CS 225

Data Structures

April 25 – Dijkstra’s Algorihtm

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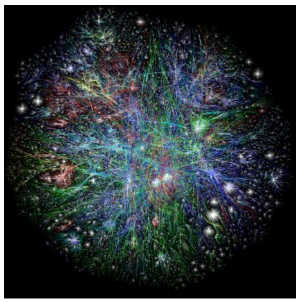
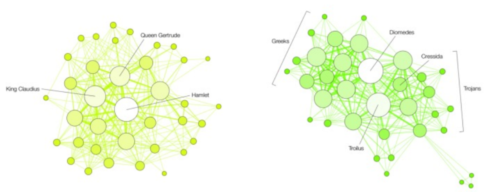
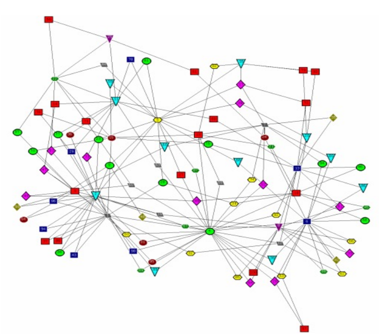
Graphs

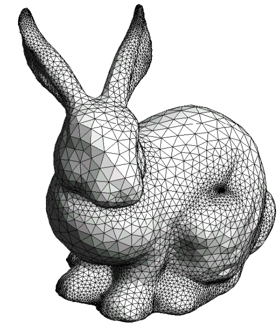




HAMLET

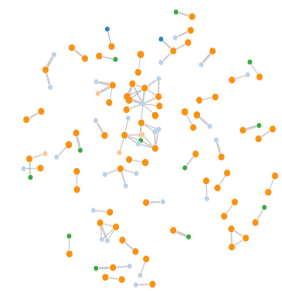
TROILUS AND CRESSIDA

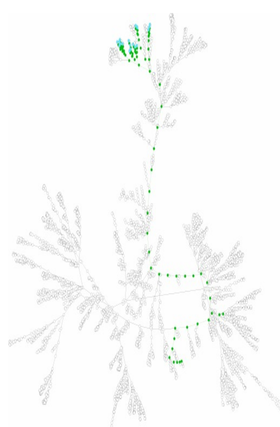


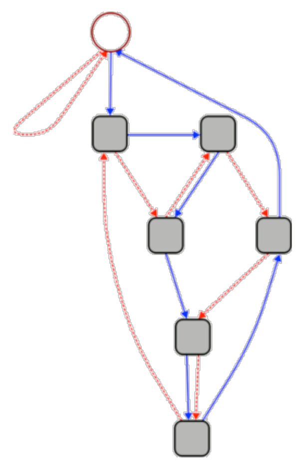


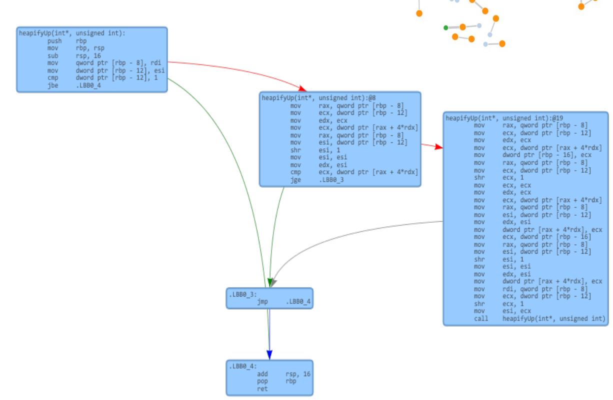
To study all of these structures:

1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms









MST Algorithm Runtime:

We know that MSTs are always run on a minimally connected graph:

$$n-1 \leq m \leq n(n-1) / 2$$

$$O(n) \leq O(m) \leq O(n^2)$$

MST Algorithm Runtime:

- Kruskal's Algorithm:
  $O(n + m \lg(n))$

Sparse Graph:

Dense Graph:

- Prim's Algorithm:
  $O(n \lg(n) + m \lg(n))$

Sparse Graph:

Dense Graph:

Suppose I have a new heap:

	Binary Heap	Fibonacci Heap
Remove Min	O(lg(n))	O(lg(n))
Decrease Key	O(lg(n))	O(1)*

What's the updated running time?

```
PrimMST(G, s):
  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T          // "labeled set"

  repeat n times:
    Vertex m = Q.removeMin()
    T.add(m)
    foreach (Vertex v : neighbors of m not in T):
      if cost(v, m) < d[v]:
        d[v] = cost(v, m)
        p[v] = m
```

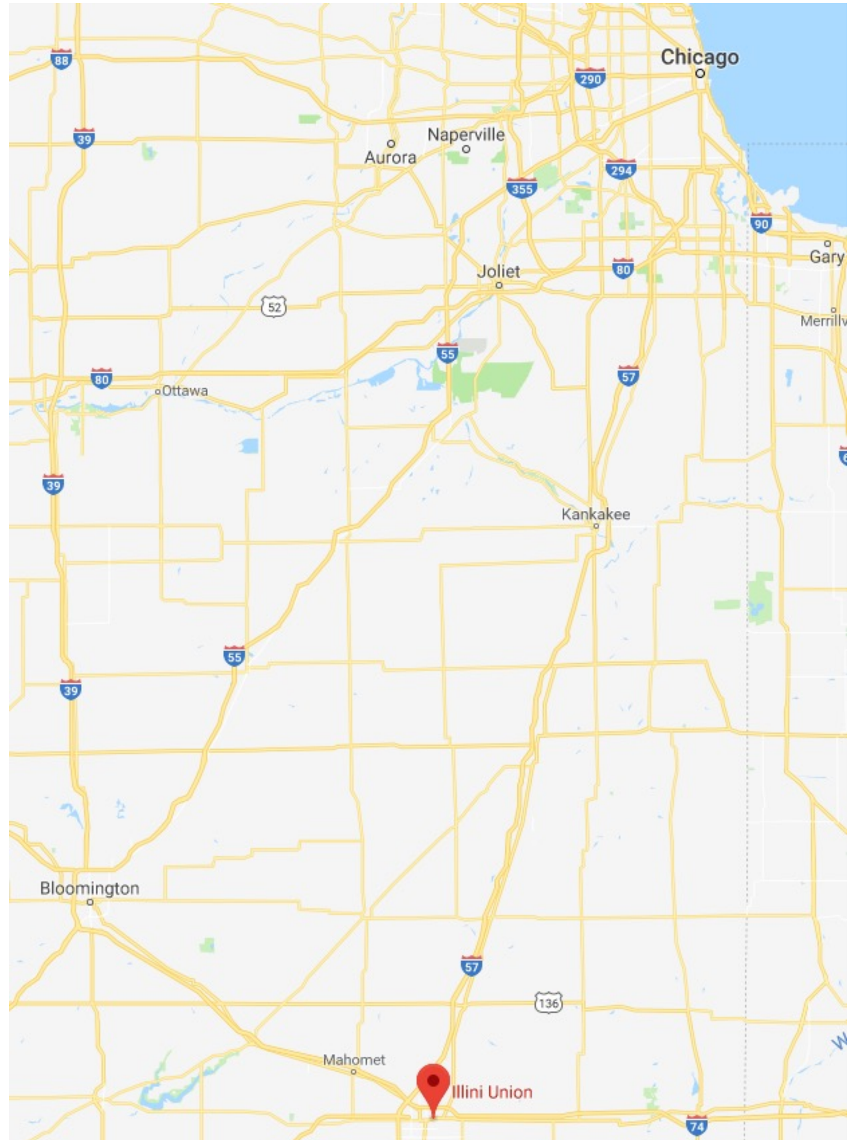
MST Algorithm Runtimes:

- Kruskal's Algorithm:
  O(m lg(n))
- Prim's Algorithm:
  O(n lg(n) + m lg(n))

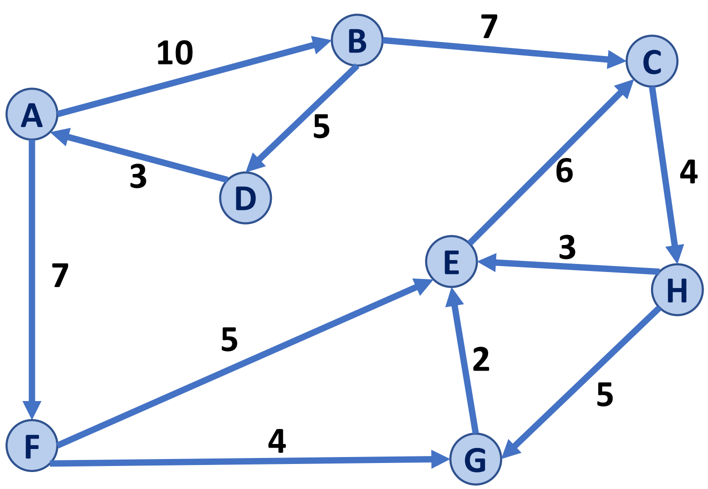
Final Big-O MST Algorithm Runtimes:

- Kruskal's Algorithm:
  O(m lg(n))
- Prim's Algorithm:
  O(n lg(n) + m)

Shortest Path



Dijkstra’s Algorithm (SSSP)




```
DijkstraSSSP(G, s):
  foreach (Vertex v : G):
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  repeat n times:
    Vertex u = Q.removeMin()
    T.add(u)
    foreach (Vertex v : neighbors of u not in T):
      if _______________ < d[v]:
        d[v] = __________________
        p[v] = m
```
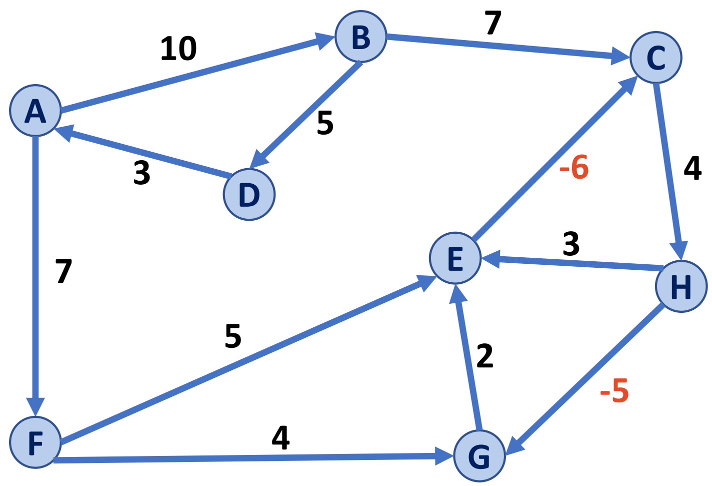
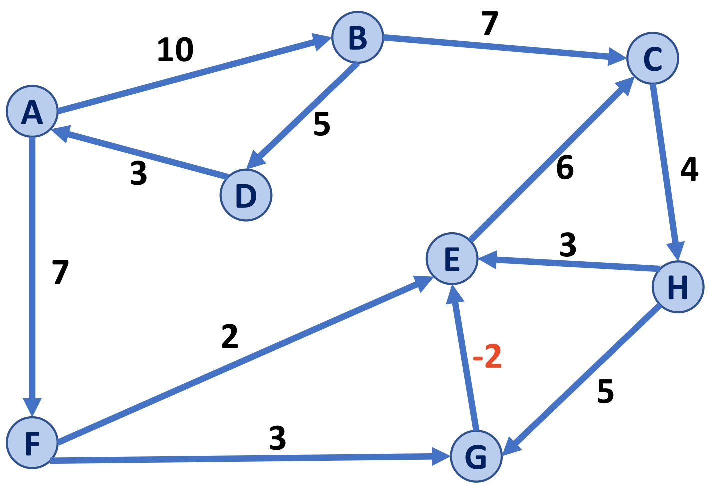

Dijkstra's Algorithm (SSSP)

What about negative weight cycles?



Dijkstra's Algorithm (SSSP)

What about negative weight edges, without negative weight cycles?



Dijkstra’s Algorithm (SSSP)

What is the running time?

```
DijkstraSSSP(G, s):
  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T         // "labeled set"

  repeat n times:
    Vertex u = Q.removeMin()
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```