

#41: Floyd-Warshall's Algorithm

2 5 April 30, 2018 · Wade Fagen-Ulmschneider

Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alternative to Dijkstra in the presence of negative-weight edges (but <u>not</u> negative weight cycles).

Algorithm Design:

• **Goal:** Find the shortest path from vertex **u** to **v**.



- **Setup:** Create an n×n matrix that maintains the best known path between every pair of vertices:
 - \circ Initialize (u, u) to 0.
 - $\circ~$ Initialize all edges present on the graph to their edge weight.
 - Initialize all other edges to +infinity.

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Α				
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D				

- For every vertex **k**, consider which of the following are shorter:
 - \circ path(u, v) or -
 - \circ path(u, **k**) + path(**k**, v)

Big Idea: _

- Store intermediate results to improve build towards an optimal solution.
- Example application of memoization.

Running Time:

```
Pseudocode for Floyd-Warshall's Algorithm
    FloydWarshall(G):
 1
 2
      Input: G, Graph;
 3
      Output: d, an adjacency matrix of distances between
 4
    all
 5
              vertex pairs
 6
 7
      Let d be an adj. matrix (2d array) initialized to +inf
 8
      foreach (Vertex v : G):
 9
        d[v][v] = 0
10
      foreach (Edge (u, v) : G):
11
        d[u][v] = cost(u, v)
12
13
      foreach (Vertex u : G):
14
        foreach (Vertex v : G):
15
          foreach (Vertex w : G):
16
            if d[u, v] > d[u, w] + d[w, v]:
17
              d[u, v] = d[u, w] + d[w, v]
18
      return d
```

Overview of Graphs:

Implementations

- Edge List
- Adjacency Matrix
- Adjacency List

Traversals

- Breadth First
- Depth First

Minimum Spanning Tree

- Kruskal's Algorithm
- Prim's Algorithm

Shortest Path

- Dijkstra's Algorithm
- Floyd-Warshall's Algorithm

...and this is just the beginning. The journey continues to CS 374!

CS 225 – Things To Be Doing:

- 1. MP7 due tonight (April 30); standard grace period applies.
- 2. Final Exam starts Thursday, May 3