CS 225

Data Structures

October 19 – Intro Kd-trees and Btrees

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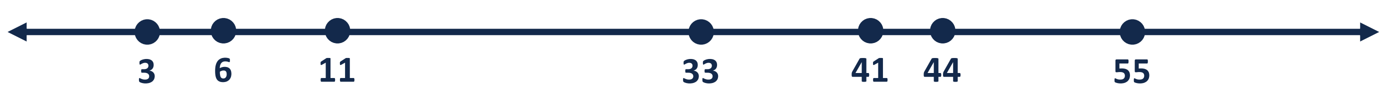
Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

…what points fall in [11, 42]?

Ex:



Range-based Searches

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Tree construction:

Range-based Searches

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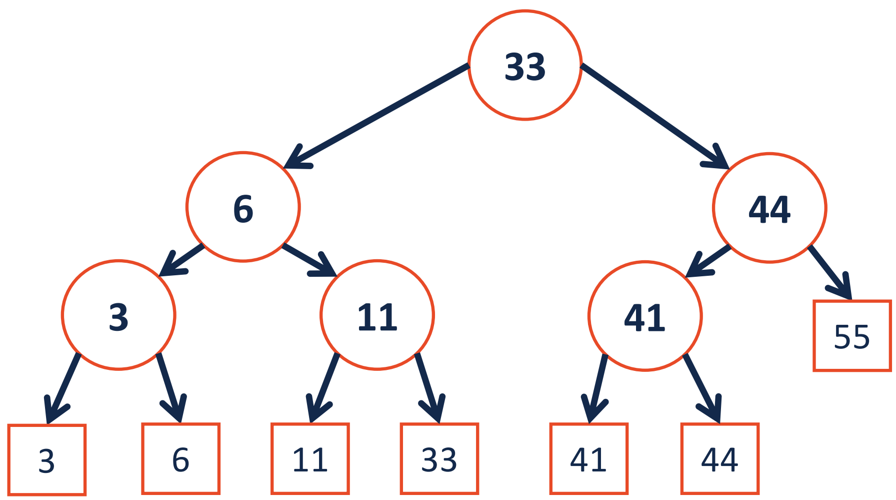


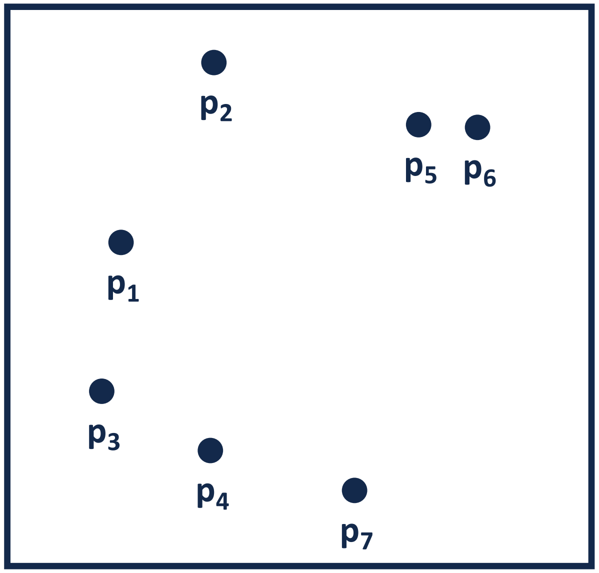
Range-based Searches

Consider points in 2D: $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$.

Q: What points are in the rectangle:

$[\ (x_1, y_1),\ (x_2, y_2)\ ]$?

Q: What is the nearest point to (x_1, y_1)?



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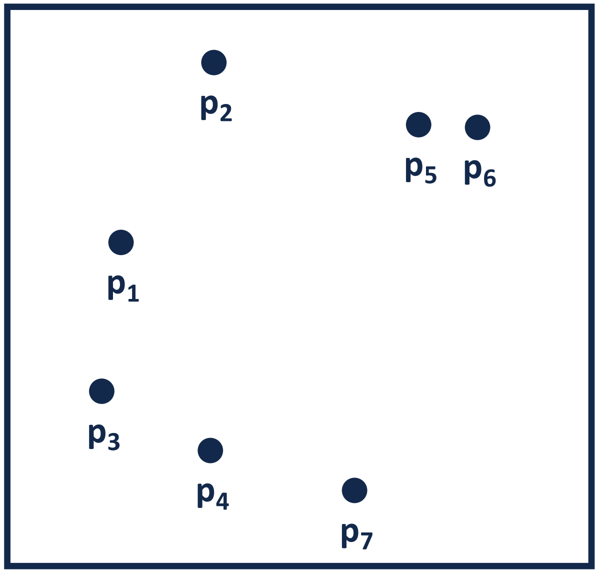
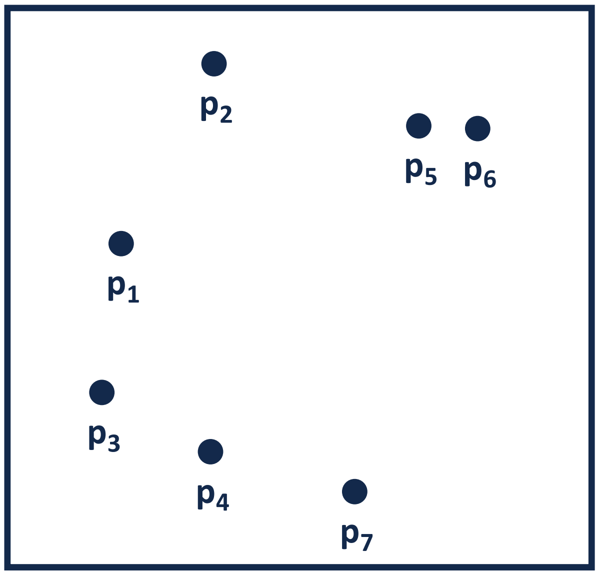
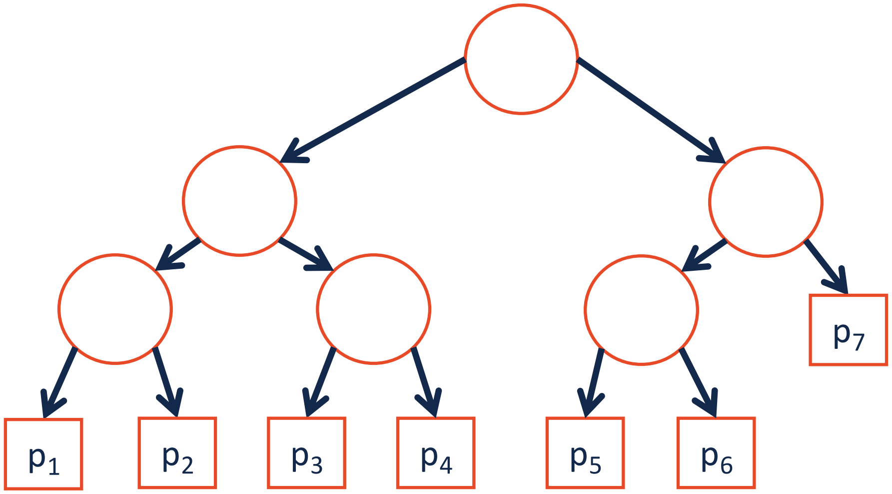
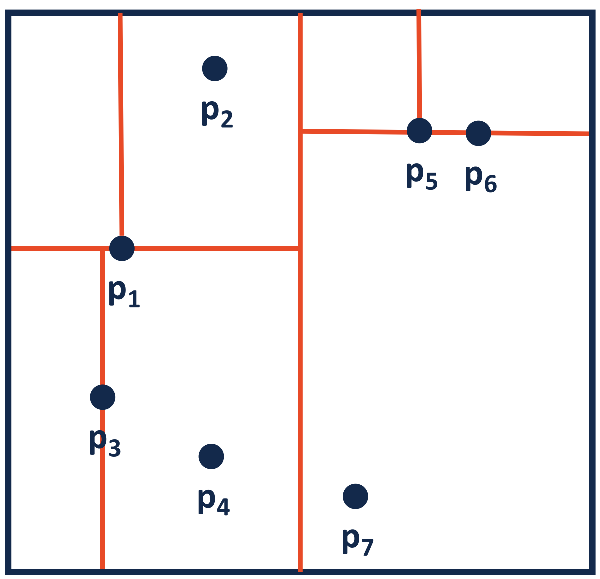


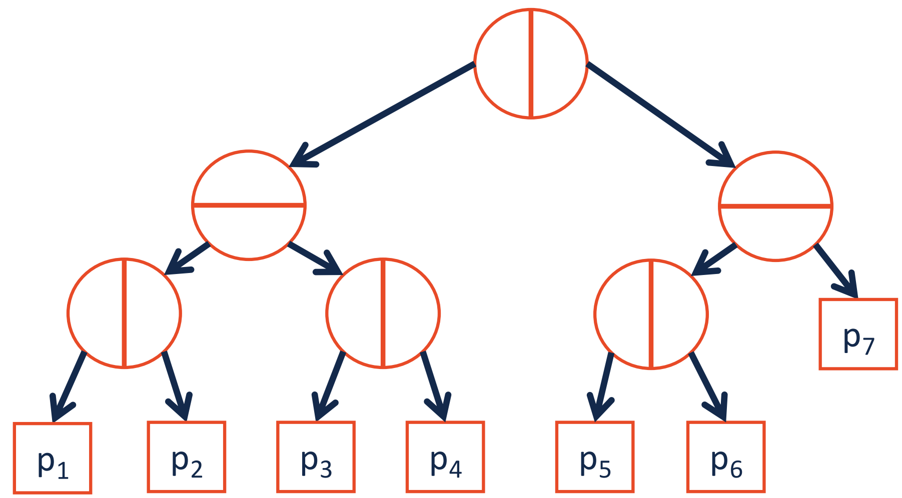
Range-based Searches





kD-Trees





B-Trees

Q: Can we always fit our data in main memory?

Q: Where else can we keep our data?

However, Our big-O has assumed uniform time for all operations.

Vast Differences in Time

A **3GHz** CPU performs 3m operations in _______.

<u>Old Argument:</u> "Disk Storage is Slow"

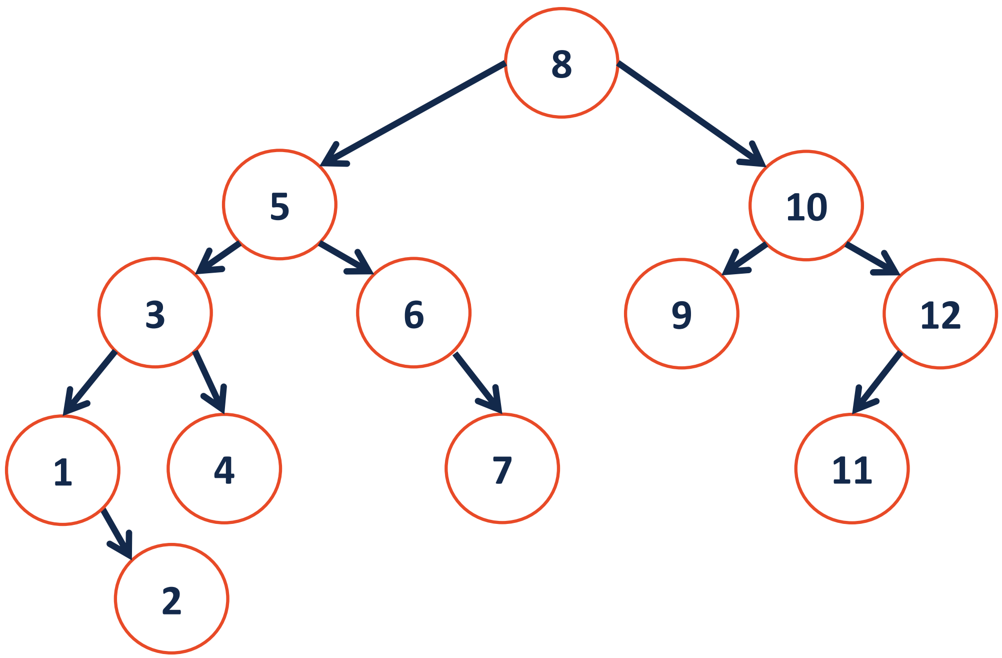
- Bleeding-edge storage is pretty fast:

    SSD

- Large Disks (25 TB+) still have slow throughout:

<u>New Argument: </u>"The Cloud is Slow!"

AVLs on Disk



Real Application

Imagine storing TicTok profiles for everyone in the US:

How many records?

How much data in total?

How deep is the AVL tree?

BTree Motivations

Knowing that we have large seek times for data, we want to:

BTree (of order m)

-3	8	23	25	31	42	43	55

m=9

Goal: Minimize the number of reads!

Build a tree that uses ______________________ / node

[1 network packet]

[1 disk block]

BTree Insertion

A **BTrees** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** keys.

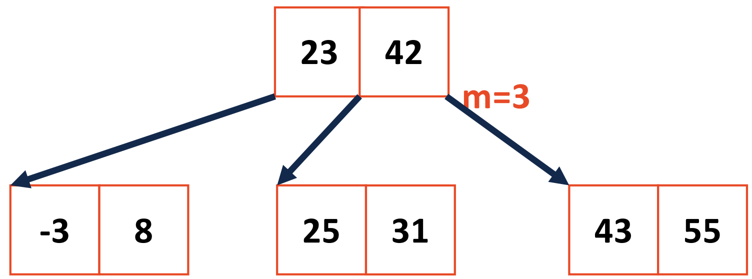
m=5

BTree Insertion

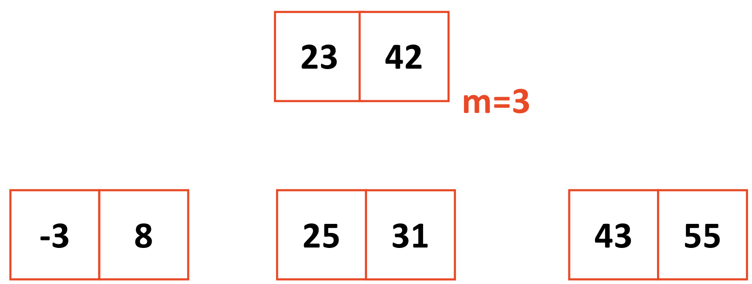
When a BTree node reaches **m** keys:

m=5

BTree Recursive Insert



BTree Recursive Insert



BTree Visualization/Tool

https://www.cs.usfca.edu/~galles/visualization/BTree.html

Btree Properties

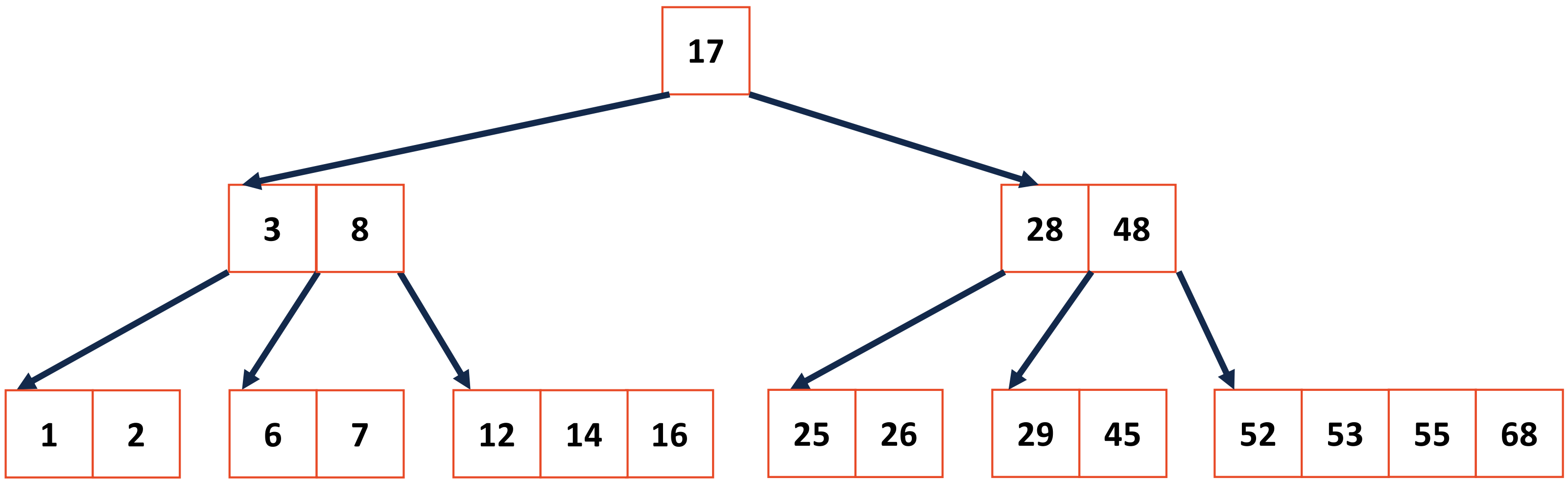
A **BTrees** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** keys.

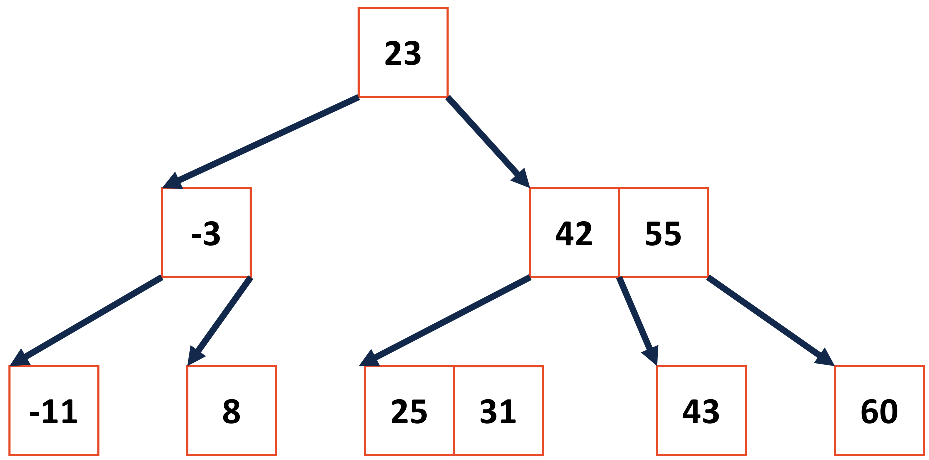
- All internal nodes have exactly **one more child than keys**
- Root nodes can be a leaf or have **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children.

- All leaves are on the same level

BTree



BTree Search



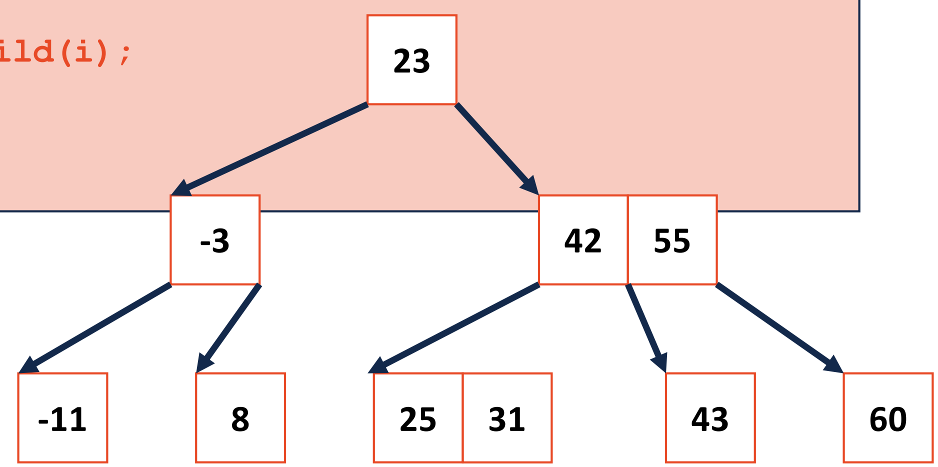
BTree Search

```
bool Btree::_exists(BTreeNode & node, const K & key) {

  unsigned i;
  for ( i = 0; i < node.keys_ct_ && key < node.keys_[i]; i++) { }

  if ( i < node.keys_ct_ && key == node.keys_[i] ) {
    return true;
  }

  if ( node.isLeaf() ) {
    return false;
  } else {
    BTreeNode nextChild = node._fetchChild(i);
    return _exists(nextChild, key);
  }
}
```



BTree Analysis

The height of the BTree determines maximum number of _____________ possible in search data.

…and the height of the structure is: _______________.

Therefore: The number of seeks is no more than __________.

…suppose we want to prove this!

BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given **n**) is the same as finding a lower bound on the nodes (given **h**).

We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

BTree Analysis

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.

BTree Analysis

The minimum number of **nodes** for a BTree of order m **at each level**:

root:

level 1:

level 2:

level 3:

…

level h:

BTree Analysis

The **total number of nodes** is the sum of all of the levels:

BTree Analysis

The **total number of keys**:

BTree Analysis

The **smallest total number of keys** is:

So an inequality about **n**, the total number of keys:

Solving for **h**, since **h** is the number of seek operations:

BTree Analysis

Given **m=101**, a tree of height **h=4** has:

Minimum Keys:

Maximum Keys: