CS 225

Data Structures

April 8 – Graphs

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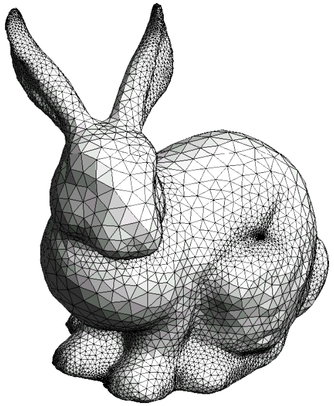
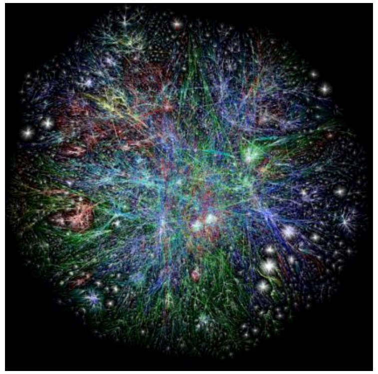
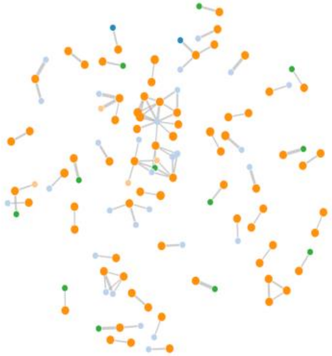
Graphs

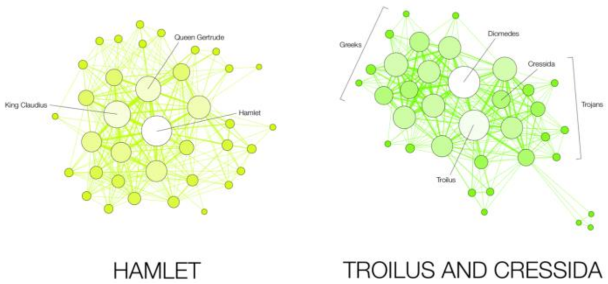


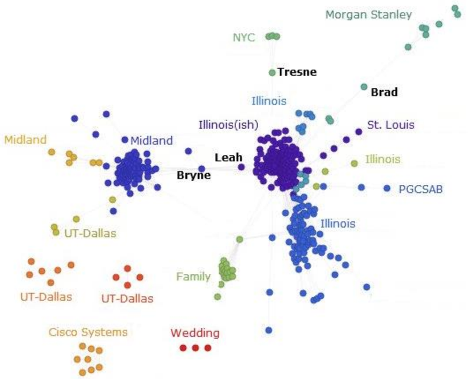
To study all of these structures:

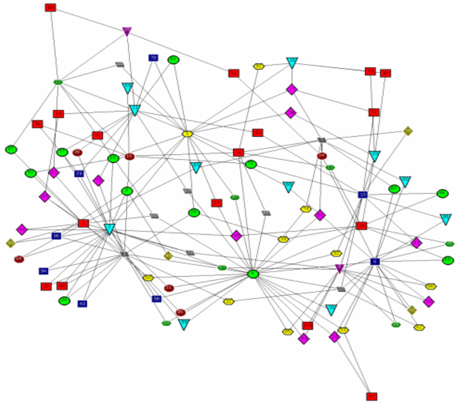
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

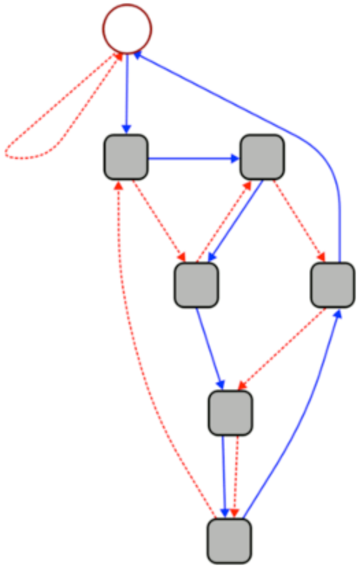


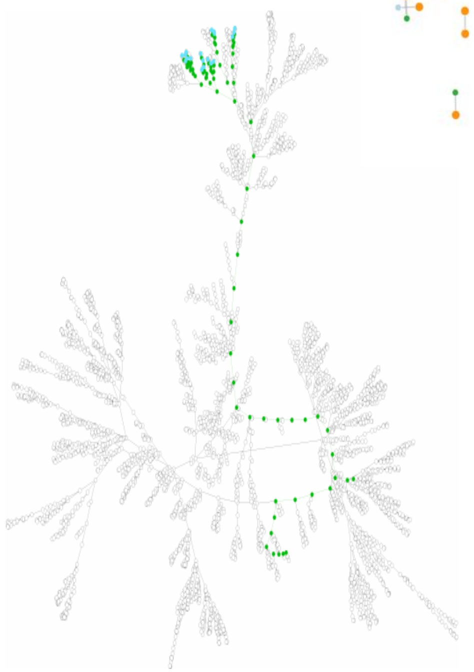










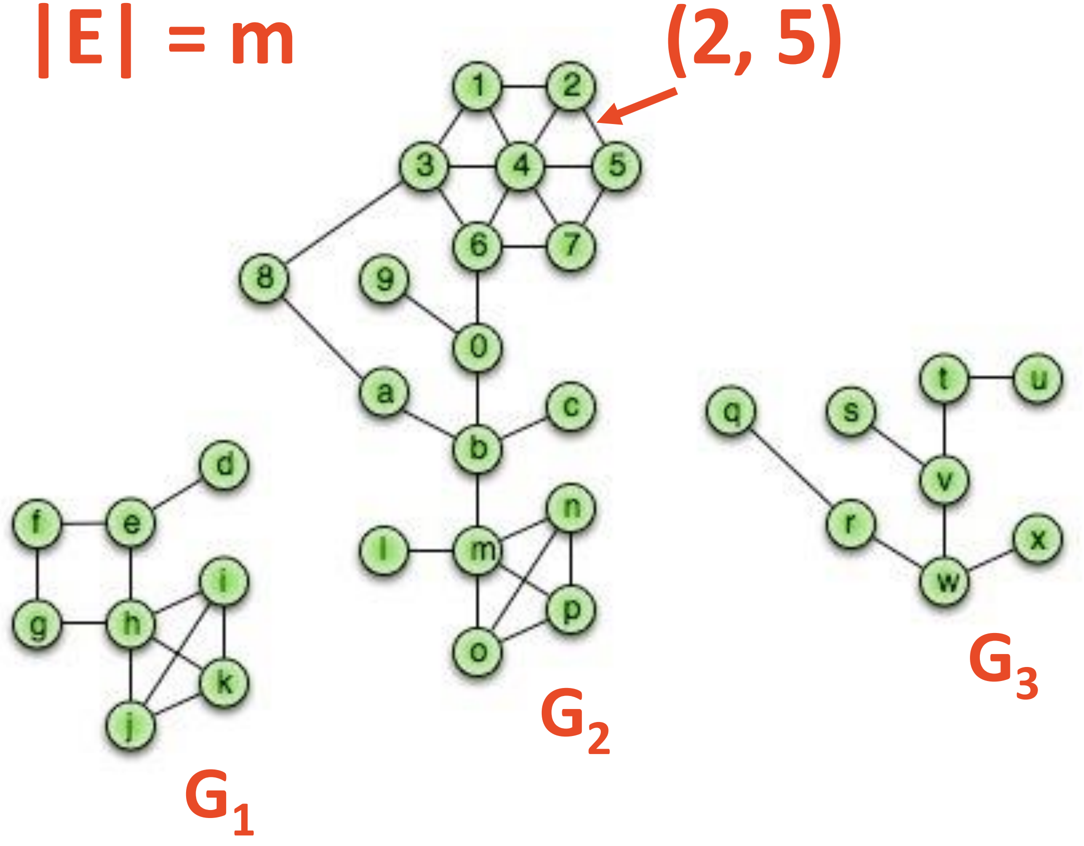


Graph Vocabulary

$G = (V, E)$

$|V| = n$

$|E| = m$



Incident Edges:
$I(v) = \{ (x, v) \text{ in } E \}$

Degree(v): $|I|$

Adjacent Vertices:
$A(v) = \{ x : (x, v) \text{ in } E \}$

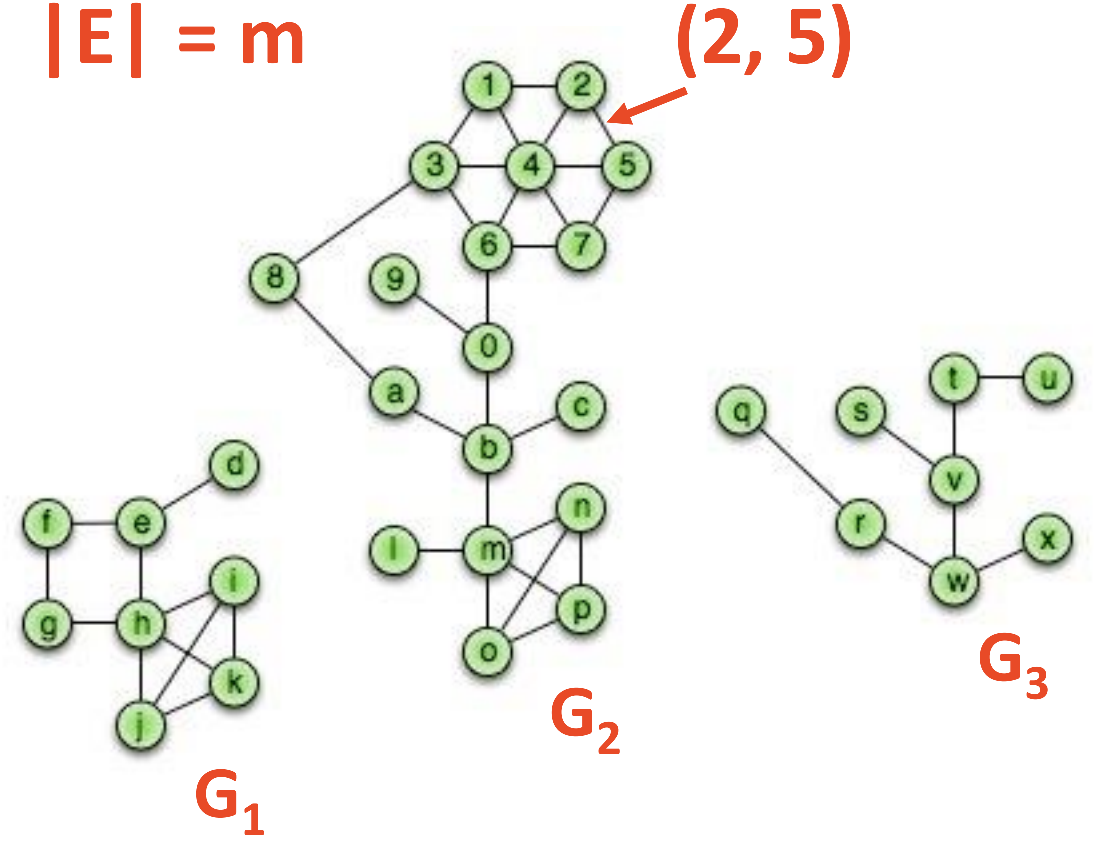
Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

$G = (V, E)$
$|V| = n$
$|E| = m$



Subgraph(G):

$G' = (V', E')$:

$V' \in V$, $E' \in E$, and

$(u, v) \in E \rightarrow u \in V', v \in V'$

Complete subgraph(G)

Connected subgraph(G)

Connected component(G)

Acyclic subgraph(G)

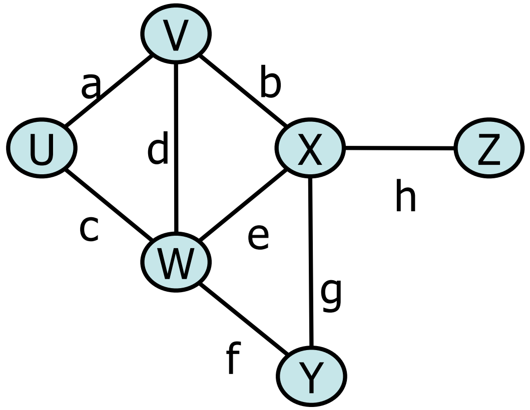
Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges? **Minimum edges:**

Not Connected:

Connected*:



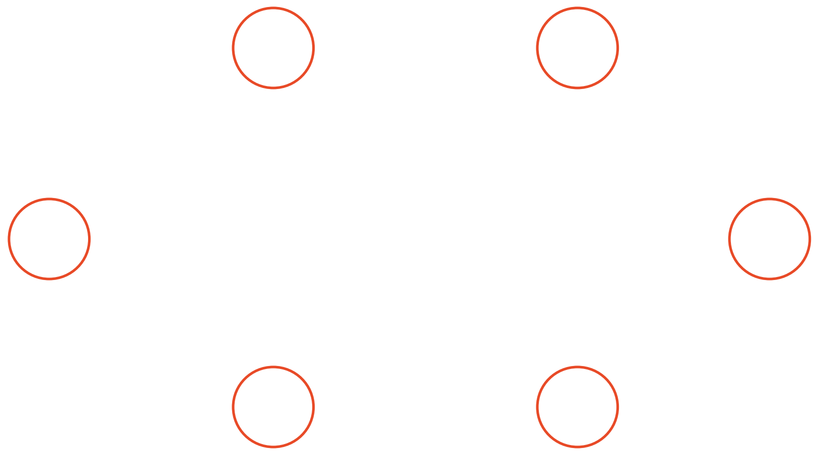
Maximum edges:

Simple:

Not simple:

$$\sum_{v \in V} \deg(v) =$$

Connected Graphs



Proving the size of a minimally connected graph

Theorem:
Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Thm: Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Proof: Consider an arbitrary, minimally connected graph **G=(V, E)**.

Lemma 1: Every connected subgraph of **G** is minimally connected.
(Easy proof by contradiction left for you.)

Suppose |V| = 1:

Definition: A minimally connected graph of 1 vertex has 0 edges.

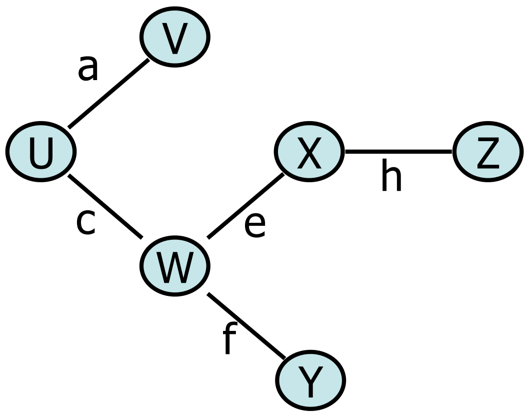
Theorem: |V|-1 edges → 1-1 = 0.

Inductive Hypothesis: For any **j < |V|**, any minimally connected graph of **j** vertices has **j-1** edges.

Suppose |V| > 1:

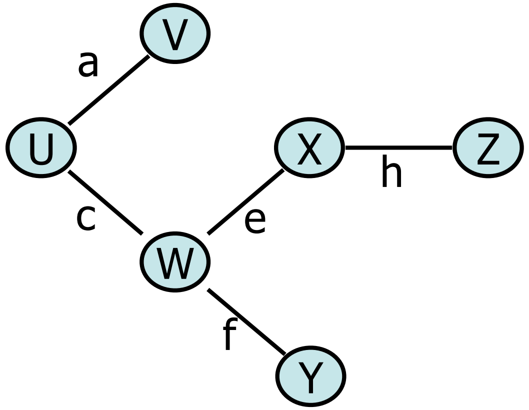
1. Choose any vertex:

2. Partition:



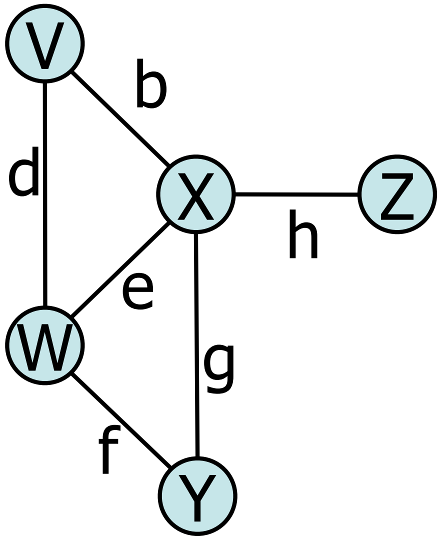
Suppose |V| > 1:

3. Count the edges



Graph ADT

Data:
- Vertices
- Edges
- Some data structure maintaining the structure between vertices and edges.



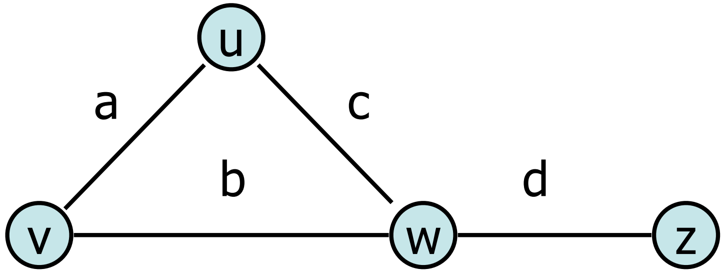
Functions:
- insertVertex(K key);
- insertEdge(Vertex v1, Vertex v2, K key);

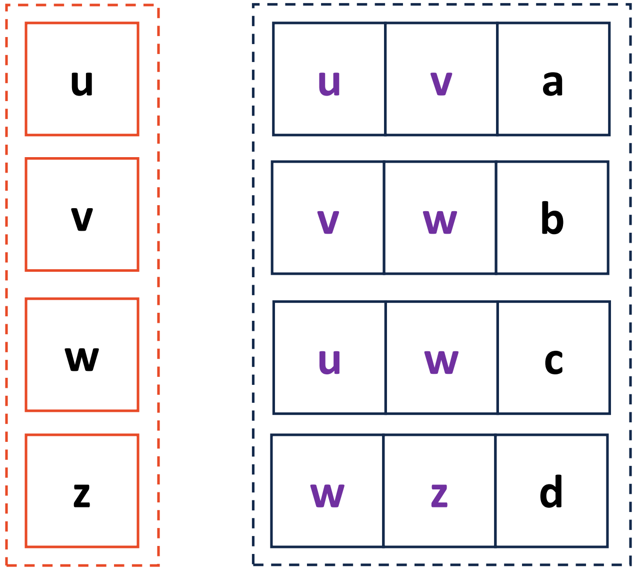
- removeVertex(Vertex v);
- removeEdge(Vertex v1, Vertex v2);

- incidentEdges(Vertex v);
- areAdjacent(Vertex v1, Vertex v2);

- origin(Edge e);
- destination(Edge e);

Graph Implementation: Edge List

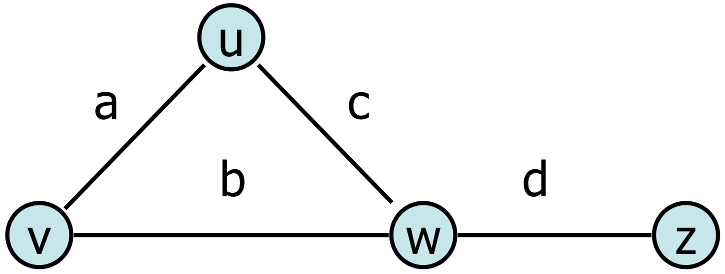


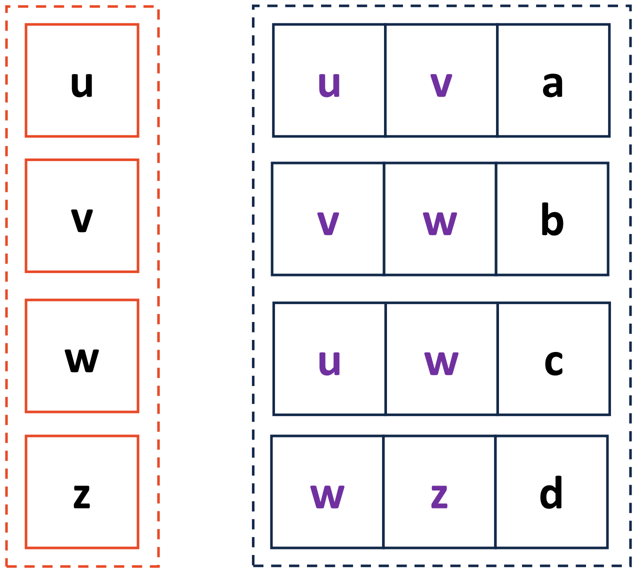


Vertex Collection:

Edge Collection:

Graph Implementation: Edge List

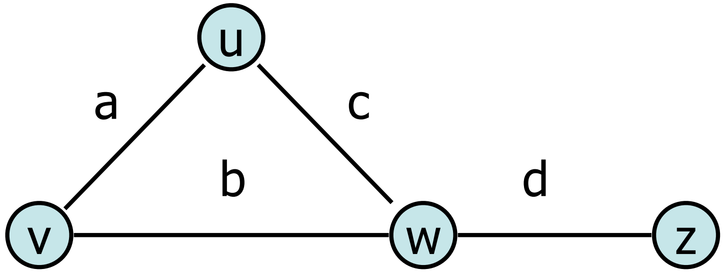


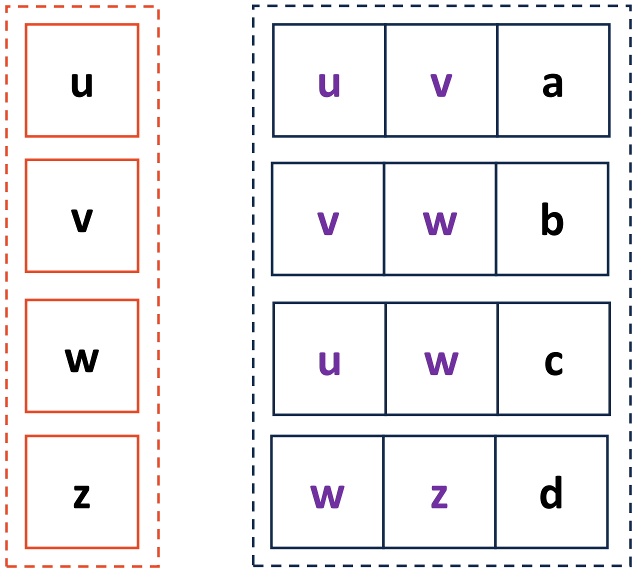


insertVertex(K key):

removeVertex(Vertex v):

Graph Implementation: Edge List



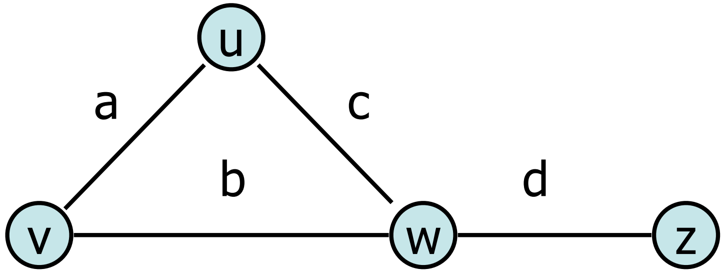


incidentEdges(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

```
G.incidentEdges(v1).contains(v2)
```

Graph Implementation: Edge List

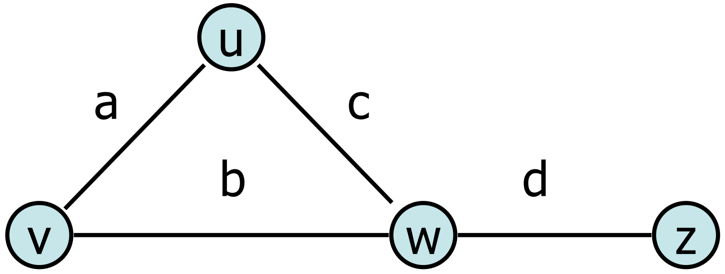


u
v
w
z

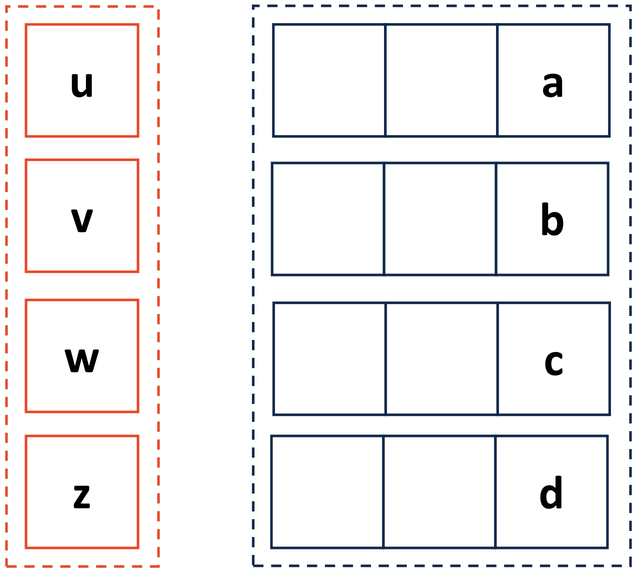
u	v	a
v	w	b
u	w	c
w	z	d

insertEdge(Vertex v1, Vertex v2, K key):

Graph Implementation: Adjacency Matrix



insertVertex(K key);
removeVertex(Vertex v);
areAdjacent(Vertex v1, Vertex v2);
incidentEdges(Vertex v);



	u	v	w	z
u				
v				
w				
z				