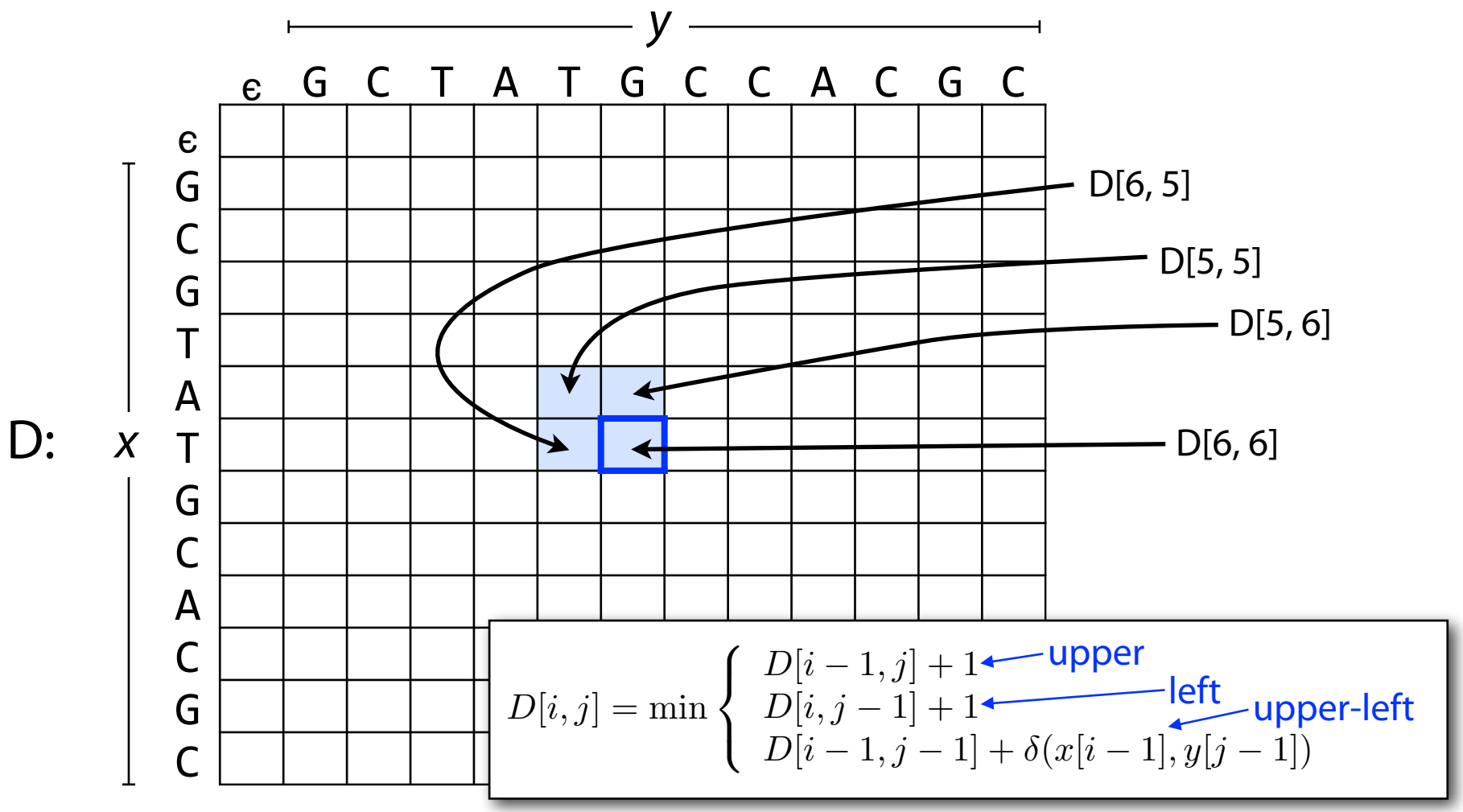
# Edit Distance: dynamic programming



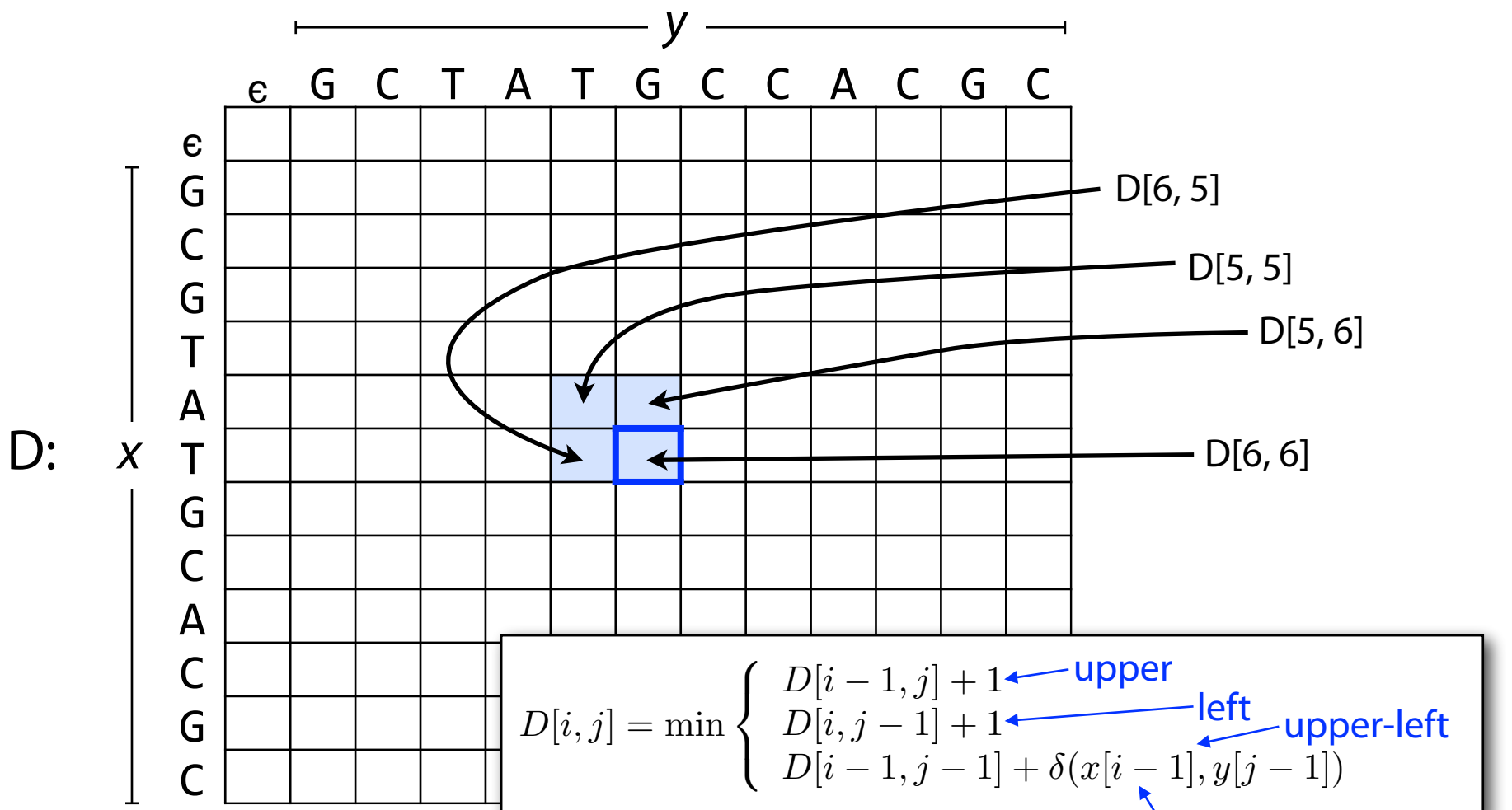

Cell depends upon its upper, left, and upper-left neighbors

# Edit Distance: dynamic programming




Cell depends upon its upper, left, and upper-left neighbors

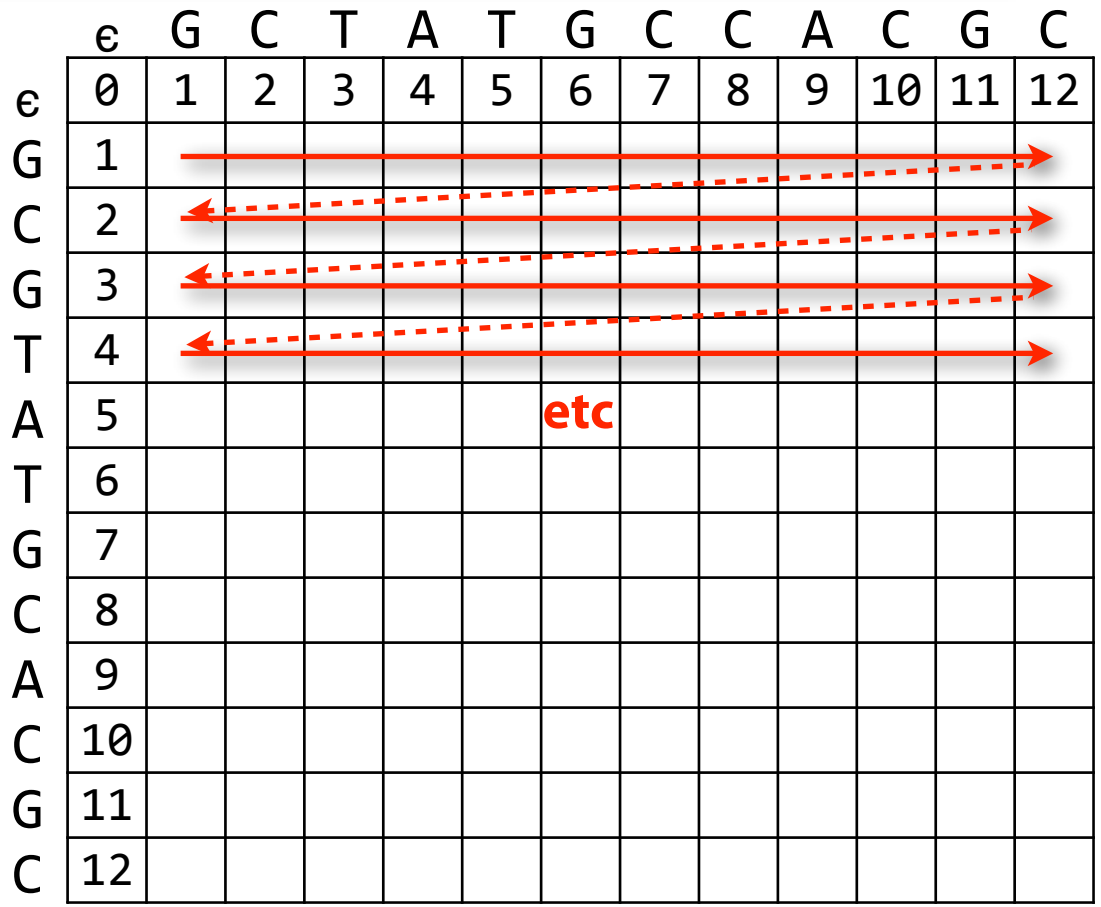
# Edit Distance: dynamic programming




Cell depends upon its upper, left, and upper-left neighbors

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$




Fill remaining cells from top row to bottom and from left to right

# Edit Distance: dynamic programming

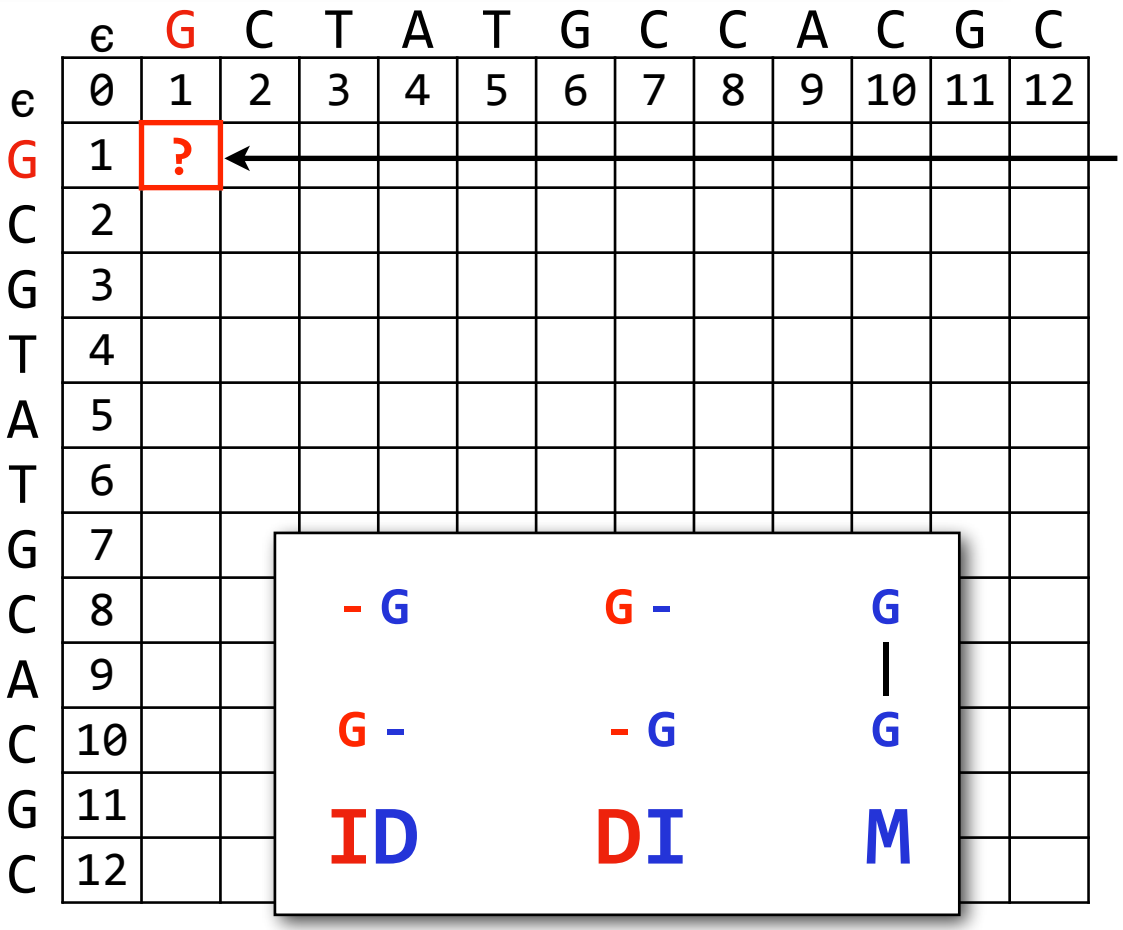
$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | ? | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1,j=1**?

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$




What goes here in **i=1,j=1**?

**x[i-1]** = '**G**',
**y[j-1]** = '**G**',
so **delt = 0**

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | ? | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1,j=1**?

**x[i-1]** = '**G**',
**y[j-1]** = '**G**',
so **delt = 0**

| - G | G - | G |
|---|---|---|
| G - | - G | G |
| **ID** | **DI** | **M** |

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(1 + 1, 1 + 1, 0 + 0)
        = 0
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | ? | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1, j=2**?

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | ? | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1,j=2**?

**x[i-1]** = '**G**',
**y[j-1]** = '**C**',
so **delt = 1**

```
--G     G-     -G
        |
GC-     GC     GC
IID     MI     IR
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ϵ | 0 | | | | | | | | | | | | |
| G | 1 | 0 | ? | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1,j=2**?

**x[i-1]** = '**G**',
**y[j-1]** = '**C**',
so **delt = 1**

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(2 + 1, 0 + 1, 1 + 1)
        = 1
```

| --G | G- | -G |
|---|---|---|
| GC- | GC | GC |
| IID | MI | IR |

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | | | | | |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | | | | | |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | ? | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=4,j=7**?

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | | | | | |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | | | | | |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=4,j=7**?

**x[i-1]** = '**T**',
**y[j-1]** = '**C**',
so **delt = 1**

```
G C - - - G T        G C - G T - -
G C T A T G C        G C T A T G C
M M I I I M R        M M I R M I I
```

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(4 + 1, 3 + 1, 3 + 1)
        = 4
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Fill remaining cells from top row to bottom and from left to right

← Edit distance for x, y

# Edit Distance

| | | *Y* | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| *X* | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | **A** | 2 | 3 |
| | A | 2 | 1 | 0 | **B** | |
| | T | 3 | 2 | **C** | | |

# Edit Distance

| | | ϵ | C | A | A | T |
|---|---|---|---|---|---|---|
| | | ***Y*** | | | | |
| | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | **1** | 2 | 3 |
| ***X*** | A | 2 | 1 | 0 | **B** | |
| | T | 3 | 2 | **C** | | |

# Edit Distance

| | | *Y* | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| *X* | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | **1** | 2 | 3 |
| | A | 2 | 1 | 0 | **1** | |
| | T | 3 | 2 | **C** | | |

# Edit Distance

| | | *Y* | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| *X* | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | **1** | 2 | 3 |
| | A | 2 | 1 | 0 | **1** | |
| | T | 3 | 2 | **1** | | |

# Edit Distance

| | | *Y* | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| *X* | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | 1 | 2 | 3 |
| | A | 2 | 1 | 0 | 1 | 2 |
| | T | 3 | 2 | 1 | 1 | 1 |

# eMatrix buildEditMatrix(X, Y)

Input:

**`string X:`** Input string X

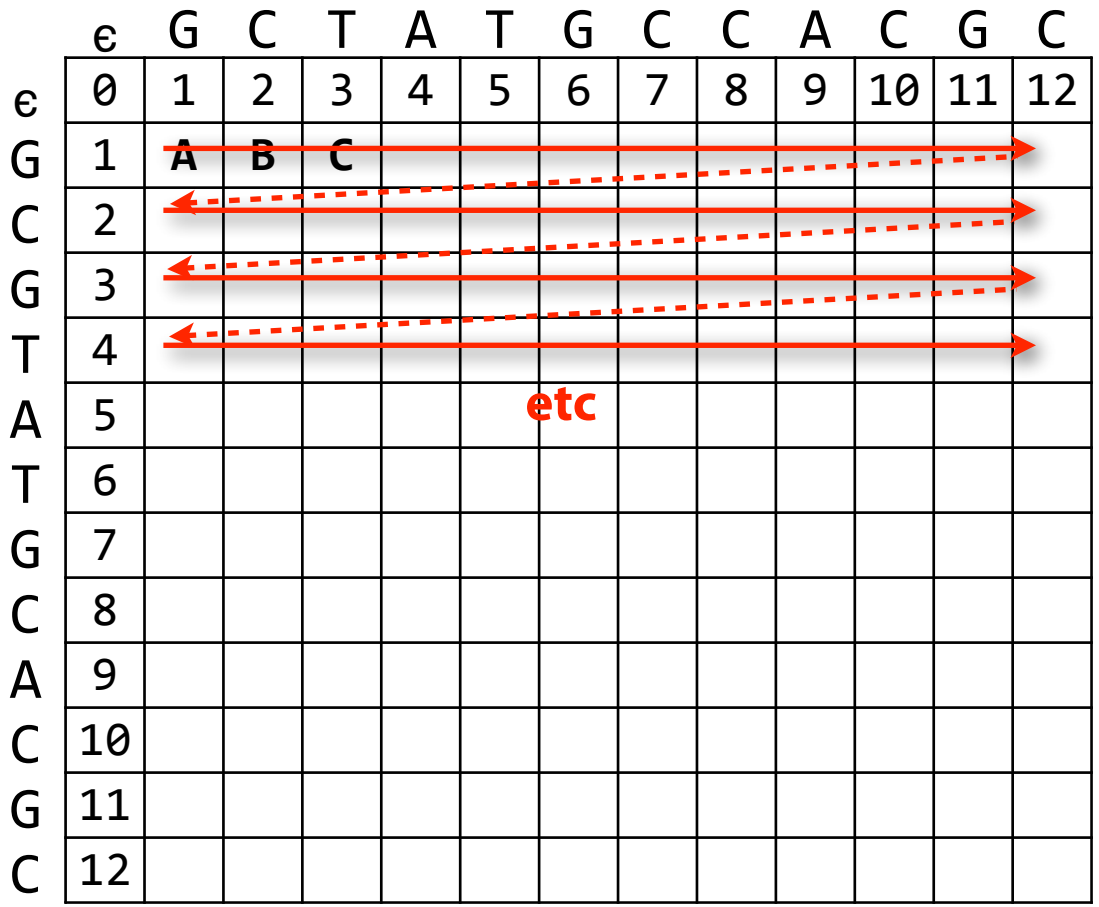
**`string Y:`** Input string Y

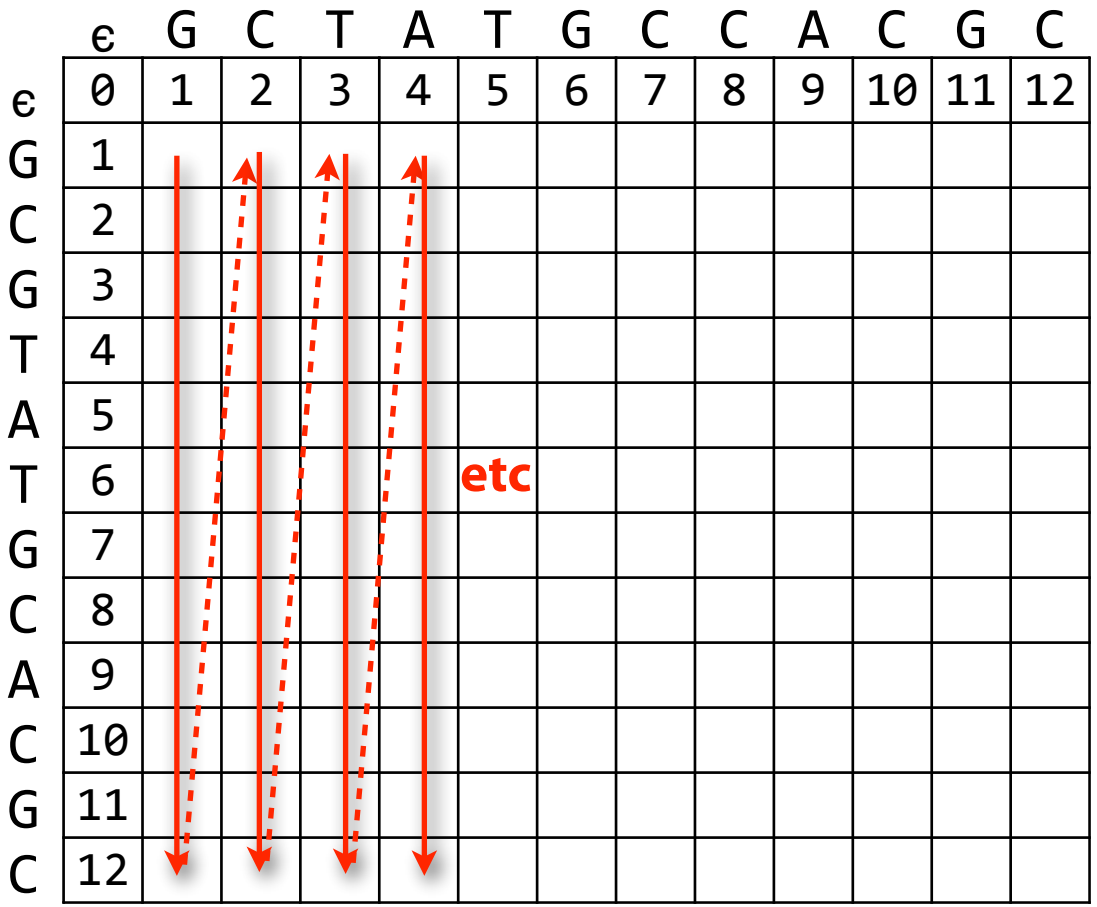
Output:

`eMatrix` : vector<vector<int>> storing all optimal edit distances

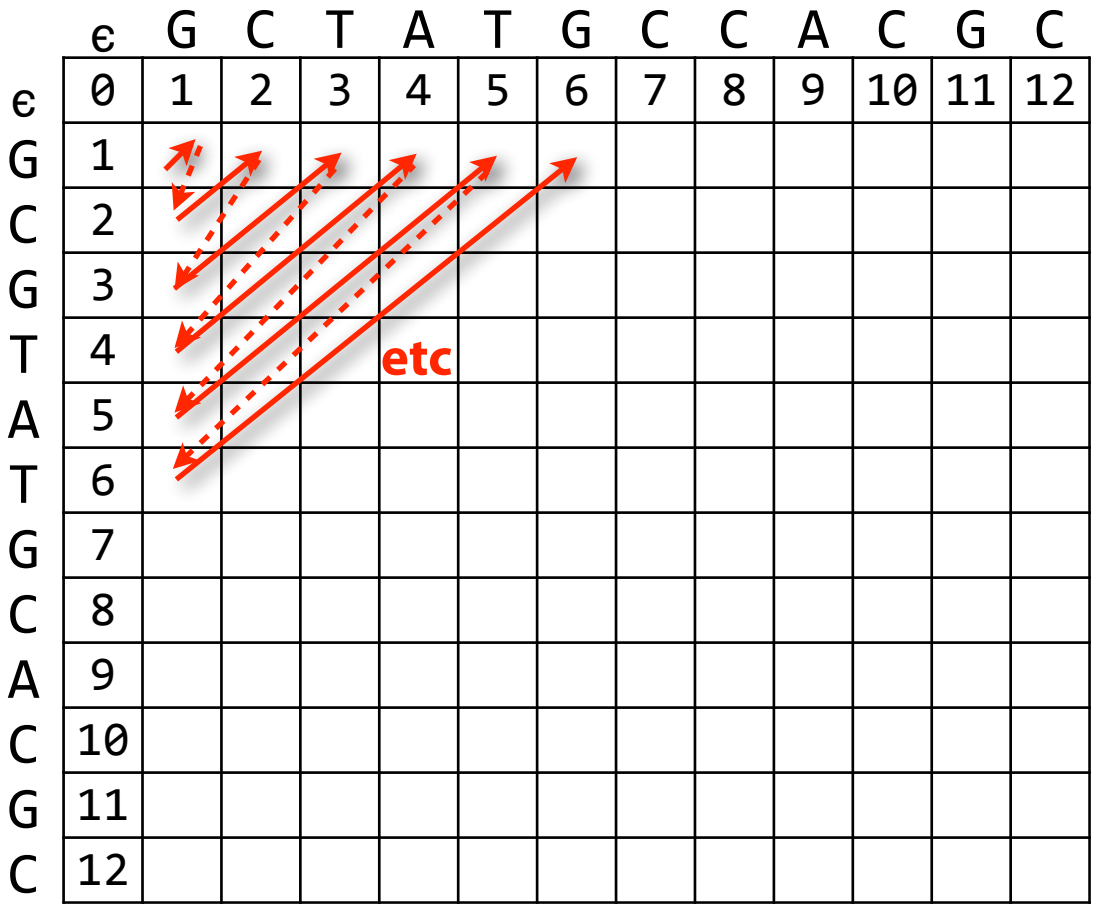
| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

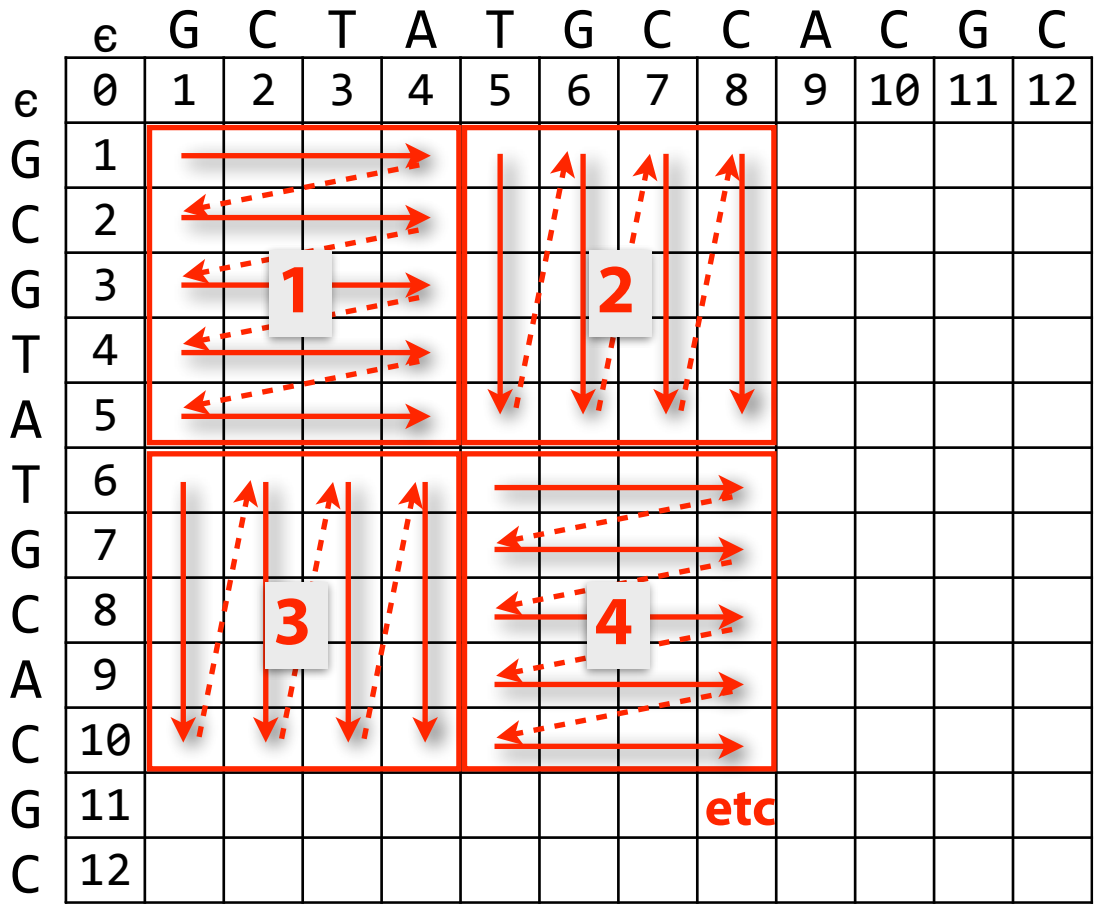
# Edit Distance: dynamic programming



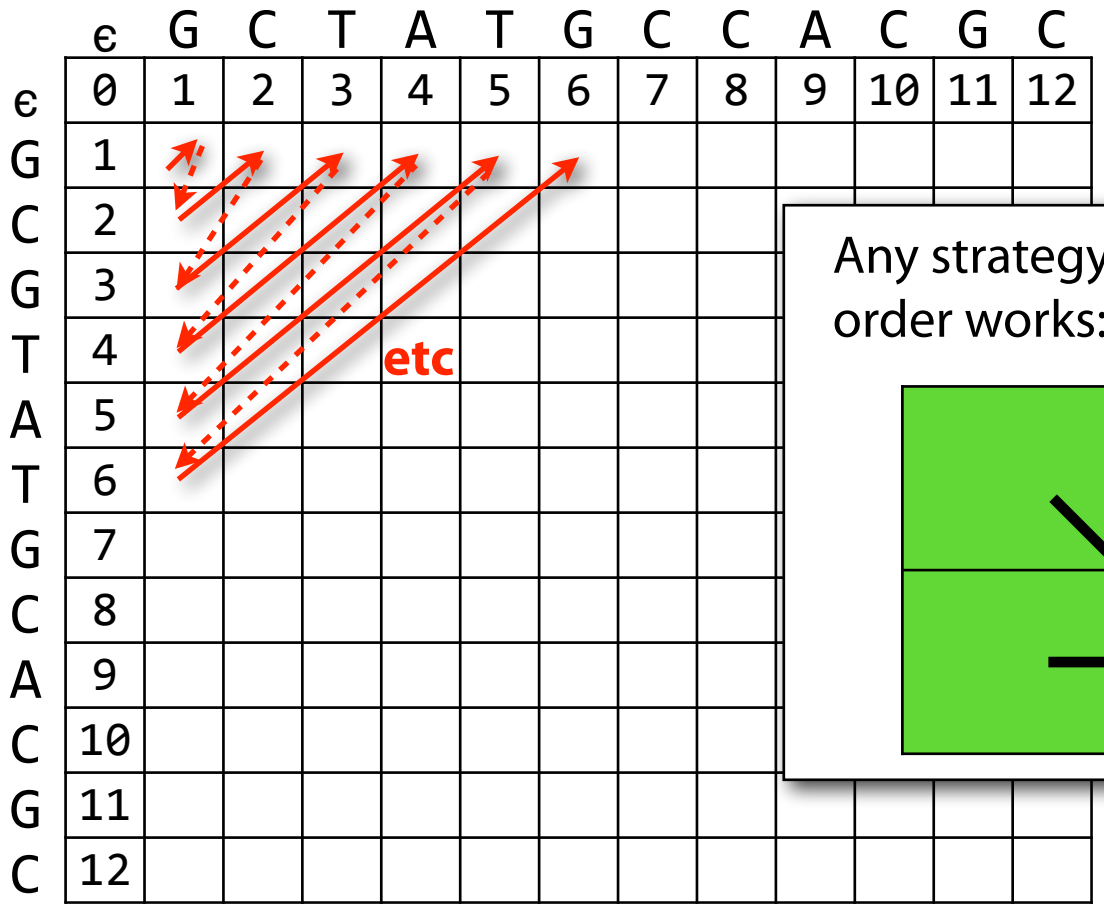



# Edit Distance: dynamic programming

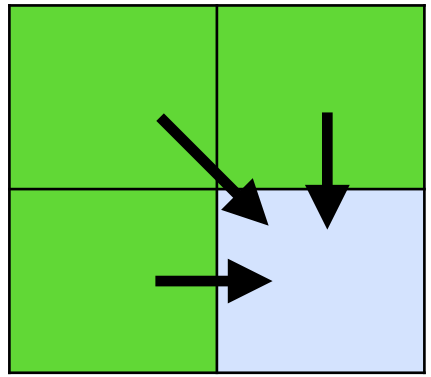


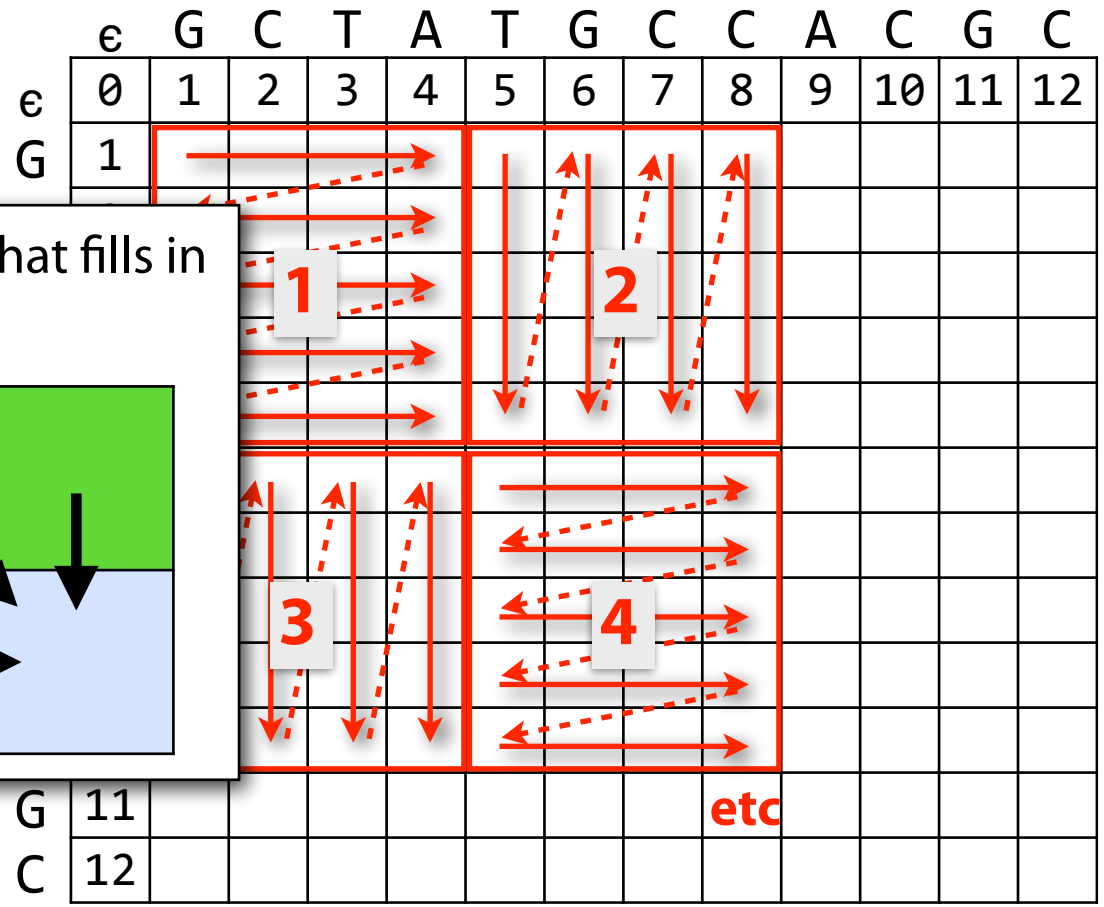



# Edit Distance: dynamic programming




Any strategy that fills in order works:





# Assignment 11: a_edist

Learning Objective:

Use dynamic programming to build an edit distance matrix

Construct an optimal edit string from the edit matrix

**Consider:** Does substitution, insertion, and deletion need to have the same 'weight' as a penalty? How could you modify the code to take a user-specified input for each?

# Edit Distance

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

← Edit distance for x, y

But where and what is the edit?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | **1** |

← Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | **2** |
| T | 3 | 2 | 1 | 1 | 1 |

$D[2,\ 4] =$ _____

← Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[3,\ 3] =$ _____

← Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | **1** | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[2,\ \ 3] =$ _____

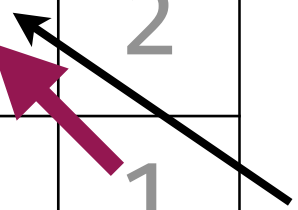
Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |



Q: How did I get here?

$D[1,\ 3] =$ _____

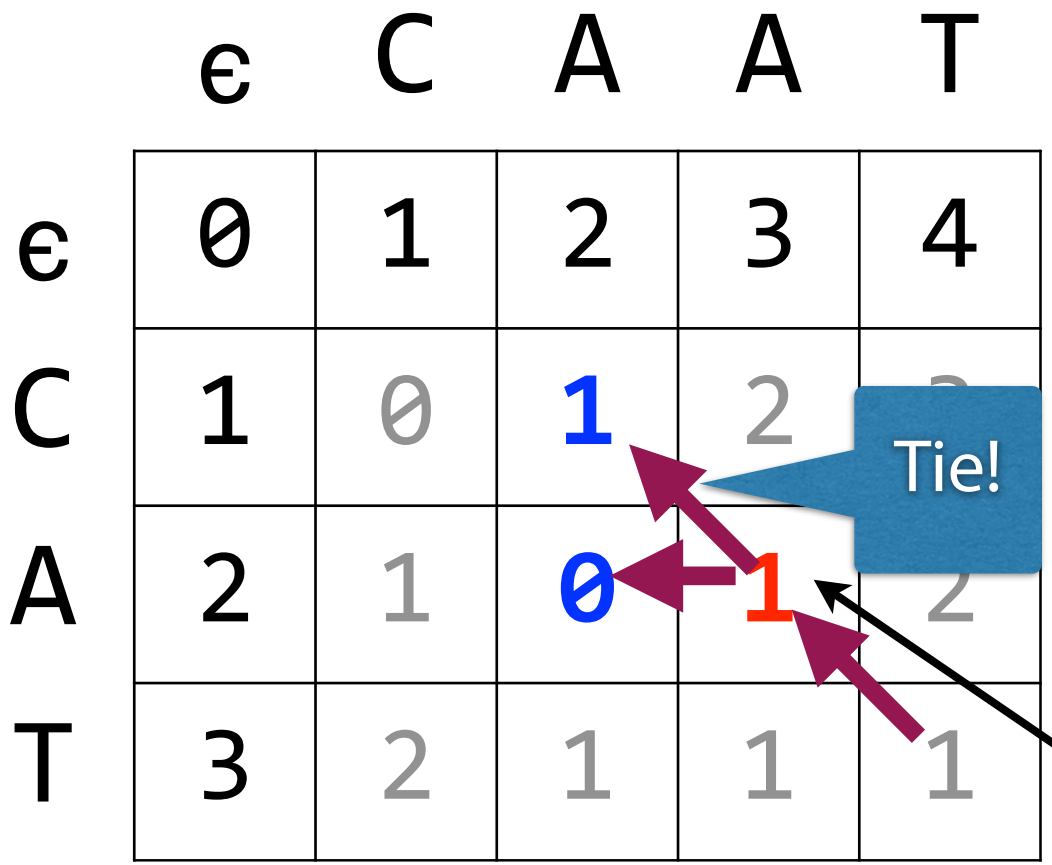
$D[1,\ 2] =$ _____

$D[2,\ 2] =$ _____

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?




$$D[1,\ 3] = 2 + 1$$

$$D[1,\ 2] = 1 + 0$$

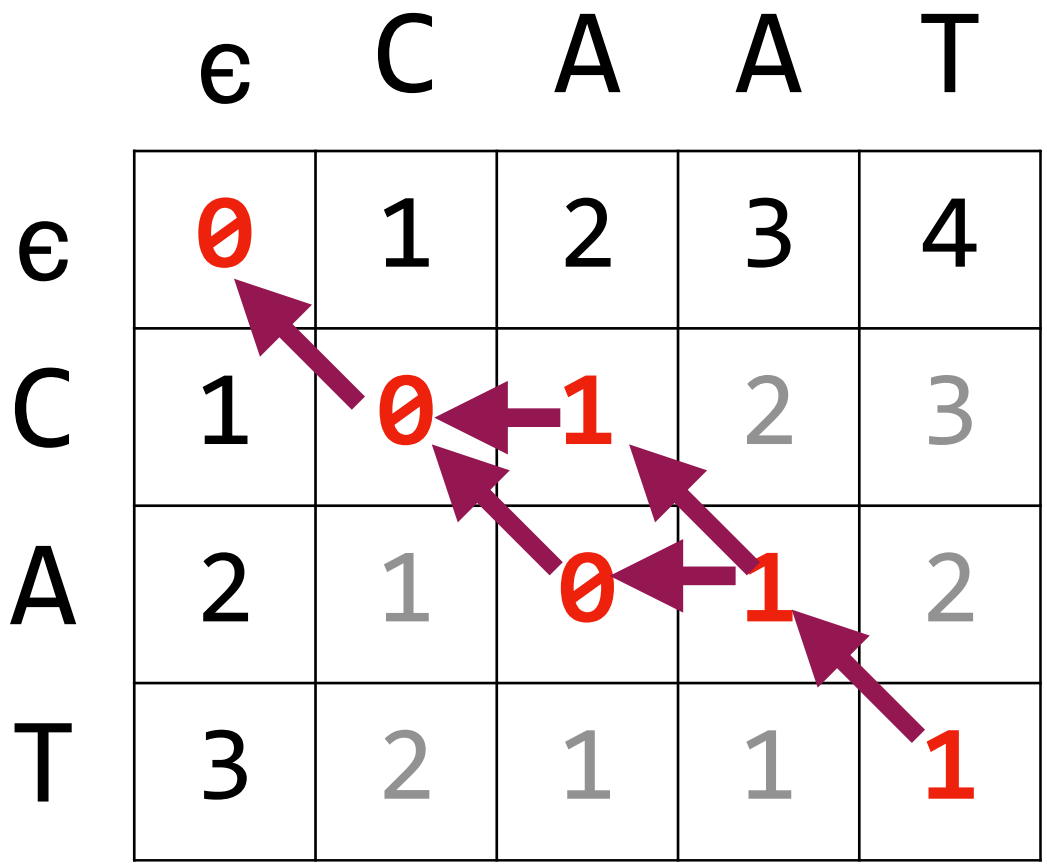
$$D[2,\ 2] = 0 + 1$$

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

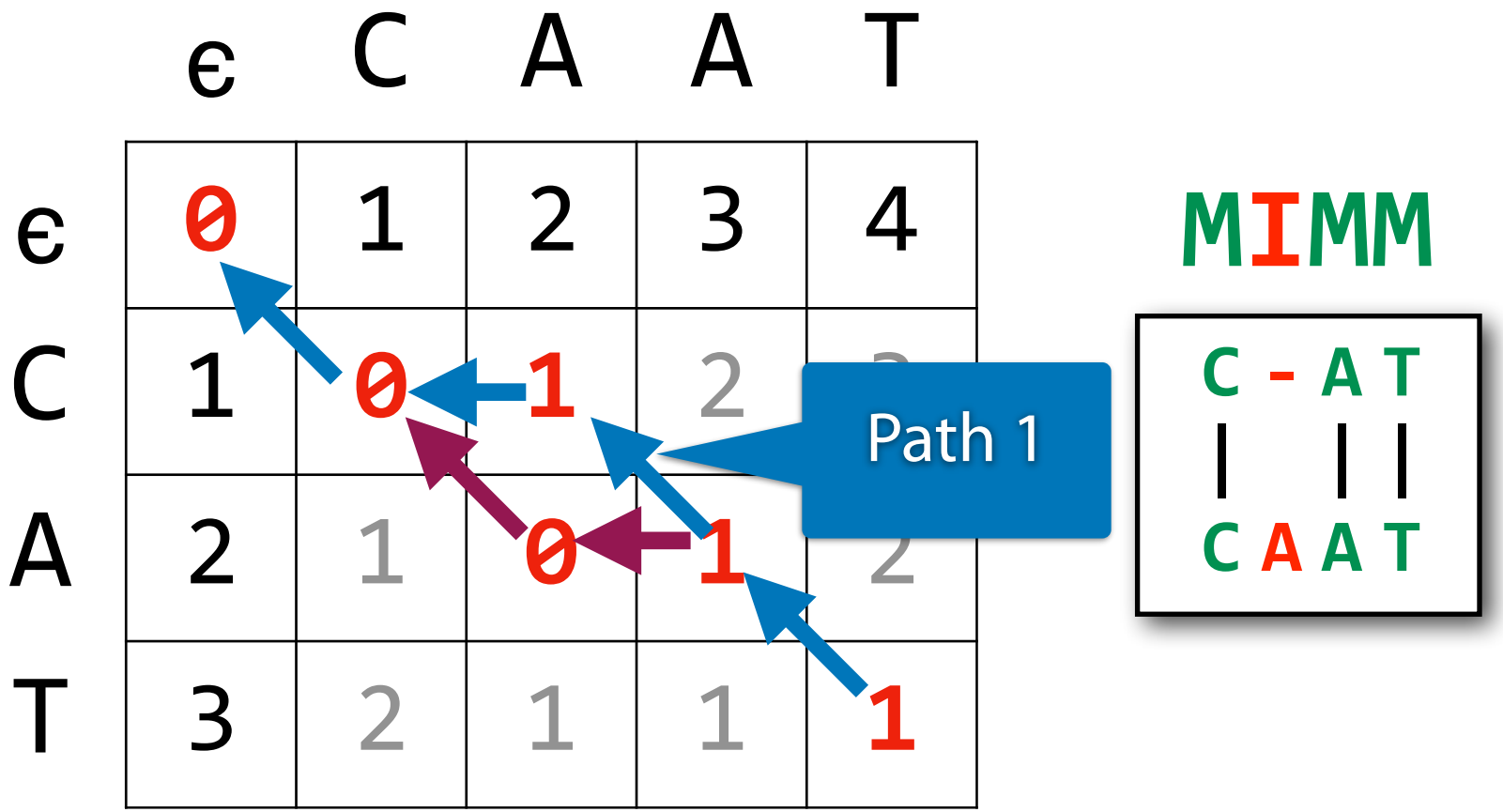
At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

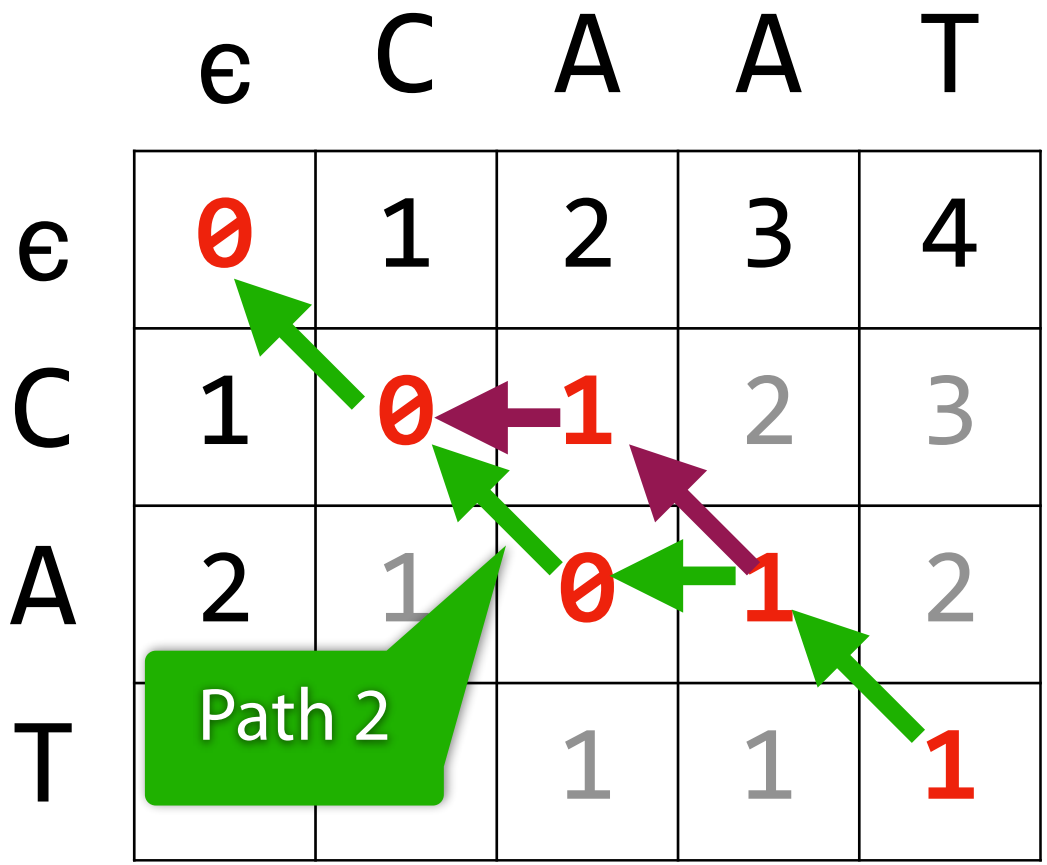
At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?



# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?




**MIMM**

```
C - A T
|   | |
C A A T
```

**MMIM**

```
C A - T
| |   |
C A A T
```

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

← Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

$$D[11,\ 12] = 3 + 1$$

$$D[11,\ 11] = 2 + 0$$

$$D[12,\ 11] = 3 + 1$$

A: From here

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

$$D[10,\ 11] = 3 + 1$$

$$D[10,\ 10] = 2 + 0$$

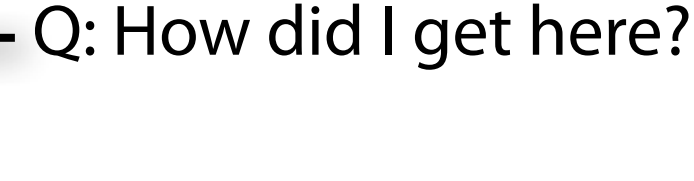
$$D[11,\ 10] = 3 + 1$$

A: From here

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |



# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

$$D[9,\ 10] = 3 + 1$$

$$D[9,\ 9] = 2 + 0$$

$$D[10,\ 9] = 3 + 1$$

A: From here

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Alignment:

```
G C G T A T G - C A C G C
|| |  |||| |  |||||
G C - T A T G C C A C G C
```

```
MMDMMMMIMMMMM
```

# Edit Distance

**Dynamic Programming** fills our table with optimal distances

**Traceback** identifies the optimal *edit string* to convert *X* to *Y*

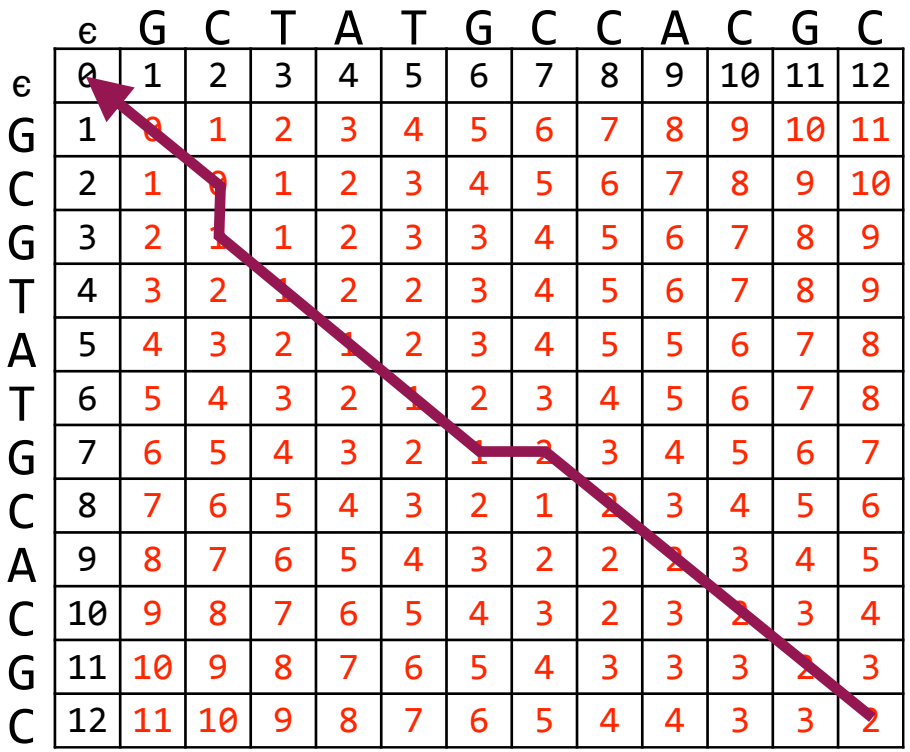
What is our efficiency? ($|X| = m, |Y| = n$)

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

# Edit Distance

**Dynamic Programming** fills our table with optimal distances

**Traceback** identifies the optimal *edit string* to convert *X* to *Y*



| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

What is our efficiency? ($|X| = m$, $|Y| = n$ )

**Table filling:** Filling (m + 1) (n + 1) cells, each requiring constant work, so $O(mn)$

**Traceback:** Each step goes ⇖, ⇐ or ⇑. Worst case: traceback never moves diagonally, requiring $m$ ⇑'s and $n$ ⇐'s, so $O(m + n)$

# `string buildEditString(X, Y)`

Input: **`string X:`** Input string X (edits with respect to X)

**`string Y:`** Input string Y (edits turn X into Y)

Output: **`string`**: An optimal edit string produced by the matrix

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

On tie: prioritize diagonal, then vertical, then horizontal

**MIMM**

```
C - A T
|   | |
C A A T
```

# Approximate Pattern Matching

"Seed and extend" works for edit distance too!