CS 225

Data Structures

May 3 – Floyd-Warshall's Algorithm

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Floyd-Warshall Algorithm

Floyd-Warshall's Algorithm is an alterative to Dijkstra in the presence of negative-weight edges (not negative weight cycles).

```
    FloydWarshall(G):
 6    Let d be a adj. matrix initialized to +inf
 7    foreach (Vertex v : G):
 8      d[v][v] = 0
 9    foreach (Edge (u, v) : G):
10      d[u][v] = cost(u, v)
11
12    foreach (Vertex w : G):
13      foreach (Vertex u : G):
14        foreach (Vertex v : G):
15          if (d[u, v] > d[u, w] + d[w, v])
16            d[u, v] = d[u, w] + d[w, v]
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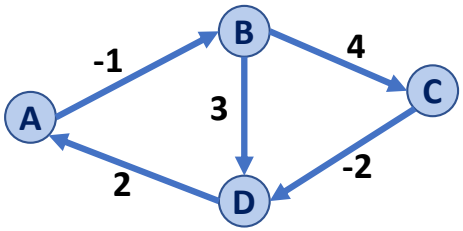
15          if d[u, v] > d[u, w] + d[w, v]:
16            d[u, v] = d[u, w] + d[w, v]
```

	A	B	C	D
A				
B				
C				
D				



Floyd-Warshall Algorithm

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12    foreach (Vertex k : G):
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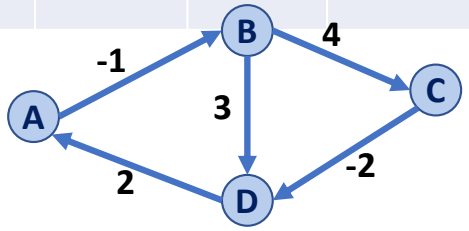
15          if d[u, v] > d[u, k] + d[k, v]:
16            d[u, v] = d[u, k] + d[k, v]
```

	A	B	C	D
A	0	-1	∞	∞
B	∞	0	4	3
C	∞	∞	0	-2
D	2	∞	∞	0



Floyd-Warshall Algorithm

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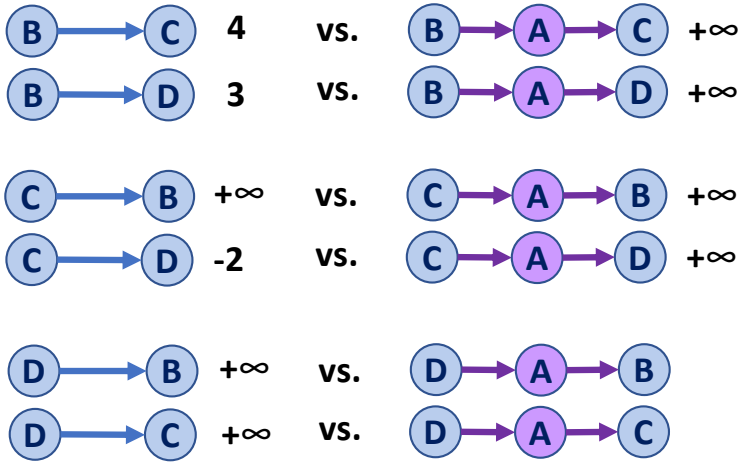
16            d[u, v] = d[u, k] + d[k, v]
```

Let us consider k=A:



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B	∞	0	4	3
C	∞	∞	0	-2
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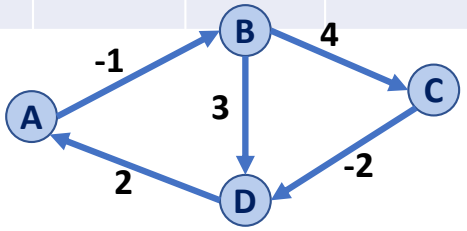

Floyd-Warshall Algorithm

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C	∞	∞	0	-2
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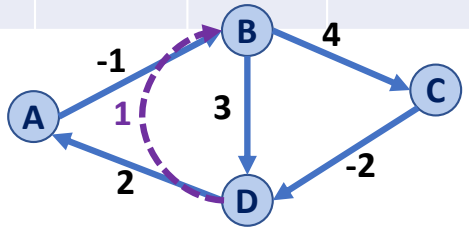

Floyd-Warshall Algorithm

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14        foreach (Vertex k : G):
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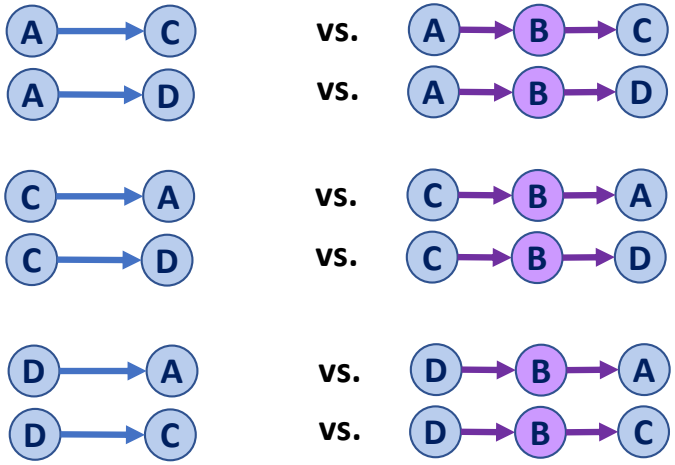
16            d[u, v] = d[u, k] + d[k, v]
```

Let us consider k=B:



	A	B	C	D
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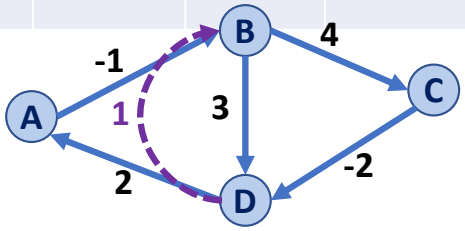
B	∞	0	4	3
C	∞	∞	0	-2
D	2	1	∞	0



Floyd-Warshall Algorithm

Running Time?

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