# CS 225

**Data Structures**

April 19 – Graph Traversals

Brad Solomon

# Mid-Project Check-ins this week!

Discuss:

Current Progress (First deliverable done?)

Future Progress (What do you have left to do?)

Group Cohesion (Any issues or concerns?)

# Learning Objectives

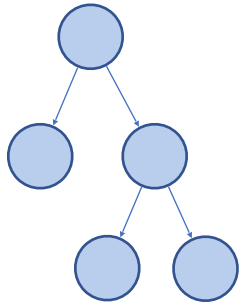
- Discuss pseudo-code for BFS and DFS on graphs
- Analyze and contrast BFS/DFS algorithm runtime and utility
- If time: Introduce Minimum Spanning Tree (MST) problem

# Traversal:

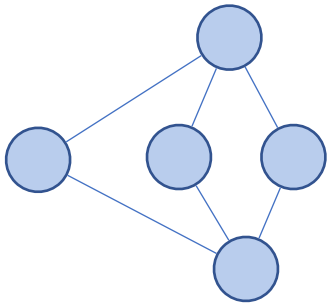
**Objective:** Visit every vertex and every edge in the graph.

**Purpose:** Search for interesting sub-structures in the graph.

We've seen traversal before ….but it's different:

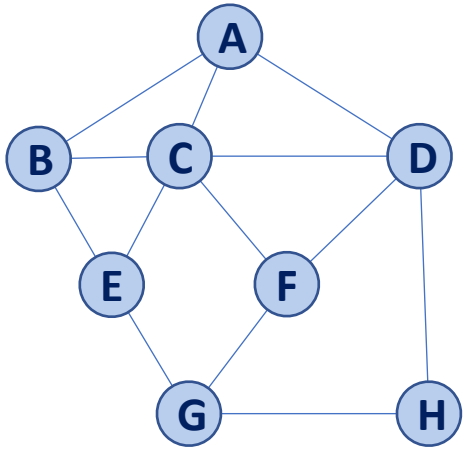


- Ordered
- Obvious Start
- Clear End



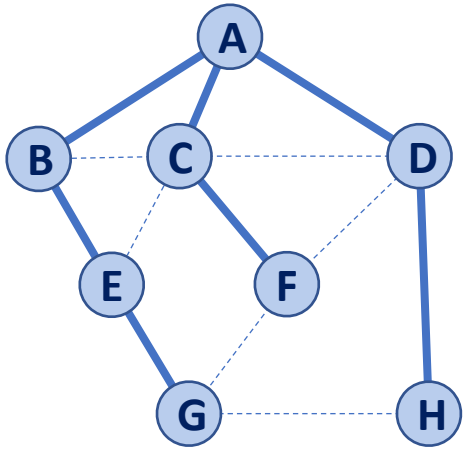
- Any Order
- Any Start
- End is not obvious

# Traversal: BFS



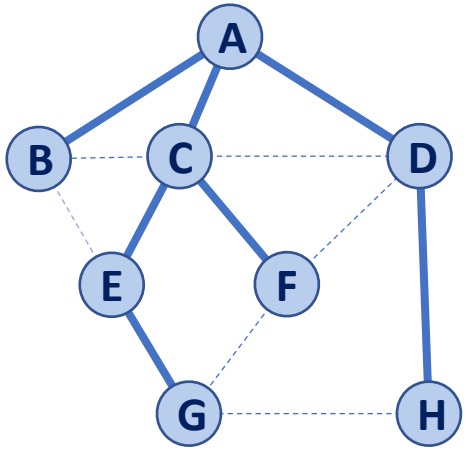

| v | d | P | Adjacent Edges |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |
| E | | | |
| F | | | |
| G | | | |
| H | | | |

# Traversal: BFS




| v | d | P | Adjacent Edges |
|---|---|---|---|
| A | 0 | - | B C D |
| B | 1 | A | A C E |
| C | 1 | A | A B D E F |
| D | 1 | A | A C F H |
| E | 2 | C | B C G |
| F | 2 | C | C D G |
| G | 3 | E | E F H |
| H | 2 | D | D G |

# Traversal: BFS

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| F | 2 | C | C D G |
| G | 3 | E | E F H |
| H | 2 | D | D G |

```
1   BFS(G):
2     Input: Graph, G
3     Output: A labeling of the edges on
4         G as discovery and cross edges
5
6     foreach (Vertex v : G.vertices()):
7       setLabel(v, UNEXPLORED)
8     foreach (Edge e : G.edges()):
9       setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12          BFS(G, v)
```

```
14  BFS(G, v):
15    Queue q
16    setLabel(v, VISITED)
17    q.enqueue(v)
18
19    while !q.empty():
20      v = q.dequeue()
21      foreach (Vertex w : G.adjacent(v)):
22        if getLabel(w) == UNEXPLORED:
23            setLabel(v, w, DISCOVERY)
24            setLabel(w, VISITED)
25            q.enqueue(w)
26        elseif getLabel(v, w) ==
27  UNEXPLORED:
              setLabel(v, w, CROSS)
```

# BFS Analysis

**Q:** Does our implementation handle disjoint graphs? If so, what code handles this?

- ***How do we use this to count components?***

**Q:** Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

**Q:** What is the running time?

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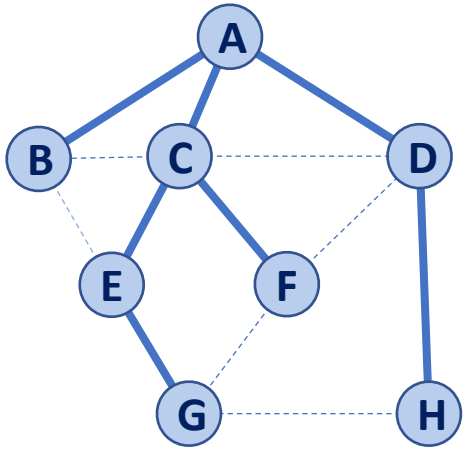
# Running time of BFS




While-loop at **:19**?

For-loop at **:21**?

| v | d | P | Adjacent Edges |
|---|---|---|---|
| A | 0 | - | C B D |
| B | 1 | A | A C E |
| C | 1 | A | B A D E F |
| D | 1 | A | A C F H |
| E | 2 | C | B C G |
| F | 2 | C | C D G |
| G | 3 | E | E F H |
| H | 2 | D | D G |

# BFS Observations

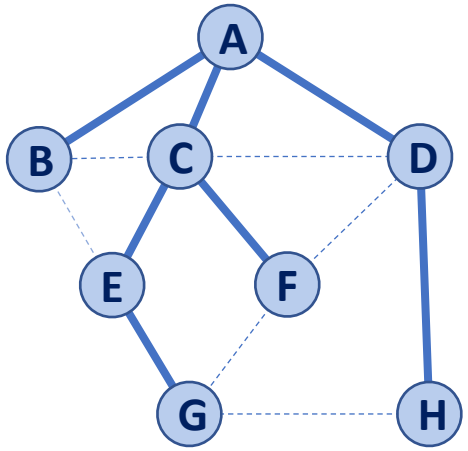
**Q:** What is a shortest path from **A** to **H**?

**Q:** What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

| v | d | P | Adjacent Edges |
|---|---|---|---|
| **A** | 0 | - | **C B D** |
| **B** | 1 | **A** | **A C E** |
| **C** | 1 | **A** | **B A D E F** |
| **D** | 1 | **A** | **A C F H** |
| **E** | 2 | **C** | **B C G** |
| **F** | 2 | **C** | **C D G** |
| **G** | 3 | **E** | **E F H** |
| **H** | 2 | **D** | **D G** |



# BFS Observations

**Obs. 1:** BFS can be used to count components.

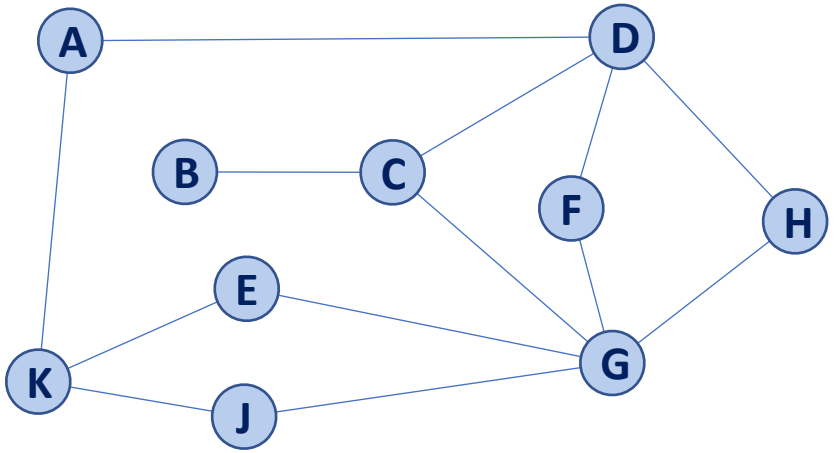
**Obs. 2:** BFS can be used to detect cycles.

**Obs. 3:** In BFS, **d** provides the shortest distance to every vertex.

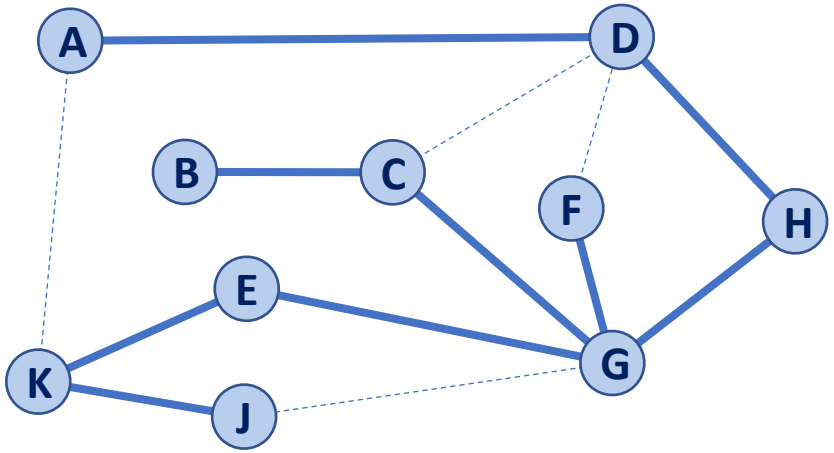
**Obs. 4:** In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| \leq 1$$

# Traversal: DFS



# Traversal: DFS




Discovery Edge

Back Edge

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27  UNEXPLORED:
              setLabel(v, w, CROSS)
```

```
1  DFS(G):
2    Input: Graph, G
3    Output: A labeling of the edges on
4        G as discovery and back edges
5
6    foreach (Vertex v : G.vertices()):
7      setLabel(v, UNEXPLORED)
8    foreach (Edge e : G.edges()):
9      setLabel(e, UNEXPLORED)
10   foreach (Vertex v : G.vertices()):
11     if getLabel(v) == UNEXPLORED:
12       DFS(G, v)
```

```
14 DFS(G, v):
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23         setLabel(v, w, DISCOVERY)
24         setLabel(w, VISITED)
25         DFS(G, w)
26       elseif getLabel(v, w) ==
27 UNEXPLORED:
           setLabel(v, w, BACK)
```