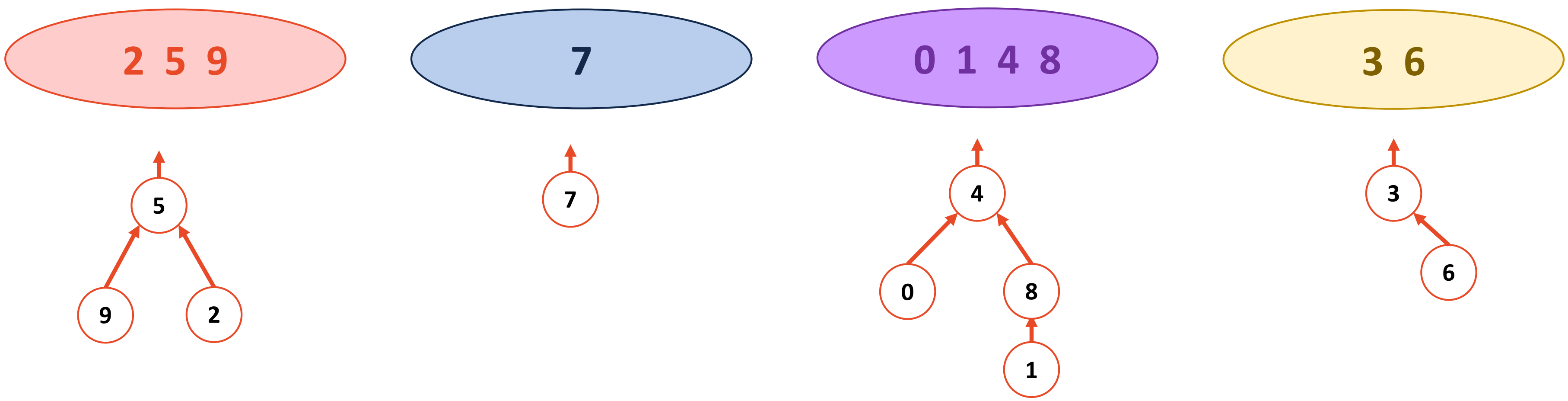
CS 225

Data Structures

March 14 – Disjoint Sets and Iterators

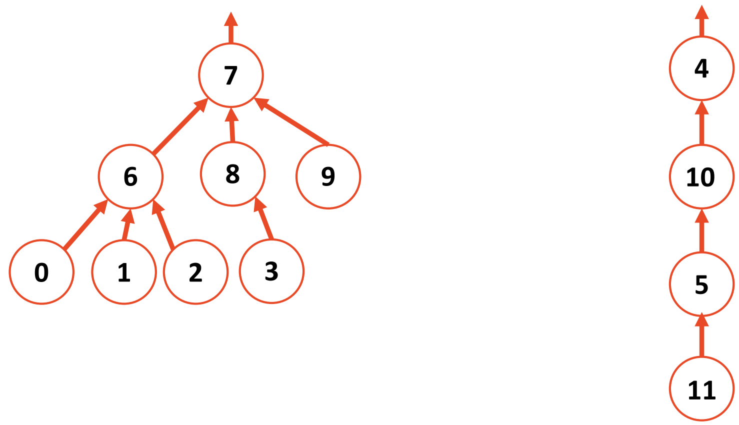
G Carl Evans

Disjoint Sets



0	1	2	3	4	5	6	7	8	9
4	8	5	-1	-1	-1	3	-1	4	5

Disjoint Sets – Smart Union



Union by height

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8	-4	10	7	-3	7	7	4	5

***Idea**: Keep the height of the tree as small as possible.*

Union by size

0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8	-8	10	7	-4	7	7	4	5

***Idea**: Minimize the number of nodes that increase in height*

We will show the height of the tree is: log(n).

Union by Size (limit on height)

To show that every tree in a disjoint set data structure using union by size has a height of at most O(log n) we will show that the inverse.

Base Case

Inductive Hypothesis

Union by Size

Case 1

Union by Size

Case 2

Union by Height (limit on height)

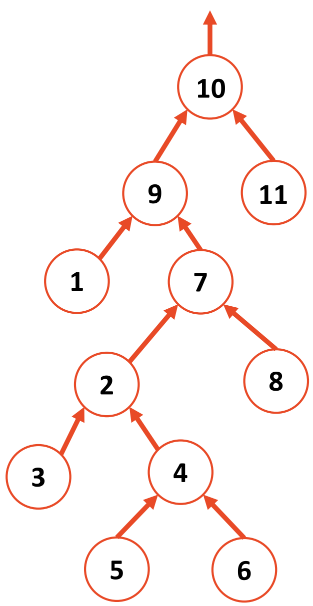
Much like before we will show the min(nodes) in a tree with a root of height $k \geq 2^k$

Base Case

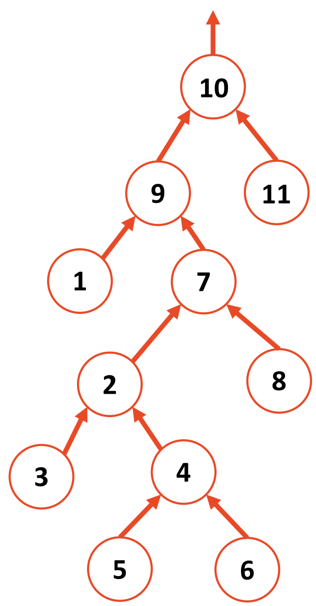
IH

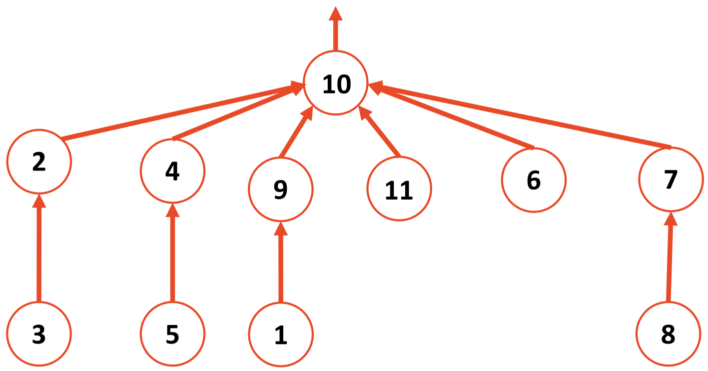
Disjoint Sets Find

```
int DisjointSets::find(int i) {
  if ( s[i] < 0 ) { return i; }
  else { return find( s[i] ); }
}
```



Path Compression





Union by Height -> Rank

Base

New UpTrees have Rank =

When you join two UpTrees

Iterated Logarithm Function (log^*n)

log^*n is piecewise defined as

0 if $n \leq 1$

otherwise

$1 + log^*(\log n)$

Buckets

- Put every non-root node in a bucket by rank
- The total number of buckets is O(log*n)
- Max nodes in a bucket is n divided by lower bound of the next bucket

Ranks	Bucket
0	0
1	1
2 - 3	2
4 - 15	3
16 – 65535	4
65536 – $2^{65536} - 1$	5

Disjoint Sets Analysis

The **iterated log** function:

The number of times you can take a log of a number.

log*(n) =

0 , $n \leq 1$

1 + log*(log(n)) , $n > 1$

What is **lg*(2^{65536})**?

Even Better

In case that seems to slow tightest bound is

$$\Theta(m\,\alpha(m,n))$$

Where $\alpha(m,n)$ is the inverse Ackermann function which grows much slower than log*n.

Proof well outside this class.

Disjoint Sets Analysis

In a Disjoint Sets implemented with smart **unions** and path compression on **find**:

Any sequence of **m union** and **find** operations result in the worse case running time of O(_____________),

where **n** is the number of items in the Disjoint Sets.