



CS 225

Data Structures

March 10 – BST Remove

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MP Extensions and Redo Clarification

- There are two free 24-hour extensions for your MPs to use these all you need to do is email cs225admin@lists.cs.illinois.edu and request an extension. If you do so you will still get the normal deadline final grade. This grade will be replaced by your extension grad which will be done on the commit you have 24 hours after the original deadline. The extension grade will be run significantly later.
- There is also one redo of an MP this semester. The redo will be done on the code in your repo on the last day of class May 5th. The redo will change your grade to the following.

$$\text{Max}(\text{old_grade}, .90 * \text{new_grade})$$



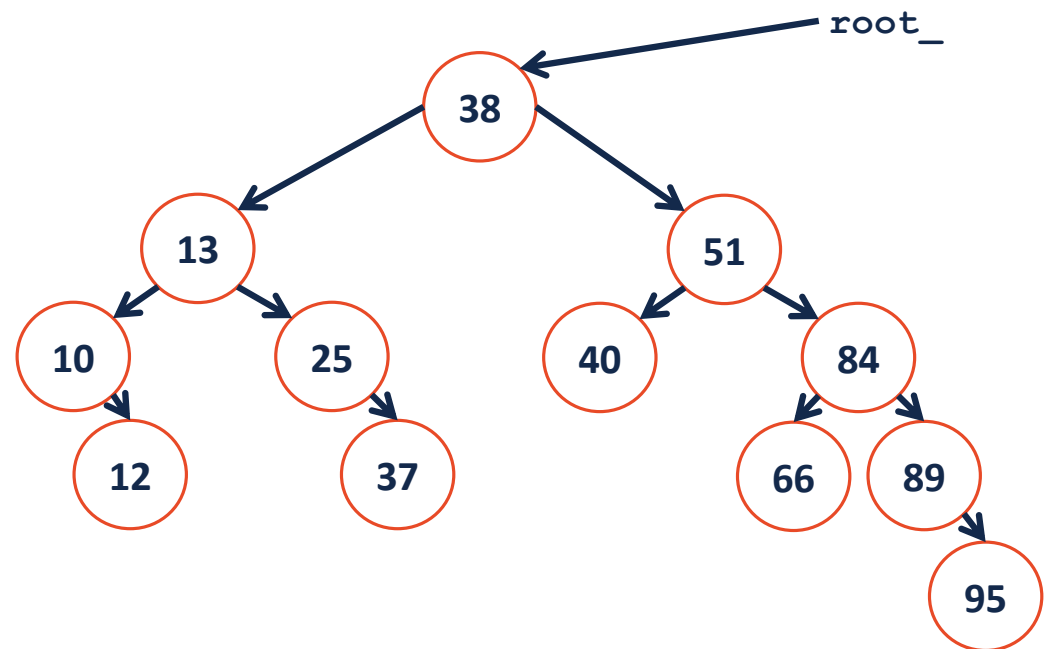
MP Mosaics AMA

When: Today 7pm-9pm

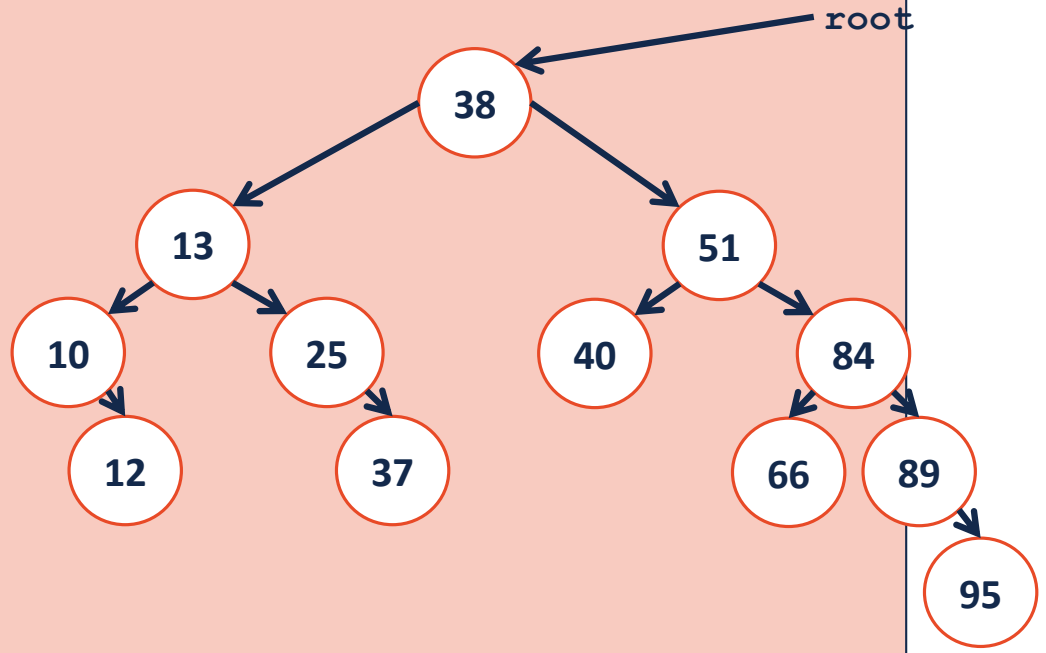
Where: <https://illinois.zoom.us/j/84203813264?pwd=dFVXZ2tUMjJnWFVBeTFxazd0aJlJOUT09>

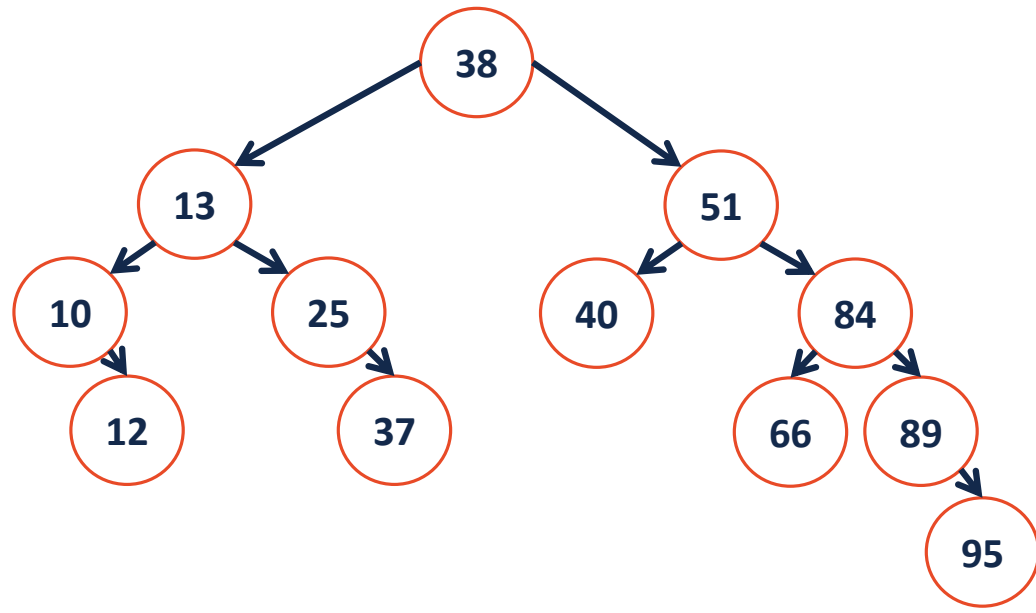
You can also find the link on piazza and the AMA will be posted to Mediaspace and ClassTranscribe later.

```
1 template<typename K, typename V>
2
3 void BST::_insert(TreeNode *& root, K & key, V & value) {
4     TreeNode * t = _find(root, key);
5     t = new TreeNode(key, value);
6 }
```

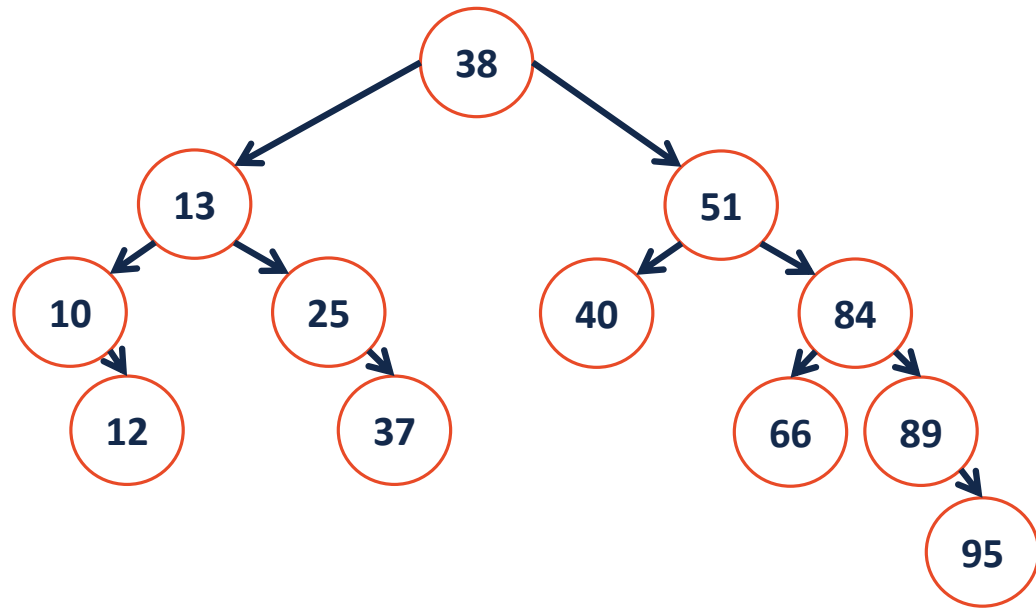


```
1  template<typename K, typename V>
2  _____ _remove(TreeNode *& root, const K & key) {
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26 }
```

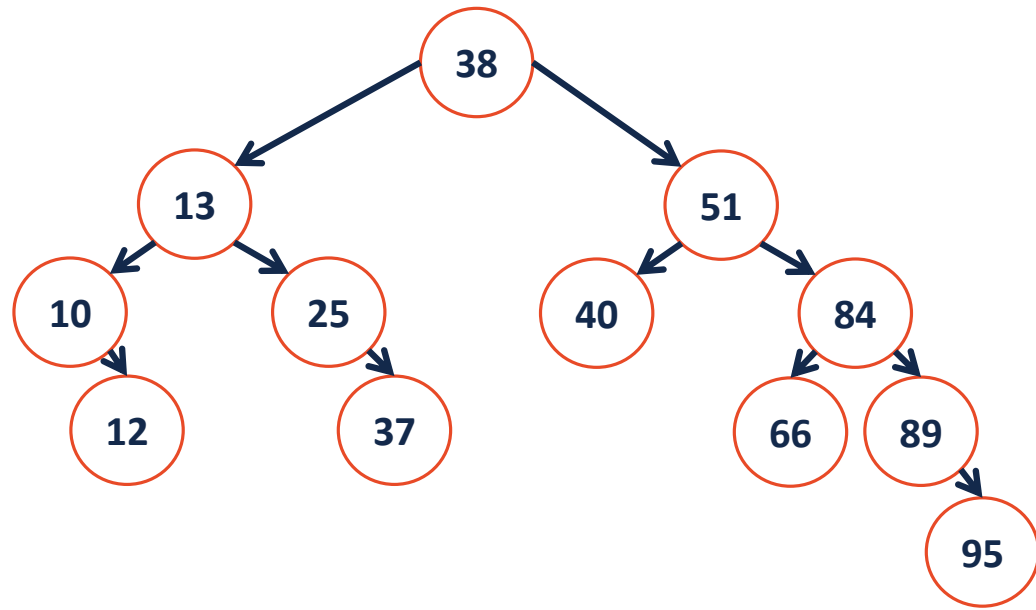




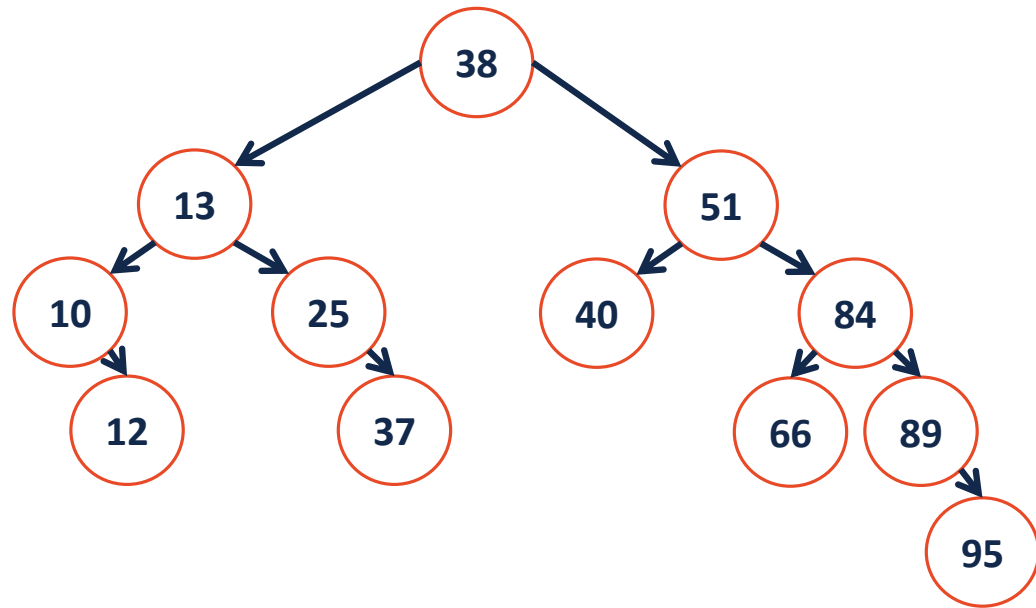
`remove (40) ;`



remove (25) ;



`remove(10);`



`remove (13) ;`

BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	



BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: **$O(h)$** .

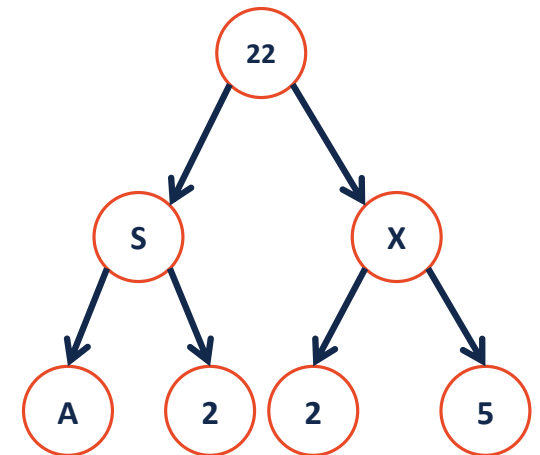
...what is this in terms of **n** , the amount of data?

We need a relationship between **h** and **n** :

$$f(h) \leq n \leq g(h)$$

BST Analysis

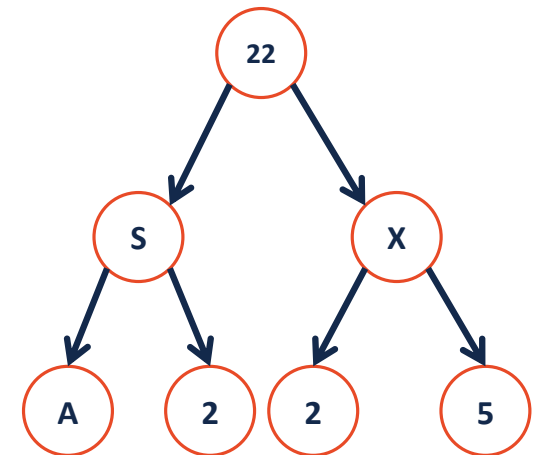
Q: What is the maximum number of nodes in a tree of height **h** ?



BST Analysis

Q: What is the minimum number of nodes in a tree of height h ?

What is the maximum height for a tree of n nodes?





BST Analysis

Therefore, for all BST:

Lower bound:

Upper bound:



BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6

vs.

4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?



BST Analysis

Q: How many different ways are there to insert keys into a BST?

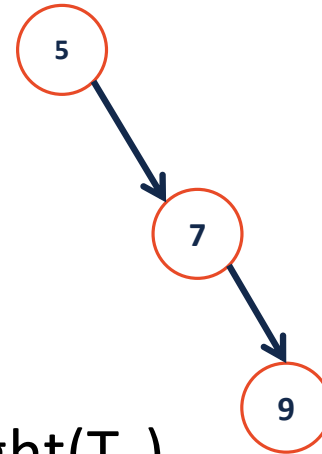
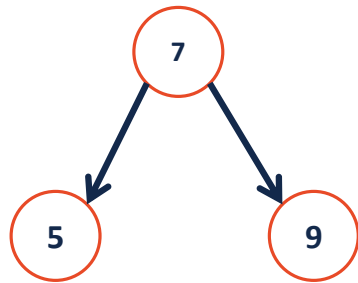
Q: What is the average height of all the arrangements?

BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
find				
insert				
delete				
traverse				

Height-Balanced Tree

What tree makes you happier?



Height balance: $b = \text{height}(T_L) - \text{height}(T_R)$

A tree is height balanced if: