CS 225

Data Structures

April 9 – Hashing II

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Team Contract and Proposal Due April 9th

Team Contract:

Be sure to ‘sign’ electronically.

Non-participants may be removed from groups!

Project Proposal:

One of your three algorithms should be completed by ***mid-project*** check-in.

Learning Objectives

- Review fundamentals of hash tables
- Introduce closed hashing approaches to hash collisions
- Determine when and how to resize a hash table
- Justify when to use different index approaches
- If time: General strategies for *creating* a hash function

A Hash Table based Dictionary

Client Code:

```
Dictionary<KeyType, ValueType> d;
d[k] = v;
```

A **Hash Table** consists of three things:

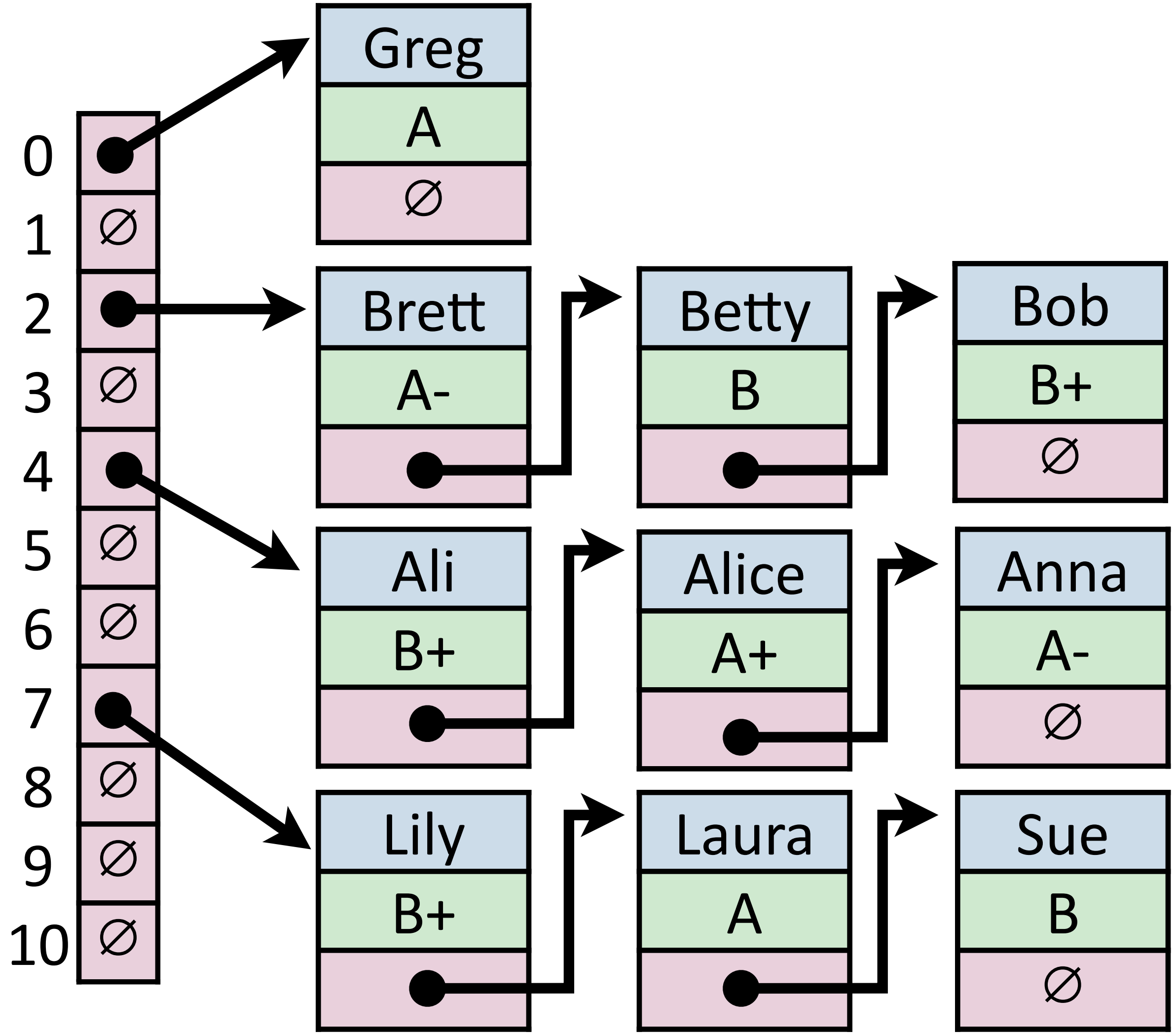
1. A hash function

2. A data storage structure

3. A method of addressing *hash collisions*

Insertion (Separate Chaining)

Key	Value	Hash
Bob	B+	2
Anna	A-	4
Alice	A+	4
Betty	B	2
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7



Hash Table (Separate Chaining)

For hash table of size *m* and *n* elements:

find runs in: ____________.

insert runs in: ____________.

remove runs in: ____________.

Simple Uniform Hashing Assumption

Given table of size m, a simple uniform hash, h, implies

$$\forall k_1, k_2 \in U \text{ where } k_1 \neq k_2, \; Pr(h[k_1] = h[k_2]) = \frac{1}{m}$$

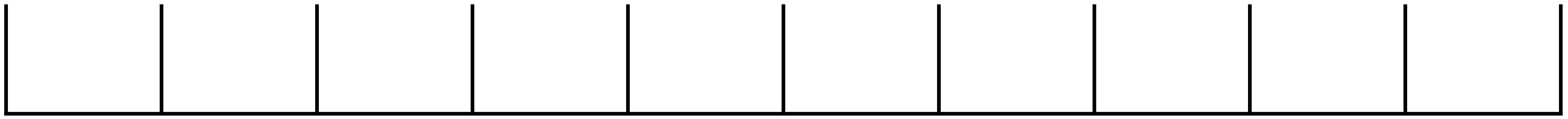
Uniform: keys are equally likely to hash to any position

Independent: key hash values are independent of other keys

Separate Chaining Under SUHA

Under SUHA, a hash table of size *m* and *n* elements:

Expected length of chain is _______________.



Separate Chaining Under SUHA

Under SUHA, a hash table of size *m* and *n* elements:

find runs in: ____________.

insert runs in: ____________.

remove runs in: ____________.

Open vs Closed Hashing

Addressing hash collisions depends on your storage structure.

- **Open Hashing:** store *k,v* pairs externally
- **Closed Hashing:** store *k,v* pairs in the hash table

(Example of closed hashing)

Collision Handling: Probe-based Hashing

S = { 1, 8 , 15}

h(k) = k % 7

|S| = n

|Array| = m

0	
1	
2	
3	
4	
5	
6	

(Example of closed hashing)

Collision Handling: Linear Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

h(k) = k % 7

|S| = n

|Array| = m

0	
1	
2	
3	
4	
5	
6	

h(k, i) = (k + i) % 7

Try h(k) = (k + 0) % 7, if full…

Try h(k) = (k + 1) % 7, if full…

Try h(k) = (k + 2) % 7, if full…

Try …

(Example of closed hashing)

Collision Handling: Linear Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

|S| = n

h(k, i) = (k + i) % 7

|Array| = m

0	
1	
2	
3	
4	
5	
6	

`_find(29)`

`_remove(16)`

	Worst Case	SUHA
Insert		
Remove/Find		

A Problem w/ Linear Probing

Primary clustering:

0	
1	**1**
2	**1’**
3	**3**
4	**1’’**
5	**3’**
6	
7	
8	
9	

Description:

Remedy:

(Example of closed hashing)

Collision Handling: Quadratic Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

|S| = n

h(k) = k % 7

|Array| = m

0	
1	**8**
2	**16**
3	
4	**4**
5	
6	**13**

h(k, i) = (k + i*i) % 7

Try h(k) = (k + 0) % 7, if full…

Try h(k) = (k + 1*1) % 7, if full…

Try h(k) = (k + 2*2) % 7, if full…

Try …

A Problem w/ Quadratic Probing

Secondary clustering:

0	**0**
1	**0’**
2	
3	
4	**0’’**
5	
6	
7	
8	
9	**0’’’**

Description:

Remedy:

(Example of closed hashing)

Collision Handling: Double Hashing

$S = \{ 16, 8, 4, 13, 29, 11, 22 \}$

$|S| = n$

$h_1(k) = k \% 7$

$|Array| = m$

$h_2(k) = 5 - (k \% 5)$

0	
1	**8**
2	**16**
3	
4	**4**
5	
6	**13**

$h(k, i) = (h_1(k) + i*h_2(k)) \% 7$

Try $h(k) = (k + 0*h_2(k)) \% 7$, if full…

Try $h(k) = (k + 1*h_2(k)) \% 7$, if full…

Try $h(k) = (k + 2*h_2(k)) \% 7$, if full…

Try …

Running Times *(Don’t memorize these equations, no need.)*

(Expectation under SUHA)

Open Hashing:

insert: ____________.

find/ remove: ____________.

Closed Hashing:

insert: ____________.

find/ remove: ____________.

Running Times *(Don't memorize these equations, no need.)*

The expected number of probes for find(key) under SUHA

Linear Probing:

- Successful: $\frac{1}{2}(1 + 1/(1-\alpha))$
- Unsuccessful: $\frac{1}{2}(1 + 1/(1-\alpha))^2$

Double Hashing:

- Successful: $1/\alpha * \ln(1/(1-\alpha))$
- Unsuccessful: $1/(1-\alpha)$

Separate Chaining:

- Successful: $1 + \alpha/2$
- Unsuccessful: $1 + \alpha$

Instead, observe:

- As α increases:

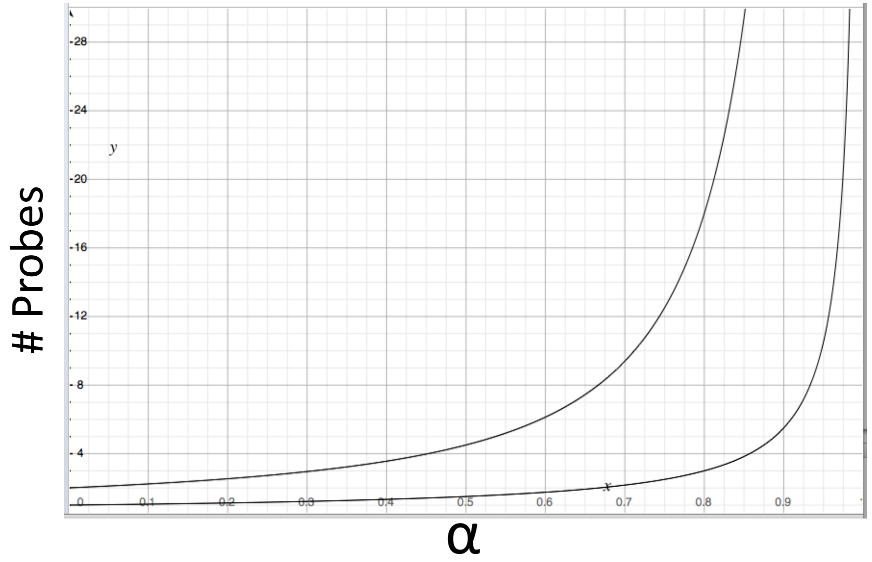
- If α is constant:

Running Times

The expected number of probes for find(key) under SUHA

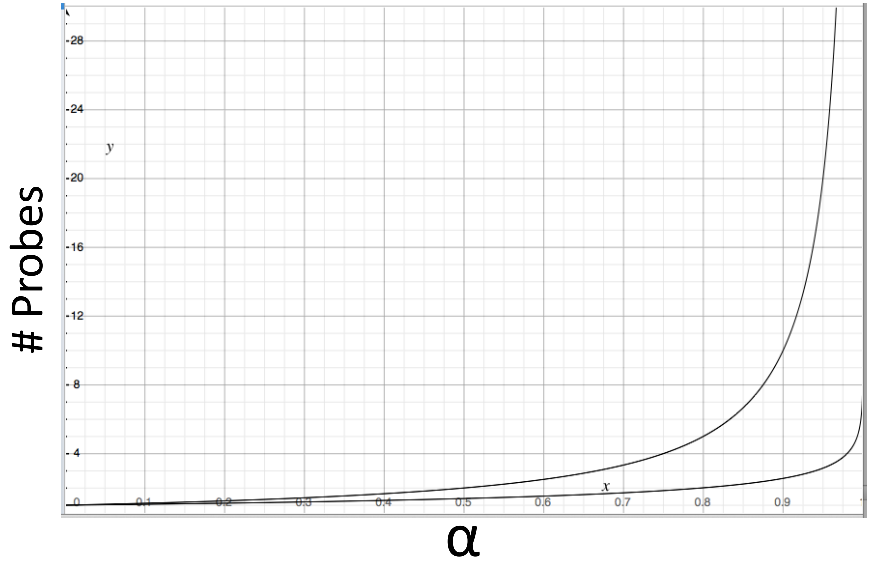
Linear Probing:

- Successful: **½(1 + 1/(1-α))**
- Unsuccessful: **½(1 + 1/(1-α))²**



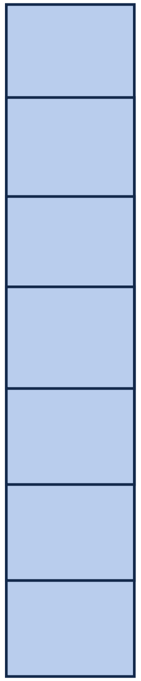
Double Hashing:

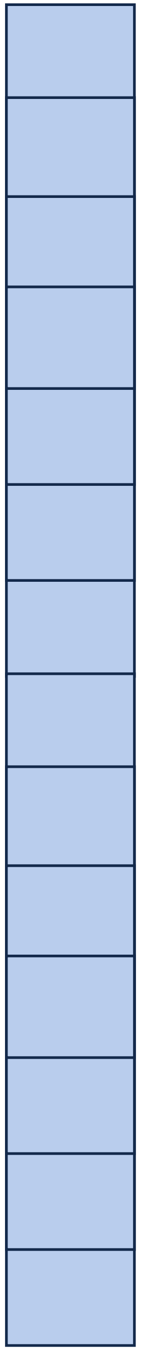
- Successful: **1/α * ln(1/(1-α))**
- Unsuccessful: **1/(1-α)**



ReHashing

What if the array fills?





Which collision resolution strategy is better?

- Big Records:
- Structure Speed:

What structure do hash tables implement?

What constraint exists on hashing that doesn't exist with BSTs?

Why talk about BSTs at all?

Running Times

	Hash Table	AVL	Linked List
Find	Amortized: Worst Case:		
Insert	Amortized: Worst Case:		
Storage Space			

std data structures

std::map

std data structures

std::map

::operator[]
::insert
::erase

::lower_bound(key) ➔ Iterator to first element ≤ key
::upper_bound(key) ➔ Iterator to first element > key

std data structures

std::unordered_map

::operator[]

::insert

::erase

~~::lower_bound(key) ➔ Iterator to first element ≤ key~~

~~::upper_bound(key) ➔ Iterator to first element > key~~

std data structures

std::unordered_map

::operator[]

::insert

::erase

~~::lower_bound(key) ➔ Iterator to first element ≤ key~~

~~::upper_bound(key) ➔ Iterator to first element > key~~

::load_factor()

::max_load_factor(ml) ➔ Sets the max load factor

Bonus Slides

Hash Function (Division Method)

Hash of form: $h(k) = k \,\%\, m$

Pro:

Con:

Hash Function (Multiplication Method)

Hash of form: $h(k) = \lfloor m(kA \,\%\, 1) \rfloor$, $0 \le A \le 1$

Pro:

Con:

Hash Function (Universal Hash Family)

Hash of form: $h_{ab}(k) = \big((ak + b)\,\%\,p\big)\,\%\,m,\;\; a, b \in Z_p^*, Z_p$

$$\forall k_1 \neq k_2,\;\; Pr_{a,b}(h_{ab}[k_1] = h_{ab}[k_2]) \leq \frac{1}{m}$$

Pro:

Con: