Data Structures

Graph Traversals

CS 225
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Announcements

EC points have been awarded for all IEF and surveys

MP_traversal survey will go out today

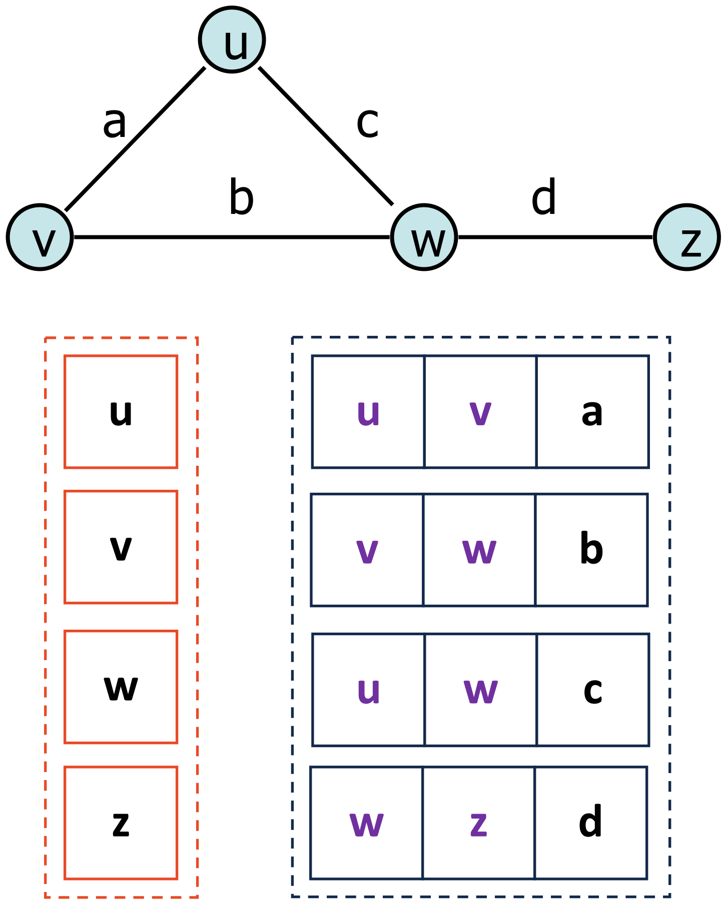
FAIR cases are in the works — remember course policies!

Learning Objectives

Discuss graph traversal algorithms

Graph Implementation: Edge List $|V| = n, |E| = m$

The equivalent of an 'unordered' data structure



Vertex Storage:

An optional list of vertices

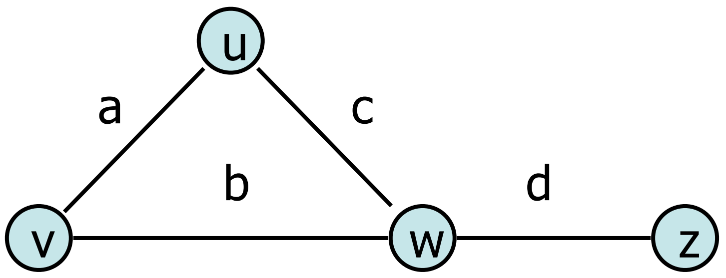
Edge Storage:

A list storing edges as (V1, V2, Weight)

Most graphs are stored as just an edge list!

Graph Implementation: Adjacency Matrix

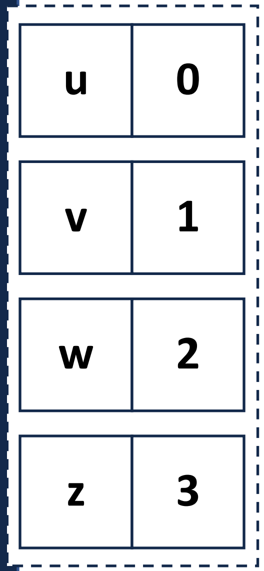
$|V| = n, |E| = m$



Vertex Storage:

A hash table of vertices

Implicitly or explicitly store index



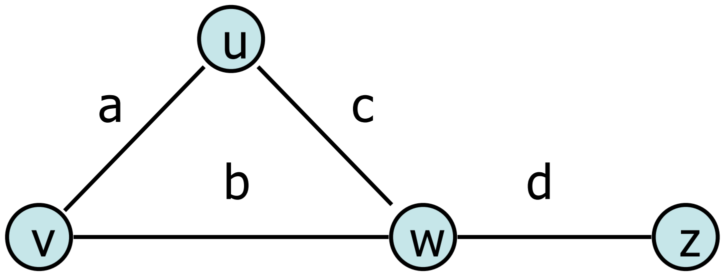
	0	1	2	3
0	-	a	c	0
1		-	b	0
2			-	d
3				-

Edge Storage:

A |V| x |V| matrix of edges

Weight is stored at position (u, v)

Adjacency List



Vertex Storage:

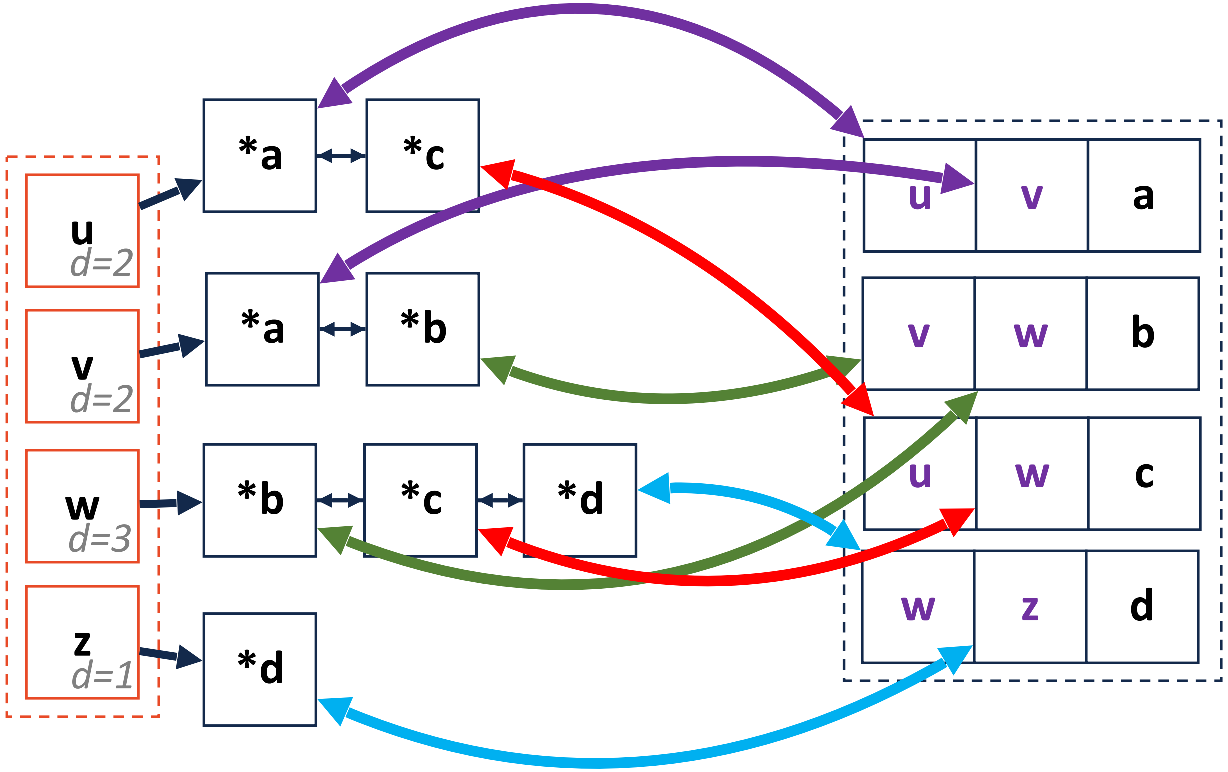
A bidirectional linked list with size variable

Each node is a pointer to edge in edge list

Edge Storage:

A list of (v1, v2, weight) edges

Also store pointers back to nodes



$|V| = n, |E| = m$

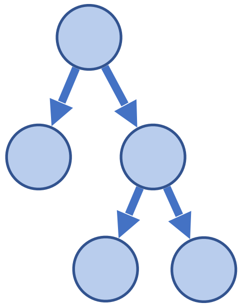
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1*	n*	1*
removeVertex(v)	n+m	n	deg(v)
insertEdge(u, v)	1	1	1*
removeEdge(u, v)	m	1	min(deg(u), deg(v))
incidentEdges(v)	m	n	deg(v)
areAdjacent(u, v)	m	1	min(deg(u), deg(v))

Graph Traversals

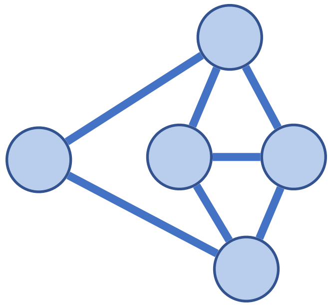
Objective: Visit every vertex and every edge in the graph.

How can we systematically go through a complex graph in the fewest steps?

Tree traversals won't work — lets compare:



- Rooted
- Acyclic
-



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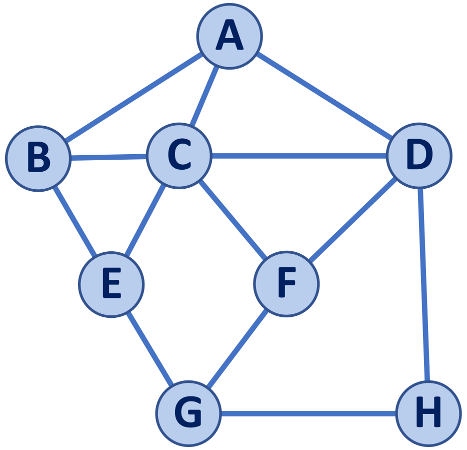
Traversal: BFS

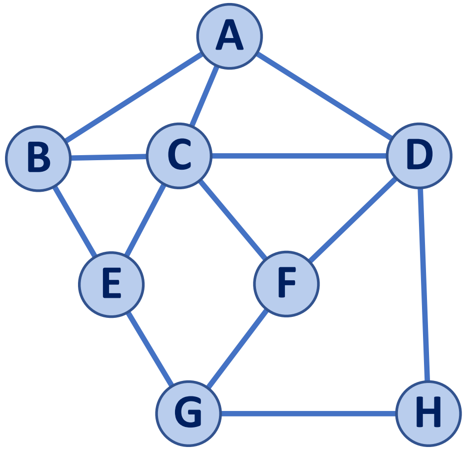


To complete a traversal what do we need?

As input:

To keep track of:

Traversal: BFS



v	d	P	Adjacent Edges
A			B C D
B			A C E
C			A B D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

Traversal: BFS

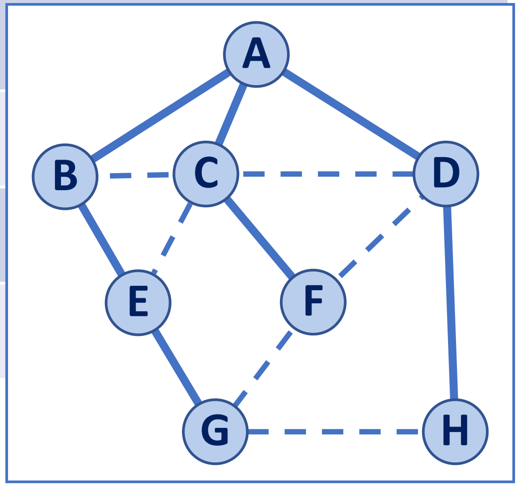
$|V| = n, |E| = m$

Initialize queue / depth / predecessor

While queue not empty:

- Remove front vertex of queue
- Check if edge connects to new vertex
  - Set dist / pred if new vertex
- Add unvisited edges to queue

v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	1	A	A B D E F
D	1	A	A C F H
E	2	B	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G

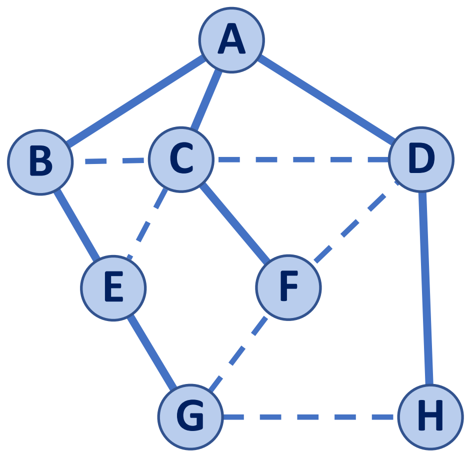


~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ ~~F~~ ~~H~~ ~~G~~

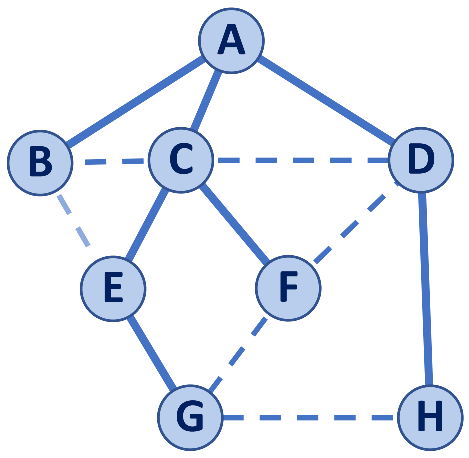
Running time?

Traversal: BFS

Traversal depends on start position as well as order of edges at each node



v	d	P	Adjacent Edges
A	0	-	**B C D**
B	1	**A**	**A C E**
C	1	**A**	**A B D E F**



v	d	P	Adjacent Edges
A	0	-	**C B D**
B	1	**A**	**A C E**
C	1	**A**	**A B D E F**

Input: Graph, G

Output: A labeling of the edges in G as discovery or cross

```
BFS(G):
  foreach (Vertex v : G.vertices()):
   setPred(v, NULL)
   setDist(v, -1)

  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)

  foreach (Vertex v : G.vertices()):
    if getDist(v) == -1:
        BFS(G, v)
```

```
BFS(G):
  foreach (Vertex v : G.vertices()):
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  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)

  foreach (Vertex v : G.vertices()):
    if getDist(v) == -1:
       BFS(G, v)
```

```
BFS(G, v):
  Queue q
  setDist(v, 0)
  q.enqueue(v)

  while !q.empty():
    v = q.dequeue()

     foreach (Vertex w : G.adjacent(v)):
        if( getDist(w) == -1):
           setLabel((v, w), DISCOVERY)
           setPred(w, v)
           setDist(w, v + 1)
           q.enqueue(w)
       else:
           setLabel((v, w), CROSS)
```

```
BFS(G):
  foreach (Vertex v : G.vertices()):
    setPred(v, NULL)
    setDist(v, -1)

  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)

  foreach (Vertex v : G.vertices()):
    if getDist(v) == -1:
        BFS(G, v)
```

Count connected components?

```
BFS(G, v):
  Queue q
  setDist(v, 0)
  q.enqueue(v)

  while !q.empty():
    v = q.dequeue()

    foreach (Vertex w : G.adjacent(v)):
        if( getDist(w) == -1):
            setLabel((v, w), DISCOVERY)
            setPred(w, v)
            setDist(w, v + 1)
            q.enqueue(w)
        else:
            setLabel((v, w), CROSS)
```

Cycle Detection?

BFS Observations

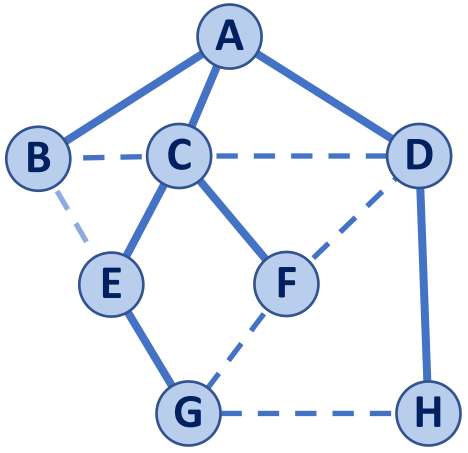
What is the shortest path from **A** to **H**?

What is the shortest path from **E** to **H**?

If my node has distance **d**, do I know anything about the nodes connected by a **cross edge**?

What structure is made from **discovery edges**?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



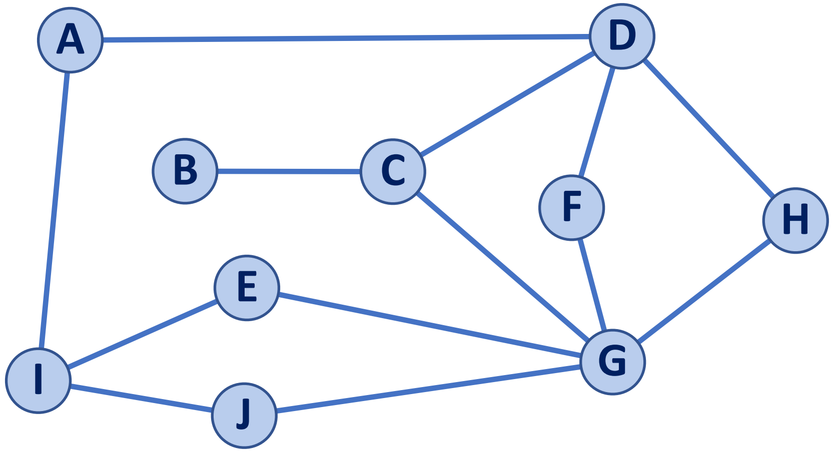
BFS Observations

1. BFS can be used to count components
2. BFS can be used to detect cycles

3. The BFS 'distance' value is always the shortest distance from source to any vertex (and the discovery edges form a MST)

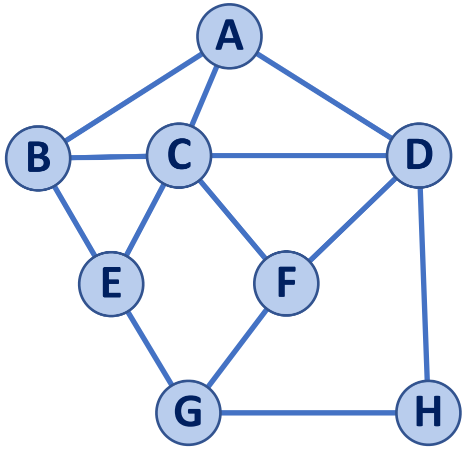
4. The endpoints of a cross edge never differ in distance by more than 1 ($|\mathbf{d(u)} - \mathbf{d(v)}| = \mathbf{1}$)

Traversal: DFS



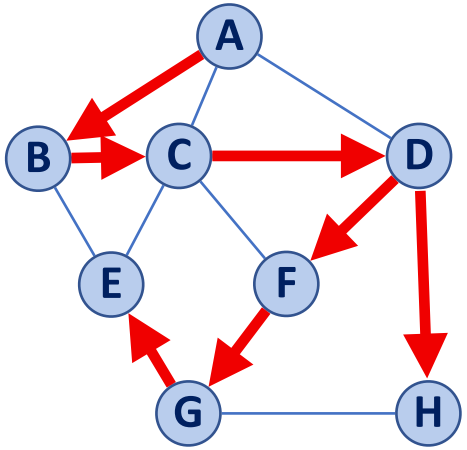
```
DFS(G):
  foreach (Vertex v : G.vertices()):
   setPred(v, NULL)
   setDist(v, -1)

  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)

  foreach (Vertex v : G.vertices()):
    if getDist(v) == -1:
       DFS(G, v)
```

```
DFS(G, v):

    foreach (Vertex w : G.adjacent(v)):
      if( getDist(w) == -1):
          setLabel((v, w), DISCOVERY)
          setPred(w, v)
          setDist(w, v + 1)
          DFS(G, w)
      else:
          setLabel((v, w), BACK)
```



```
DFS(G, v):

    foreach (Vertex w : G.adjacent(v)):
        if( getDist(w) == -1):
            setLabel((v, w), DISCOVERY)
            setPred(w, v)
            setDist(w, v + 1)
            DFS(G, w)
        else:
            setLabel((v, w), BACK)
```

v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	2	B	A B D E F
D	3	C	A C F H
E	6	G	B C G
F	4	D	C D G
G	5	F	E F H
H	4	D	D G

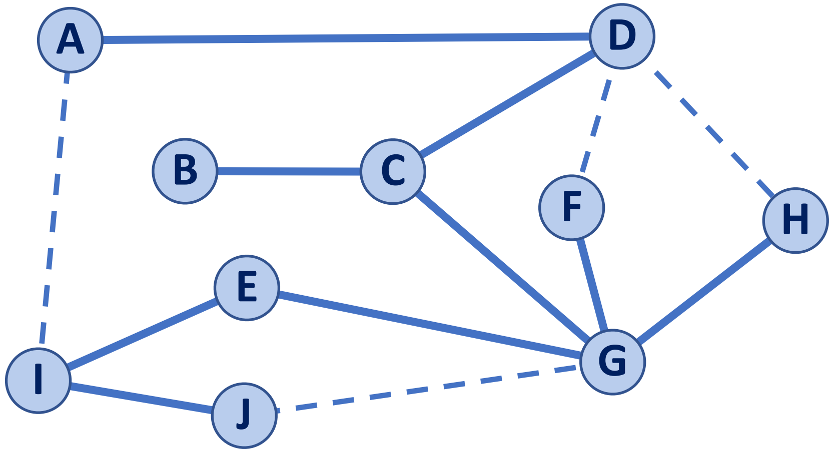


A B C D F G E H

Running time?

$|V| = n, |E| = m$

Traversal: DFS



Do we still make a spanning tree?

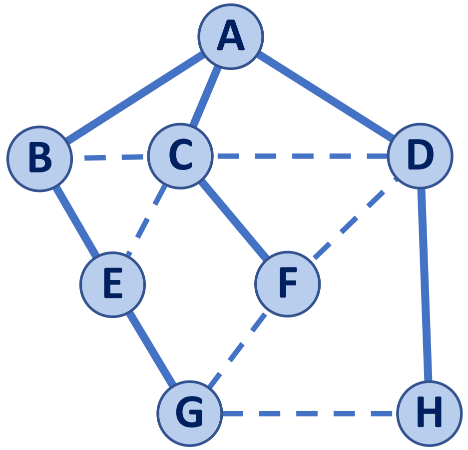
Does distance have meaning here?

Do our edge labels have meaning here?

Efficiency: DFS vs BFS

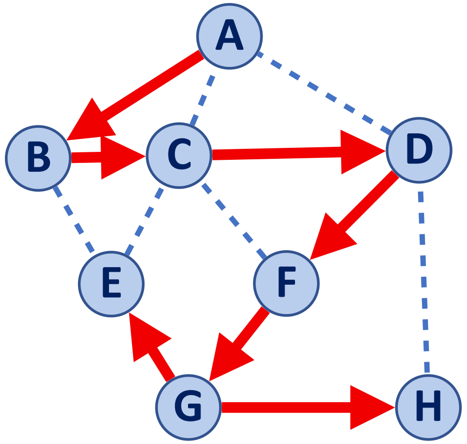
$|V| = n, |E| = m$

BFS:



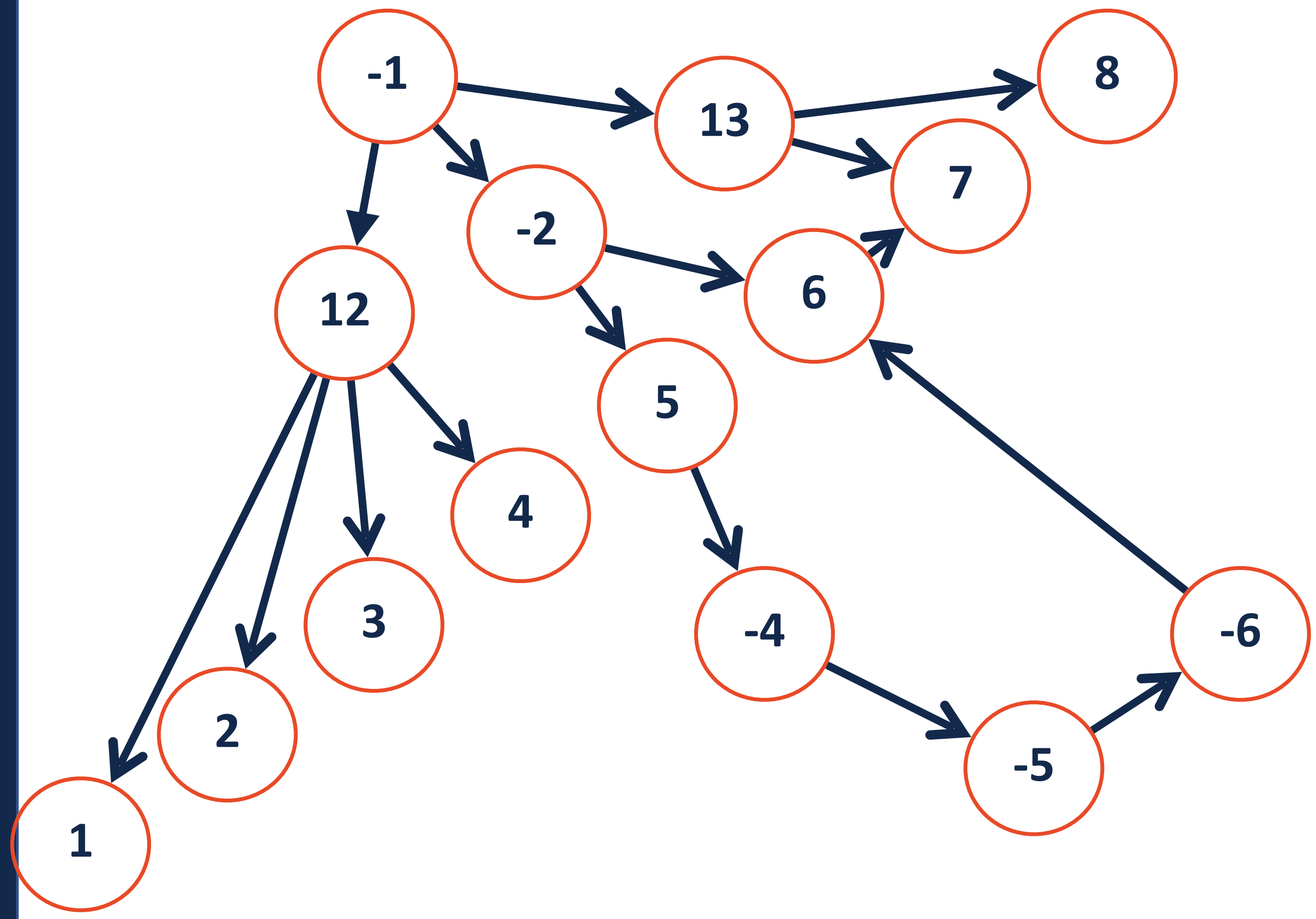
A B C D E F H G

DFS:



A B C D F G E H

Space Efficiency: DFS vs BFS



Summary: DFS and BFS

$|V| = n, |E| = m$

Both are **O(n+m)** traversals! They label every edge and every node

BFS

Solves unweighted MST

Solves shortest path

Solves cycle detection

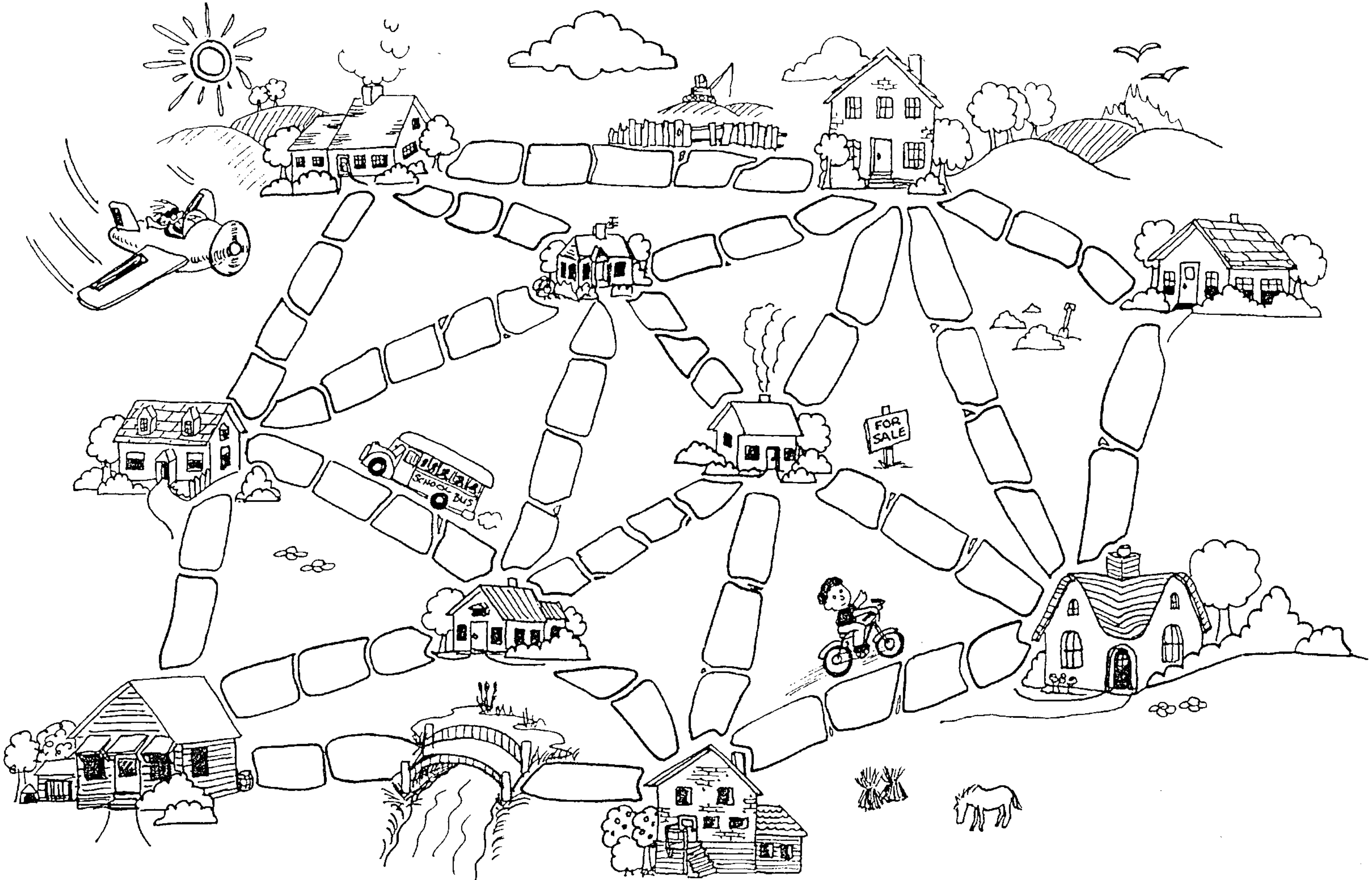
Memory bounded by width

DFS

Solves unweighted MST

Solves cycle detection

Memory bounded by longest path


SCHOOL BUS
FOR SALE

Minimum Spanning Tree Algorithms

Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

Output: A graph G' with the following properties:

- G' is a spanning graph of G
- G' is a tree (connected, acyclic)
- G' has a minimal total weight among all spanning trees