

# Errors

# Scientific Notation

In **scientific notation**, a number can be expressed in the form

$$x = \pm r \times 10^m$$

where  $r$  is a coefficient in the range  $1 \leq r < 10$  and  $m$  is the exponent.

1165.7 =

0.0004728 =

# Error in Numerical Methods

- Every result we compute in Numerical Methods contain errors!
- We always have them... so our job? Reduce the impact of the errors
- How can we model the error?

- Absolute errors can be misleading, depending on the magnitude of the true value  $x$ .
- Example:
- Relative error is independent of magnitude!

You are tasked with measuring the height of a tree which is known to be exactly 170 ft tall. You later realized that your measurement tools are somewhat faulty, up to a relative error of 10%. What is the maximum measurement for the tree height (numbers rounded to 3 sig figs)?

- A) 153 ft
- B) 155 ft
- C) 187 ft
- D) 189 ft

You are tasked with measuring the height of a tree and you get the measurement as 170 ft tall. You later realized that your measurement tools are somewhat faulty, up to a relative error of 10%. What is the minimum height of the tree (numbers rounded to 3 sig figs) ?

- A) 153 ft
- B) 155 ft
- C) 187 ft
- D) 189 ft

# Significant digits

**Significant figures** of a number are digits that carry meaningful information. They are digits beginning to the leftmost nonzero digit and ending with the rightmost “correct” digit, including final zeros that are exact.

The number 3.14159 has \_\_\_\_\_ significant digits.

The number 0.00035 has \_\_\_\_\_ significant digits.

The number 0.000350 has \_\_\_\_\_ significant digits.

**Accurate to  $n$  significant digits** means that you can trust a total of  $n$  digits. *Accurate digits* is a measure of relative error.

Suppose  $x$  is the true value and  $\tilde{x}$  the approximation.

The number of significant digits tells us about how many positions of  $x$  and  $\tilde{x}$  agree.

$\tilde{x}$  has  **$n$  significant figures** of  $x$  if  $|x - \tilde{x}|$  has zeros in the first  $n$  decimal places counting from the leftmost nonzero (leading) digit of  $x$ , followed by a digit from 0 to 4.

**Example:**

$$x = 5.1 \text{ and } \tilde{x} = 5$$

$$x = 0.51 \text{ and } \tilde{x} = 0.5$$

$$x = 5 \text{ and } \tilde{x} = 4.992$$

$$x = 5 \text{ and } \tilde{x} = 4.996$$



Suppose  $x$  is the true value and  $\tilde{x}$  the approximation.

The number of significant digits tells us about how many positions of  $x$  and  $\tilde{x}$  agree.

$\tilde{x}$  has  **$n$  significant figures** of  $x$  if  $|x - \tilde{x}|$  has zeros in the first  $n$  decimal places counting from the leftmost nonzero (leading) digit of  $x$ , followed by a digit from 0 to 4.

**Example:**

$$x = 3.141592653$$

**Accurate to  $n$  significant digits** means that you can trust a total of  $n$  digits. *Accurate digits* is a measure of relative error.

**Relative error:**  $error = \frac{|x_{exact} - x_{approx}|}{|x_{exact}|} \leq 10^{-n+1}$

$n$  is the number of accurate significant digits

After rounding, the resulting number has 5 accurate digits. What is the tightest estimate of the upper bound on my relative error?

*A)*  $10^5$

*B)*  $10^{-5}$

*C)*  $10^4$

*D)*  $10^{-4}$

# Sources of Error

Main source of errors in numerical computation:

- **Rounding error:** occurs when digits in a decimal point ( $1/3 = 0.3333\dots$ ) are lost ( $0.3333$ ) due to a limit on the memory available for storing one numerical value.
- **Truncation error:** occurs when discrete values are used to approximate a mathematical expression (eg. the approximation  $\sin(\theta) \approx \theta$  for small angles  $\theta$ )