CS 225

Data Structures

February 28 – BST Remove

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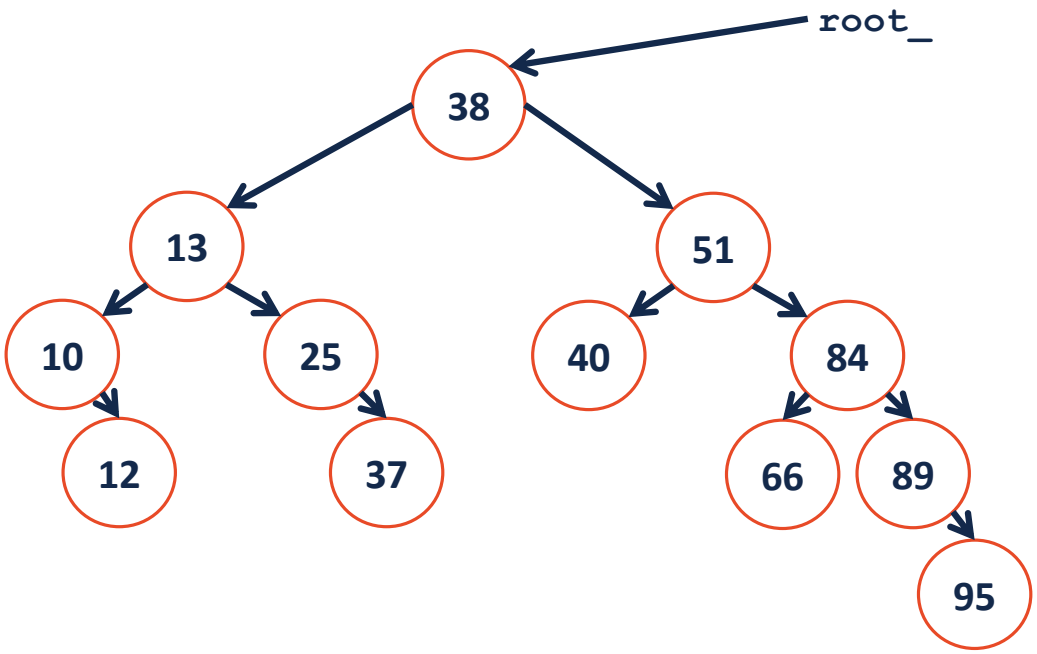
```
1  template<typename K, typename V>
2
3  void BST::_insert(TreeNode *& root, K & key, V & value) {
4    TreeNode * t = _find(root, key);
5    t = new TreeNode(key, value);
6  }
```



```
1  template<typename K, typename V>
2  _____________________________ _remove(TreeNode *& root, const K & key) {
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26 }
```
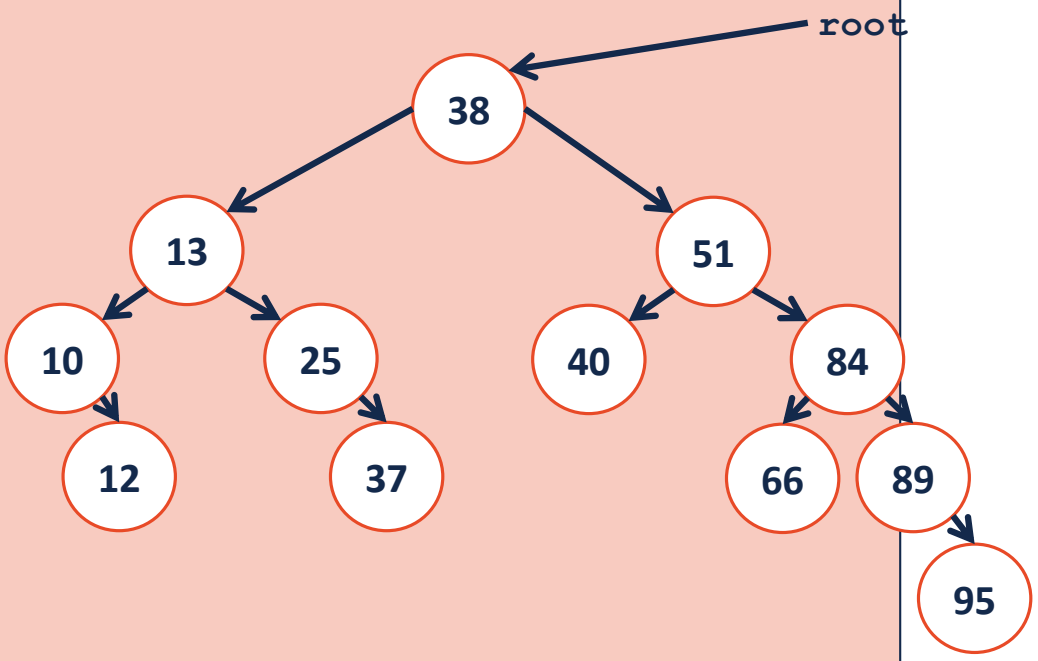
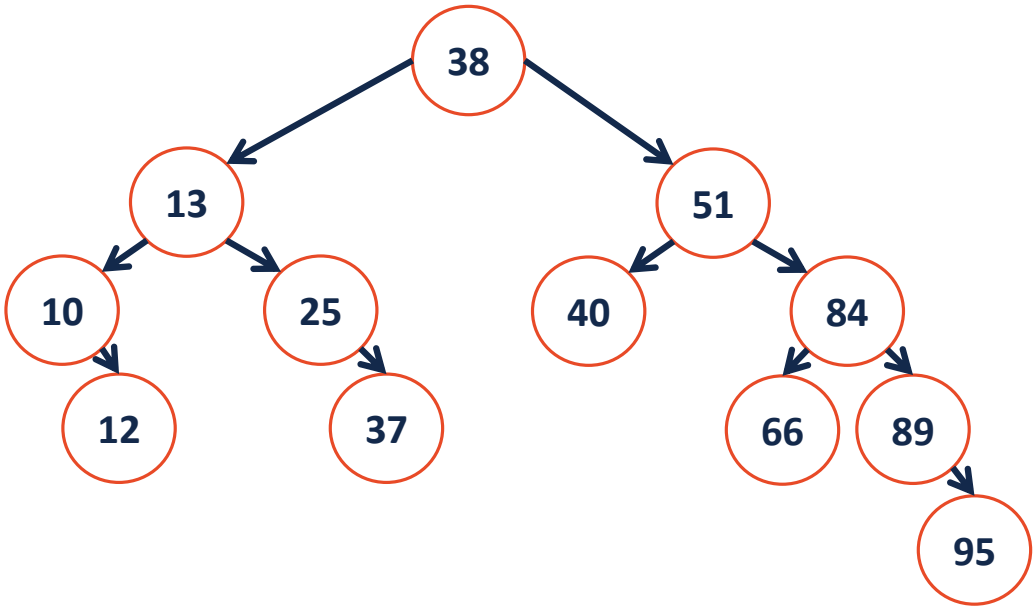
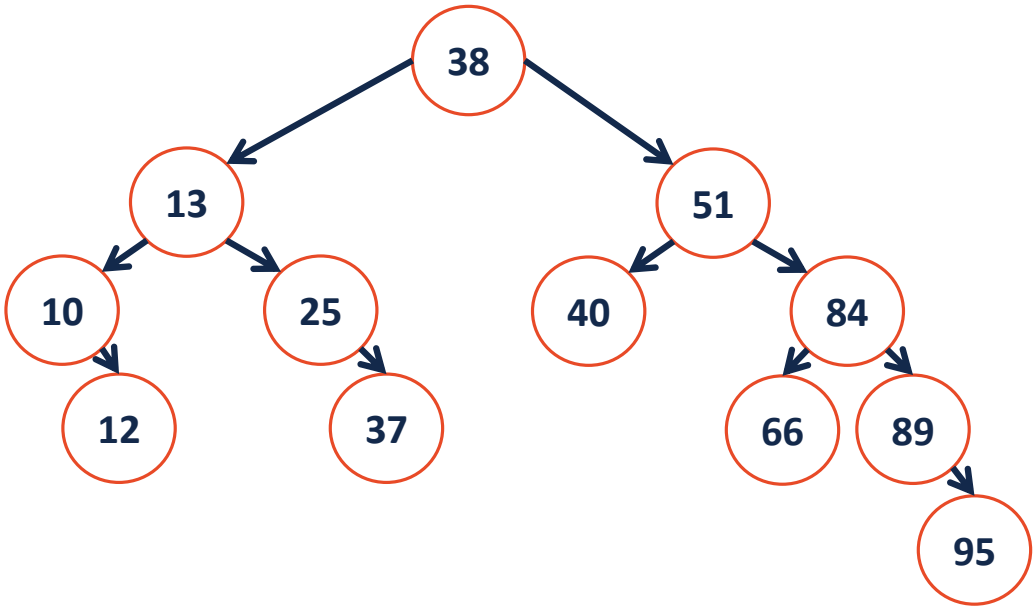
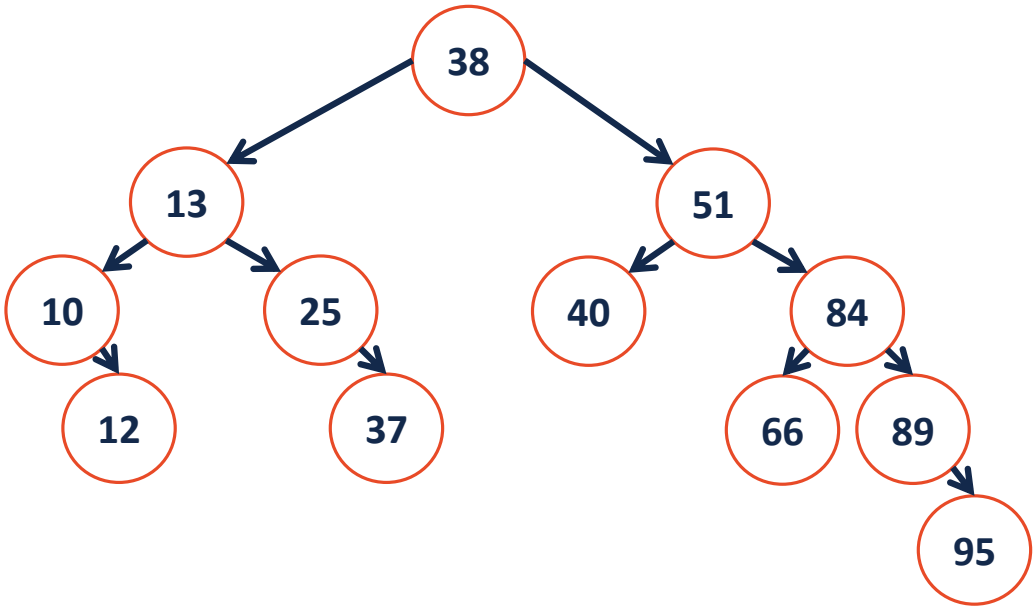





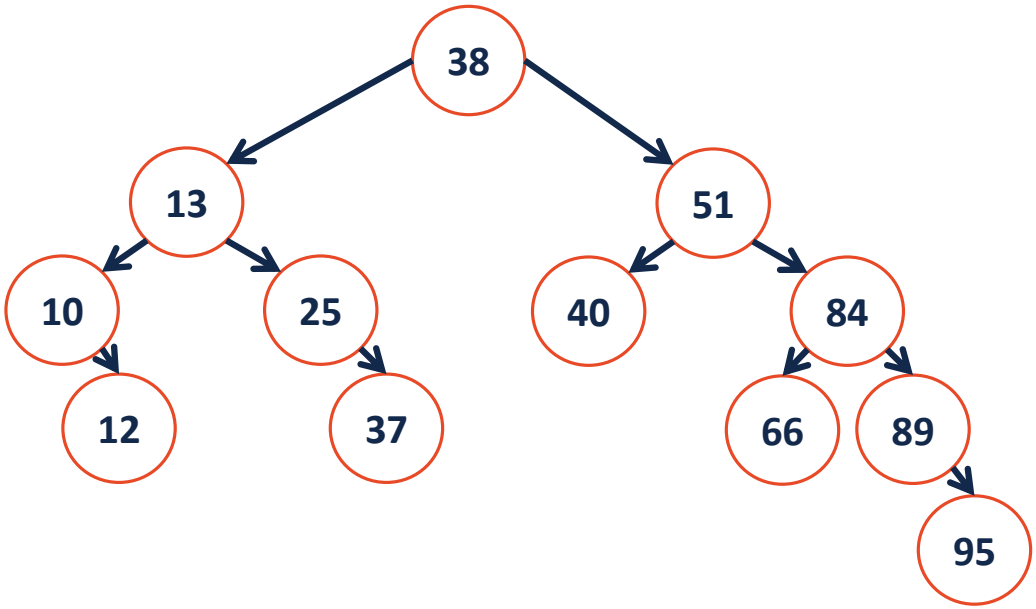

```
remove(40);
```




```
remove(25);
```




```
remove(10);
```




```
remove(13);
```

BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	

BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: **O(h)**.

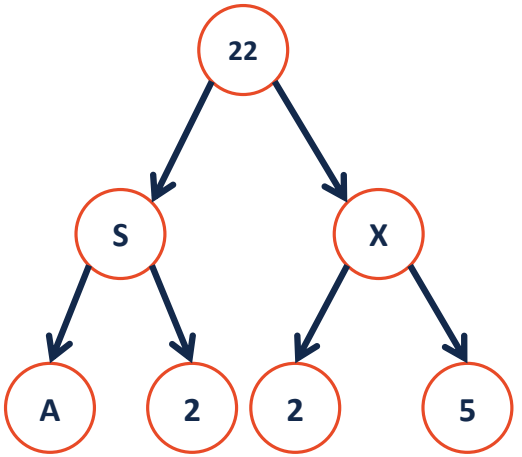
…what is this in terms of **n**, the amount of data?

We need a relationship between **h** and **n**:

$f(h) \leq n \leq g(h)$

BST Analysis

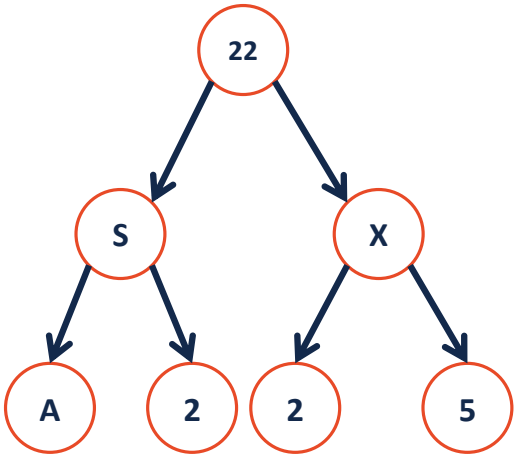
Q: What is the maximum number of nodes in a tree of height **h**?



BST Analysis

Q: What is the minimum number of nodes in a tree of height **h**?

What is the maximum height for a tree of **n** nodes?



BST Analysis

Therefore, for all BST:

Lower bound:

U**pper bound:**

BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?

BST Analysis

Q: How many different ways are there to insert keys into a BST?

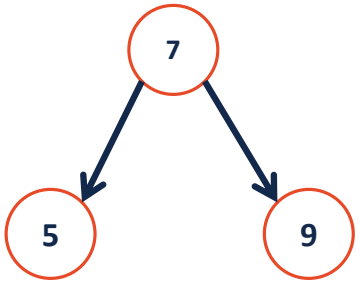
Q: What is the average height of all the arrangements?

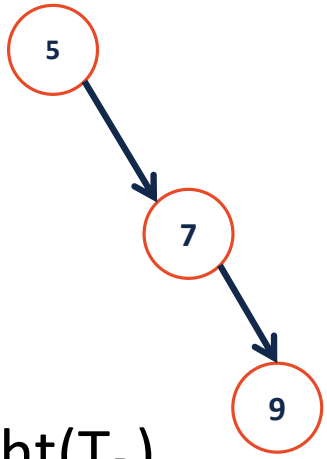
BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
find				
insert				
delete				
traverse				

Height-Balanced Tree

What tree makes you happier?





Height balance: $b = height(T_L) - height(T_R)$

A tree is height balanced if: