



# CS 225

## Data Structures

*October 28 – Hashing Analysis*

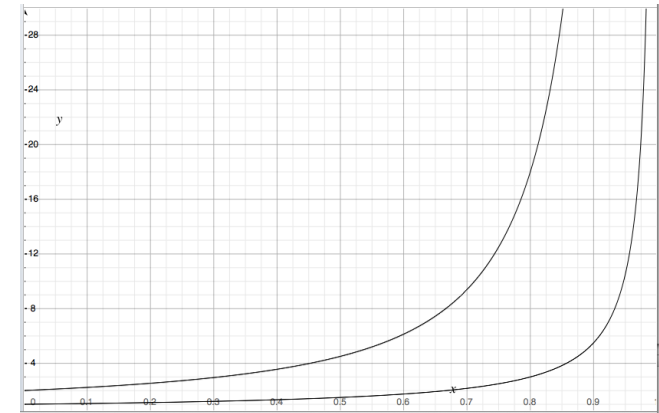
*G Carl Evans*

# Running Times

*The expected number of probes for find(key) under SUHA*

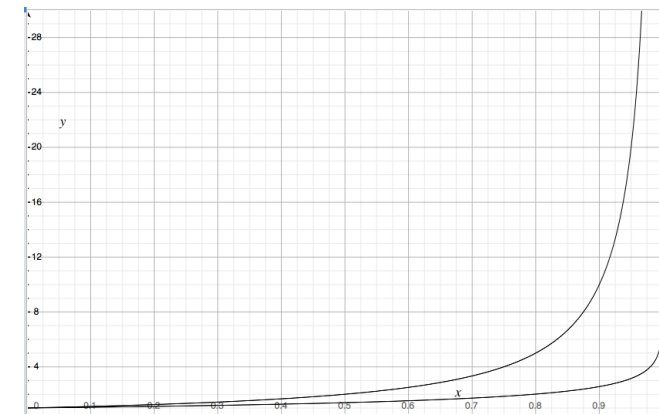
## Linear Probing:

- Successful:  $\frac{1}{2}(1 + \frac{1}{1-\alpha})$
- Unsuccessful:  $\frac{1}{2}(1 + \frac{1}{1-\alpha})^2$



## Double Hashing:

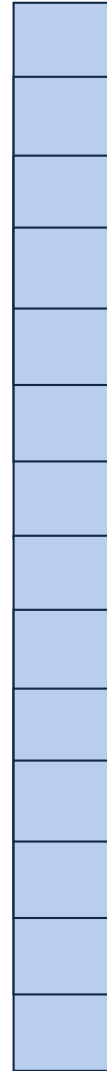
- Successful:  $\frac{1}{\alpha} * \ln(\frac{1}{1-\alpha})$
- Unsuccessful:  $\frac{1}{1-\alpha}$





# ReHashing

What if the array fills?





**Which collision resolution strategy is better?**

- Big Records:
- Structure Speed:

**What structure do hash tables replace?**

**What constraint exists on hashing that doesn't exist with BSTs?**

**Why talk about BSTs at all?**

# Running Times

	Hash Table	AVL	Linked List
<b>Find</b>	SUHA:  Worst Case:		
<b>Insert</b>	SUHA:  Worst Case:		
<b>Storage Space</b>			



# std data structures

**std::map**



# std data structures

## **std::map**

::operator[]

::insert

::erase

::lower\_bound(key) → Iterator to first element  $\leq$  key

::upper\_bound(key) → Iterator to first element  $>$  key



# std data structures

## **std::unordered\_map**

::operator[]

::insert

::erase

~~— ::lower\_bound(key) → Iterator to first element  $\leq$  key~~

~~— ::upper\_bound(key) → Iterator to first element  $>$  key~~



# std data structures

## **std::unordered\_map**

::operator[]

::insert

::erase

~~::lower\_bound(key) → Iterator to first element  $\leq$  key~~

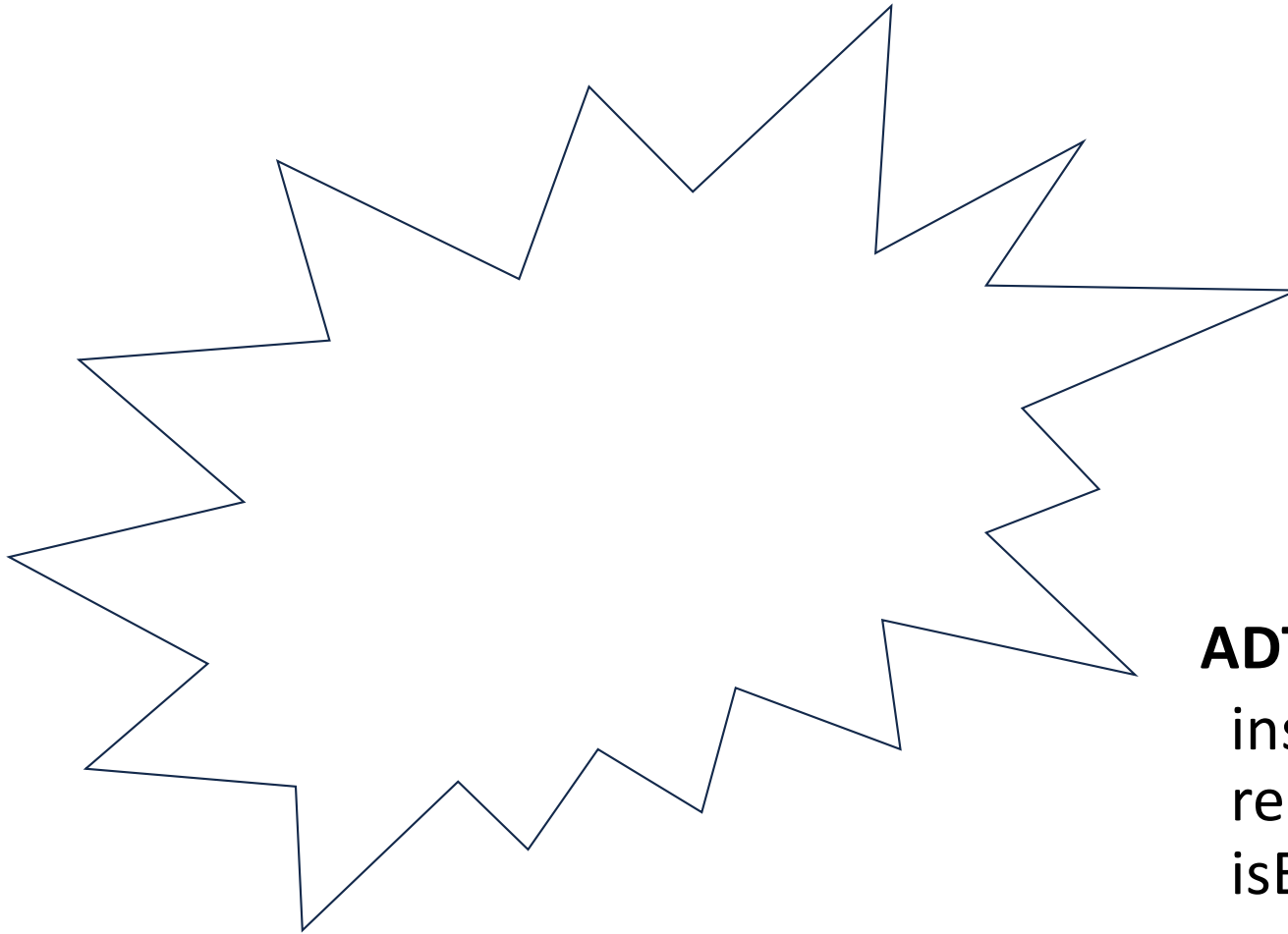
~~::upper\_bound(key) → Iterator to first element  $>$  key~~

::load\_factor()

::max\_load\_factor(ml) → Sets the max load factor



# Secret, Mystery Data Structure



**ADT:**

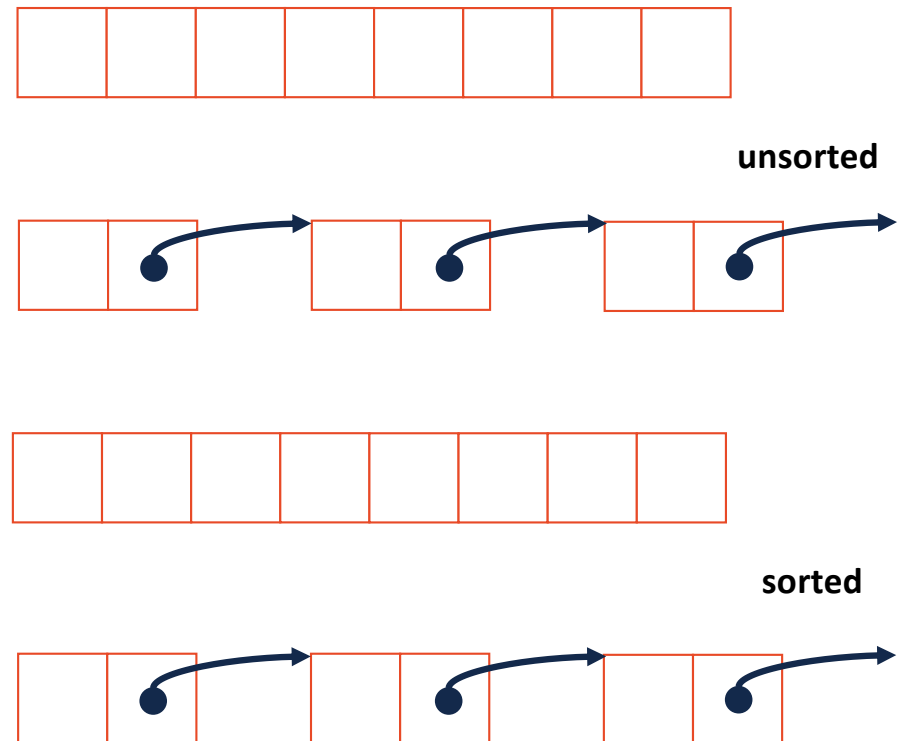
insert

remove

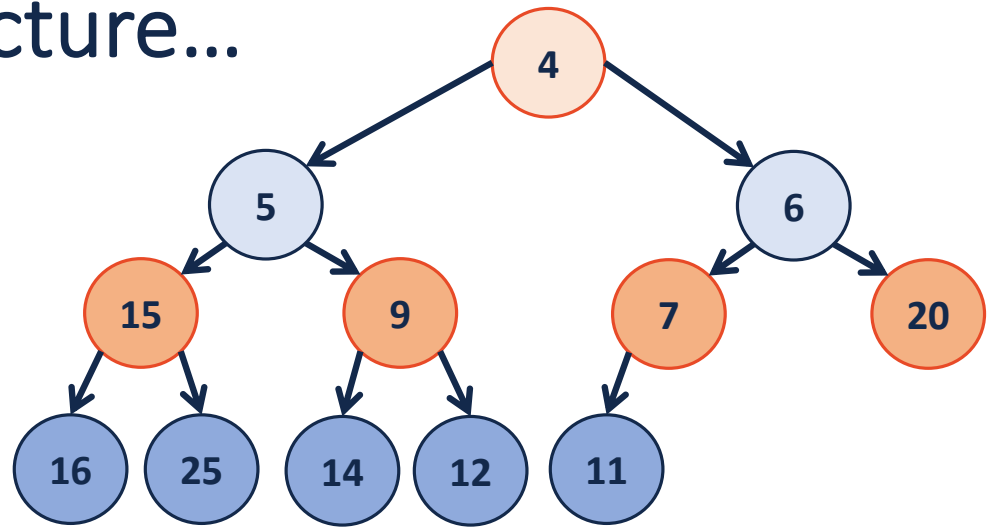
isEmpty

# Priority Queue Implementation

insert	removeMin
$O(n)$	$O(n)$
$O(1)$	$O(n)$
$O(\lg(n))$	$O(1)$
$O(\lg(n))$	$O(1)$



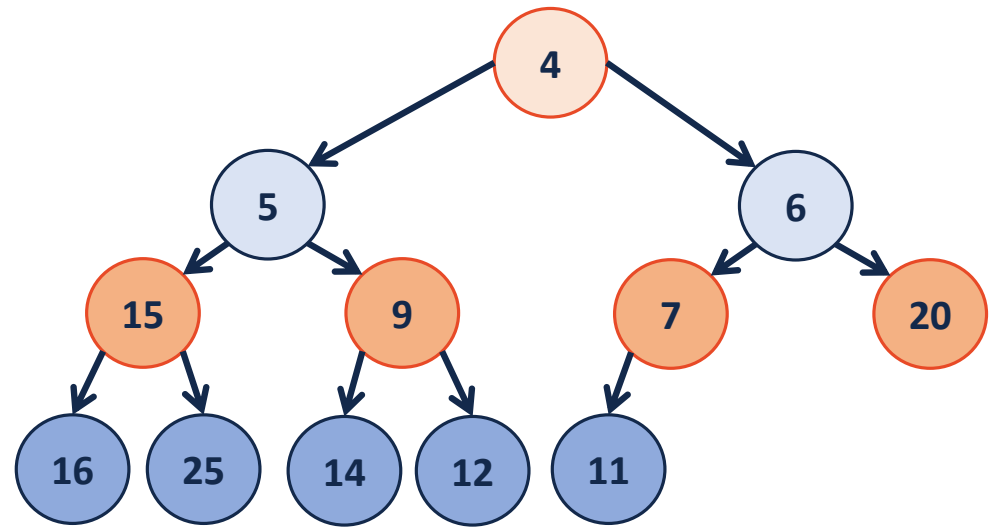
Another possibly structure...



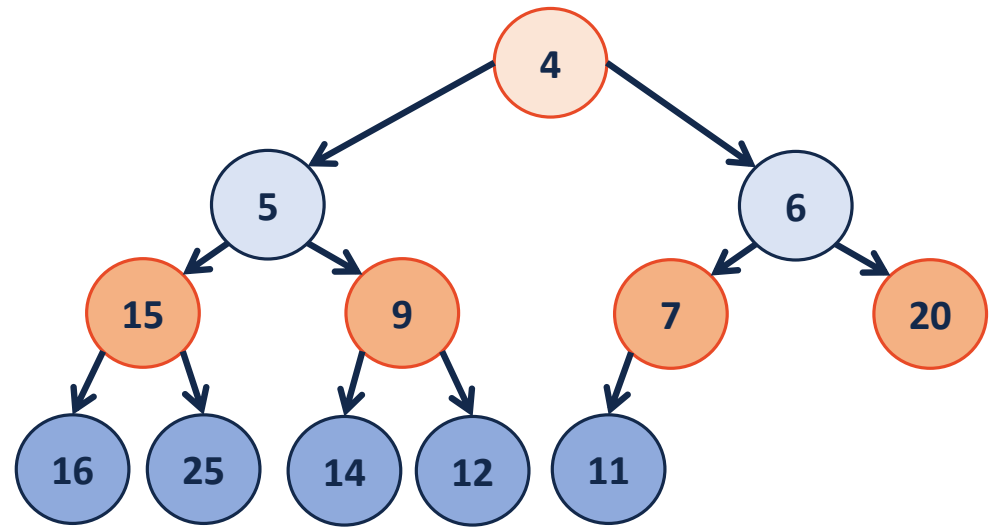
# (min)Heap

A complete binary tree  $T$  is a min-heap if:

- $T = \{\}$  or
- $T = \{r, T_L, T_R\}$ , where  $r$  is less than the roots of  $\{T_L, T_R\}$  and  $\{T_L, T_R\}$  are min-heaps.

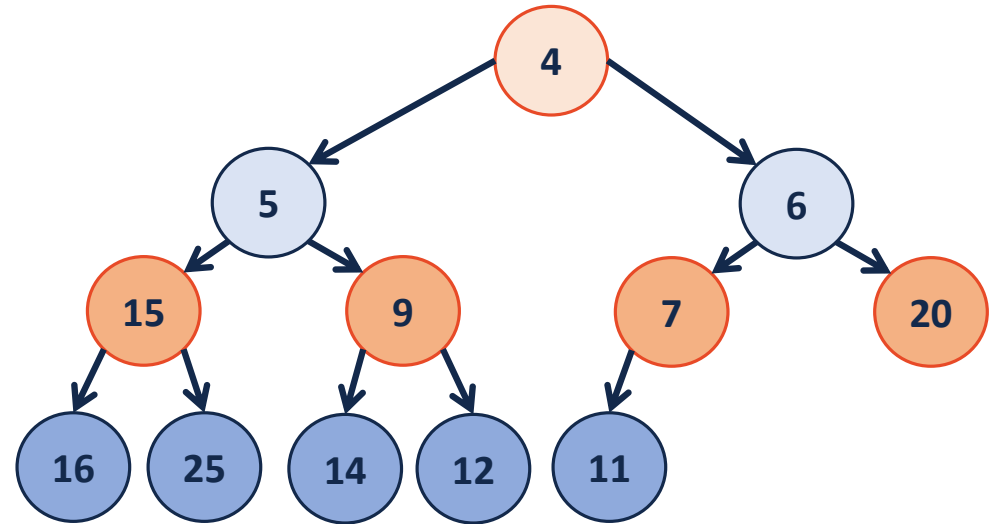


# (min)Heap



4	5	6	15	9	7	20	16	25	14	12	11			
---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

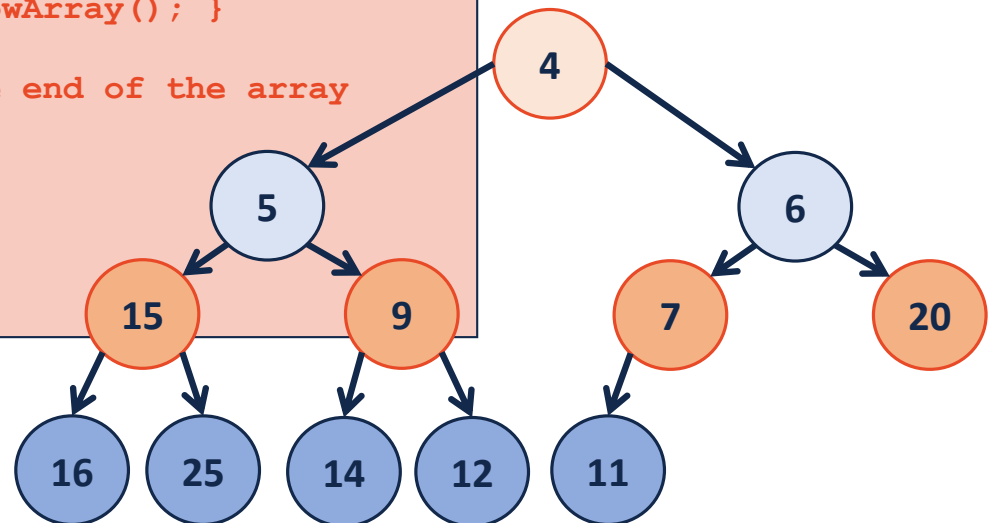
insert



	4	5	6	15	9	7	20	16	25	14	12	11			
--	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

# insert

```
1 template <class T>
2 void Heap<T>::_insert(const T & key) {
3     // Check to ensure there's space to insert an element
4     // ...if not, grow the array
5     if ( size_ == capacity_ ) { _growArray(); }
6
7     // Insert the new element at the end of the array
8     item_[++size] = key;
9
10    // Restore the heap property
11    _heapifyUp(size);
12 }
```





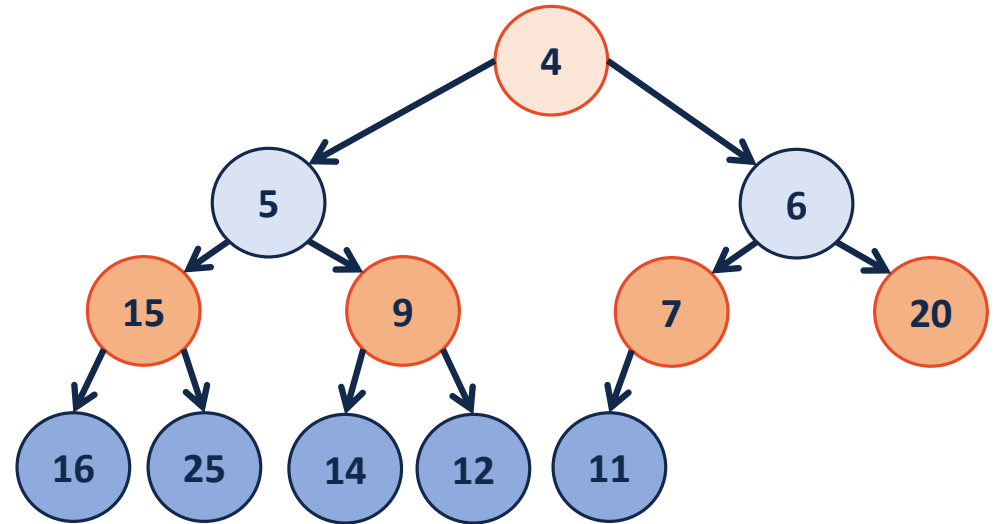


# insert - heapifyUp

```
1  template <class T>
2  void Heap<T>::_insert(const T & key) {
3      // Check to ensure there's space to insert an element
4      // ...if not, grow the array
5      if ( size_ == capacity_ ) { _growArray(); }
6
7      // Insert the new element at the end of the array
8      item_[++size] = key;
9
10     // Restore the heap property
11     _heapifyUp(size);
12 }
```

```
1  template <class T>
2  void Heap<T>::_heapifyUp( _____ ) {
3      if ( index > _____ ) {
4          if ( item_[index] < item_[ parent(index) ] ) {
5              std::swap( item_[index], item_[ parent(index) ] );
6              _heapifyUp( _____ );
7          }
8      }
9  }
```

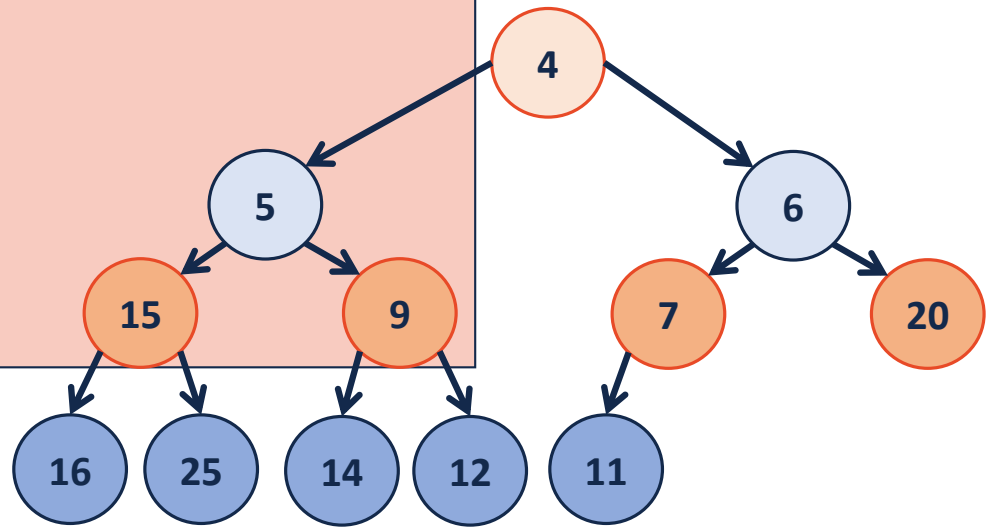
# removeMin



	4	5	6	15	9	7	20	16	25	14	12	11			
--	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

# removeMin

```
1 template <class T>
2 void Heap<T>::_removeMin() {
3     // Swap with the last value
4     T minValue = item_[1];
5     item_[1] = item_[size_];
6     size--;
7
8     // Restore the heap property
9     heapifyDown();
10
11     // Return the minimum value
12     return minValue;
13 }
```



# removeMin - heapifyDown

```
1  template <class T>
2  void Heap<T>::_removeMin() {
3      // Swap with the last value
4      T minValue = item_[1];
5      item_[1] = item_[size_];
6      size--;
7
8      // Restore the heap property
9      _heapifyDown();
10
11     // Return the minimum value
12     return minValue;
13 }
```

```
1  template <class T>
2  void Heap<T>::_heapifyDown(int index) {
3      if ( !_isLeaf(index) ) {
4          T minChildIndex = _minChild(index);
5          if ( item_[index] > item_[minChildIndex] ) {
6              std::swap( item_[index], item_[minChildIndex] );
7              _heapifyDown( minChildIndex );
8          }
9      }
10 }
```

# Array Abstractions

