CS 225

#32: Dijkstra's Algorithm

April 26, 2020 · *G Carl Evans*

Best Running Time of MST Algorithms (so far):

Kruskal's MST	Prim's MST
O(n + m lg(n))	**O(n lg(n) + m lg(n))**

...however, we know that, for an MST algorithm, the graph must be at least minimally connected. This means there must be at least one edge on every vertex. The number of edges must be:

$$n - 1 \leq m \leq n(n-1) / 2$$

...or in terms of big-O...

$$O(n) \leq O(m) \leq O(n^2)$$

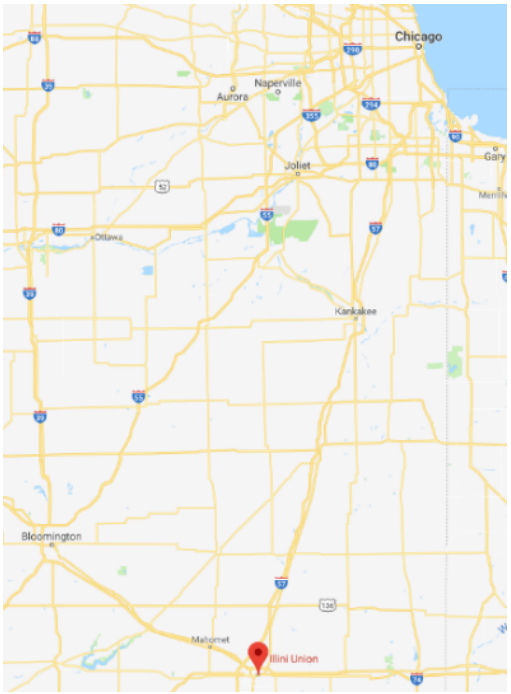
Using the fact that they are connected:

Kruskal's MST	Prim's MST
Sparse Graph (m = n):	**Sparse Graph (m = n):**
Dense Graph (m = n²):	**Dense Graph (m = n2):**
All Graphs:	**All Graphs:**

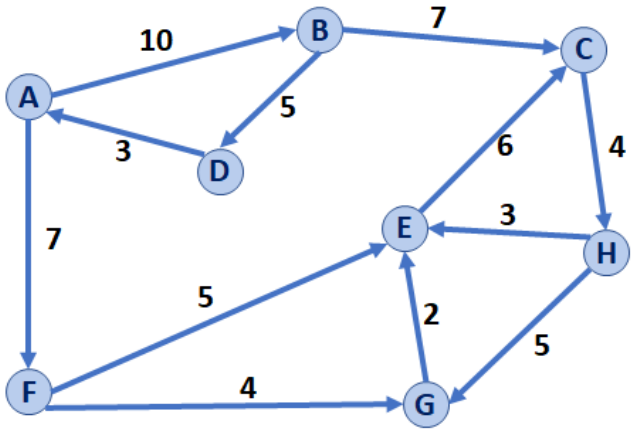
Q: Suppose we built a new heap that optimized the decrease-key operation, where decreasing the value of a key in a heap updates the heap in amortized constant time, or O(1)*. How does that change Prim's Algorithm runtime?

Kruskal's MST	Prim's MST

Shortest Path Home:



Dijkstra's Algorithm (Single Source Shortest Path)



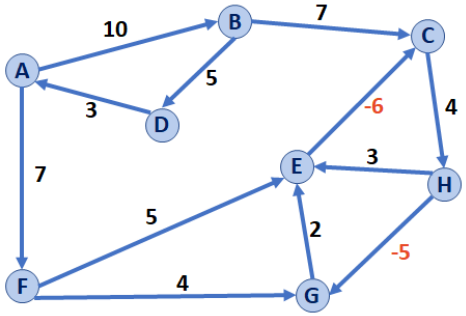
Dijkstra's Algorithm Overview:

- The overall logic is the same as Prim's Algorithm
- We will modify the code in only two places – both involving the update to the distance metric.
- The result is a directed acyclic graph or DAG

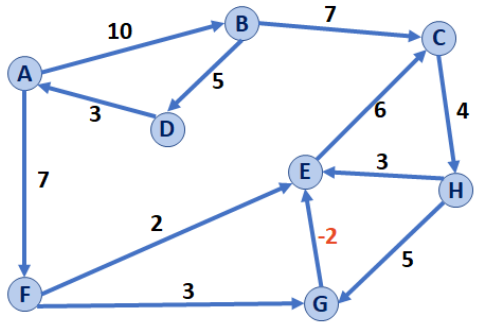
Pseudocode for Dijkstra's SSSP Algorithm

```
1  DijkstraSSSP(G, s):
2    Input: G, Graph;
3           s, vertex in G, starting vertex of algorithm
4    Output: T, DAG w/ shortest paths (and distances) to s
5
6    foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9    d[s] = 0
10
11   PriorityQueue Q   // min distance, defined by d[v]
12   Q.buildHeap(G.vertices())
13   Graph T           // "labeled set"
14
15   repeat n times:
16     Vertex m = Q.removeMin()
17     T.add(m)
18     foreach (Vertex v : neighbors of m not in T):
19       if _____________________________ < d[v]:
20         d[v] = _____________________________
21         p[v] = m
22
23   return T
```

Dijkstra: What if we have a negative-weight cycle?



Dijkstra: What if we have a minimum-weight edge, without having a negative-weight cycle?



Dijkstra makes an assumption:

Dijkstra: What is the running time?

CS 225 – Things To Be Doing:
1. **mp_mazes due today!** 2. Practice for last exam on PrairieLearn