

#24: BTree Analysis + Hashing Intro

March 14, 2018 · Wade Fagen-Ulmschneider

BTree Properties

For a BTree of order **m**:

- 1. All keys within a node are ordered.
- 2. All leaves contain no more than **m-1** nodes.
- 3. All internal nodes have exactly **one more child than keys**.
- 4. Root nodes can be a leaf or have **[2, m]** children.
- 5. All non-root, internal nodes have [ceil(m/2), m] children.
- 6. All leaves are on the same level.

BTree Analysis

The height of the BTree determines maximum number of ______ possible in search data.

...and the height of our structure:

Therefore, the number of seeks is no more than: _____.

...suppose we want to prove this!

BTree Proof #1

In our AVL Analysis, we saw finding an **upper bound** on the height (**h** given **n**, aka **h** = f(n)) is the same as finding a **lower bound** on the keys (**n** given **h**, aka $f^{-1}(n) == g(h)$).

Goal: We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

BTree Strategy:

- 1. Define a function that counts the minimum number of nodes in a BTree of a given order.
 - a. Account for the minimum number of keys per node.
- 2. Proving a minimum number of nodes provides us with an upper-bound for the maximum possible height.

Proof:

1a. The minimum number of <u>nodes</u> for a BTree of order **m** at each level is as follows:

root:

level 1:

level 2:

level 3:

level h:

1b. The minimum total number of <u>nodes</u> is the sum of all levels:

2. The minimum number of keys:

3. Finally, we show an upper-bound on height:

So, how good are BTrees?

Given a BTree of order 101, how much can we store in a tree of height=4?

Minimum:

Maximum:

Hashing

Locker Number	Name
103	
92	
330	
46	
124	

...how might we create this today?

Goals for Understanding Hashing:

1. We will define a **keyspace**, a (mathematical) description of the keys for a set of data.

2. We will define a function used to map the **keyspace** into a small set of integers.



All hash tables consists of three things:

- 1.
- 2.
- 3.

A Perfect Hash Function



... characteristics of this function?

A Second Hash Function



0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

... characteristics of this function?

	CS 225 – Things To Be Doing:
1.	Programming Exam B is ongoing
2.	MP5 has been released; EC deadline is Monday back from break
3.	lab_btree released this week; due Tuesday, March 27 th at 11:59pm
	(That's the Tuesday evening after spring break)
4.	Daily POTDs are ongoing!