Revising Big-Oh notation

Let f and g be two functions. Then

$$f(x) = \mathcal{O}(g(x)) \quad \text{as } x \to \infty$$

if and only if there exists a value M and some x_0 so that

$$|f(x)| \leq M|g(x)| \quad \text{for all } x \geq x_0$$

Want to bound $f(x)$ by a function $g(x)$ as $x \to \infty$

- complexity
- time

(and how the grow as dimension of problem increases)

$f(x) = x^2 + \underbrace{3x^4}$

$x \to \infty$

this term grows faster (dominant)

$|f(x)| \leq M x^4 \longrightarrow \boxed{f(x) = O(x^4)}$

Revising Big-Oh notation

most often we have
$x \to 0$

or ... think about $x \to a$

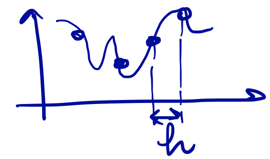
Let f and g be two functions. Then

$$f(x) = \mathcal{O}(g(x)) \quad \text{as } x \to a$$

if and only if there exists a value M and some δ so that

$$|f(x)| \le M|g(x)| \quad \text{for all } x \text{ where } 0 < |x-a| < \delta$$

(For example, how does the error decay when we decrease the value of the interval h ($h \to 0$) between points when interpolating a function?)



$$f(x) = \underbrace{x^2}_{\text{dominant term}} + \underbrace{3x^4}_{\text{this term goes to zero faster}}, \quad x \to 0$$

$$|f(x)| \le M x^2 \longrightarrow \boxed{|f(x)| = O(x^2)}$$

Another example…

Consider the function $f(x) = 2x^2 + 27x + 1000$

1) $x \to 0$: $f(x) = \underbrace{2x^2 + 27x}_{\text{go to zero faster}} + \underbrace{1000}_{\text{constant (dominant)}}$ $\longrightarrow$ $f(x) = O(1)$, $x \to 0$

2) $x \to \infty$: $f(x) = \underbrace{2x^2}_{\text{dominant term when } x \to \infty} + 27x + 1000$ $\longrightarrow$ $f(x) = O(x^2)$, $x \to \infty$

Iclicker question

Suppose that the truncation error of a numerical method is given by the following function:

goes to zero faster

dominant

$$E(h) = \overbrace{5h^2}^{\text{goes to zero faster}} + \boxed{3h}$$

$|E(h)| \leq M h$

Which of the following functions are Oh-estimates of $E(h)$ as $h \to 0$

1) $O(5h^2)$
2) $O(h)$ ✓
3) $O(5h^2 + 3h)$ ✓
4) $O(h^2)$

Mark the correct answer:

A) 1 and 2
B) 2 and 3
C) 2 and 4
D) 3 and 4
E) NOTA

$|5h^2 + 3h| \leq M(5h^2)$ NO

$|5h^2 + 3h| \leq M h$ ✓

$|5h^2 + 3h| \leq M(5h^2 + 3h)$ ✓

$|5h^2 + 3h| \leq M h^2$ NO

Iclicker question

Suppose that the complexity of a numerical method is given by the following function:

$$c(n) = \underbrace{\boxed{5n^2}}_{\text{dominant term}} + 3n \qquad |c(n)| \leq M n^2$$

Which of the following functions are Oh-estimates of $c(n)$ as $n \to \infty$

1) $O(5n^2 + 3n)$ ✓
2) $O(n^2)$ ✓
3) $O(n^3)$ ✓
4) $O(n)$ ✗

Mark the correct answer:

A) 1,2,3

B) 1,2,3,4

C) 4

D) 3

E) NOTA

*sorry about lack of thick marks

Select the function that best represents the decay of the error as n increases

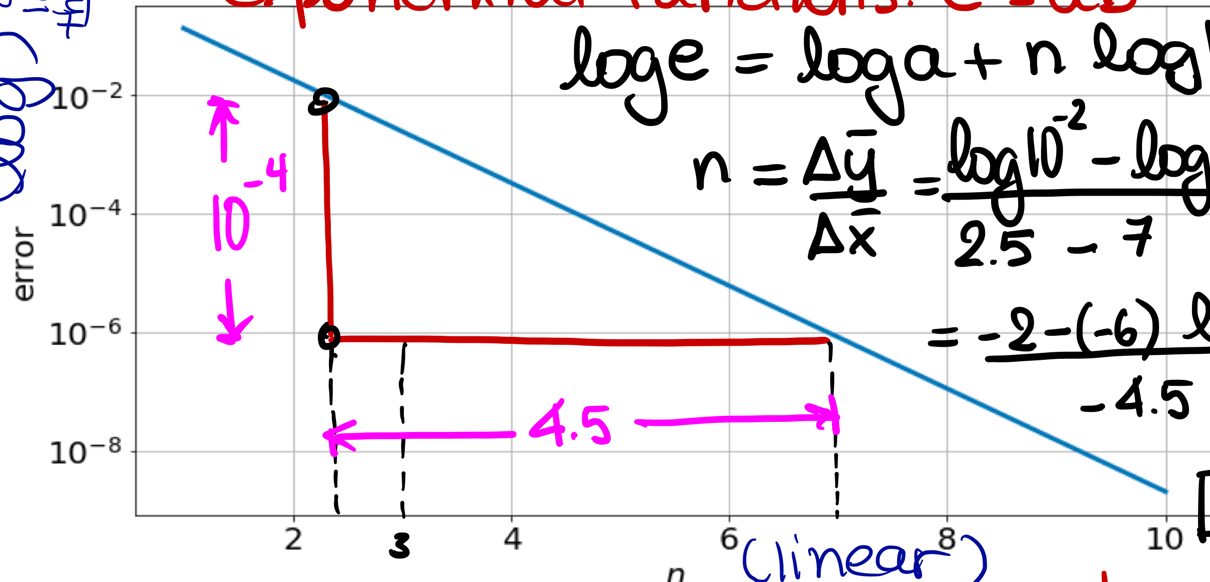
exponential functions: $e = ab^n \rightarrow$ semi-logy plot

$\log e = \log a + n \log b$

$n = \frac{\Delta \bar{y}}{\Delta \bar{x}} = \frac{\log 10^{-2} - \log 10^{-6}}{2.5 - 7}$

$= \frac{-2-(-6) \log 10}{-4.5}$

$\boxed{n \cong -2.04}$



A) e^{-2n}

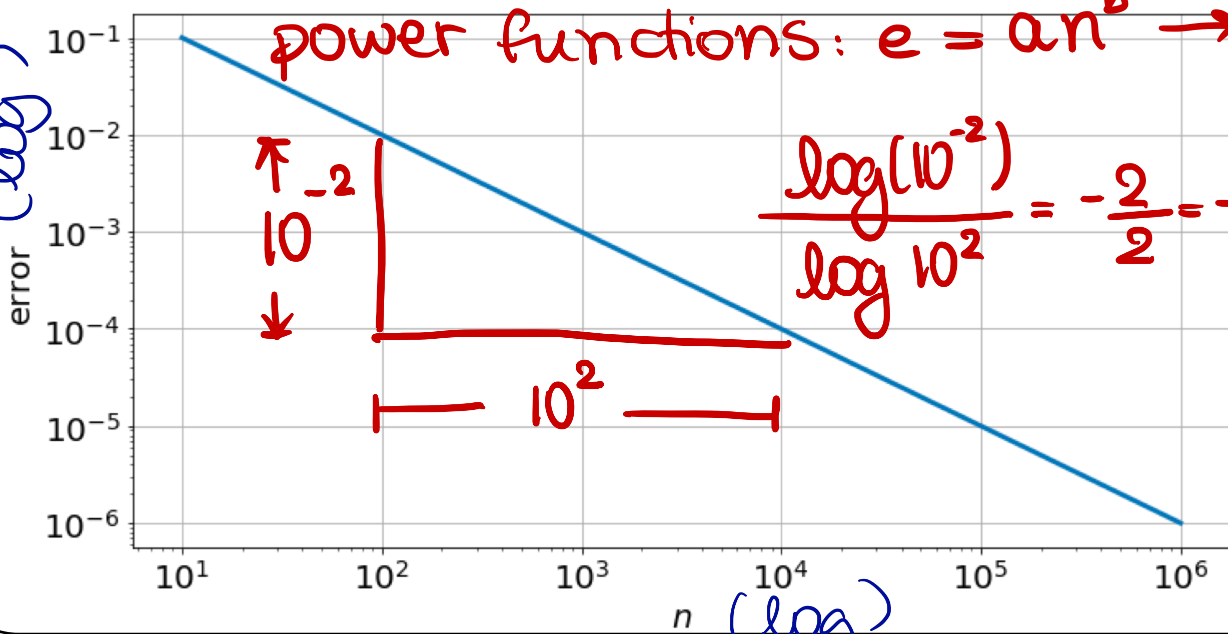
B) e^{-n}

C) n^{-1}

D) n^{-2}

power functions: $e = an^b \rightarrow$ log-log plot

$\frac{\log(10^{-2})}{\log 10^2} = -\frac{2}{2} = -1$



A) e^{-2n}

B) e^{-n}

C) n^{-1}

D) n^{-2}

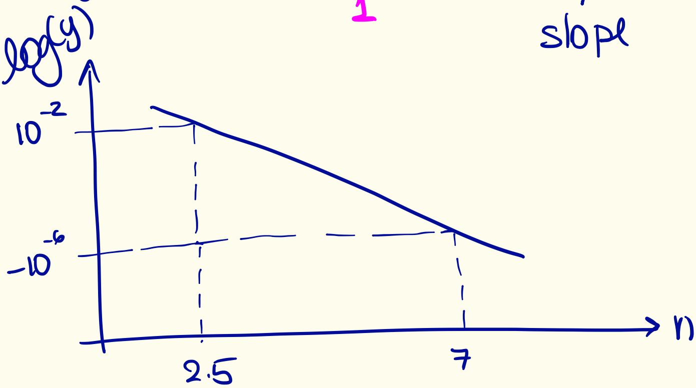
$y = e^{\alpha n}$

if log is base e

$\log(y) = \alpha n \log e$

$\log(y) = \alpha \underbrace{\log(e)}_{1} n = \underline{\underline{\alpha}}\, n$

slope



$$\alpha = \frac{\log 10^{-2} - \log 10^{-6}}{2.5 - 7} = \frac{(-2+6)\log(10)}{-4.5}$$

$\boxed{\alpha = -2.04}$

"

if considering base 10 . . .

this is now the slope

$\log(y) = \underbrace{\alpha \log(e)}\, n$

$$\alpha \log(e) = \frac{4 \log(10)}{-4.5} \Rightarrow \alpha = \frac{4 \overbrace{\log(10)}^{1}}{-4.5 \underbrace{\log(e)}_{0.434}}$$

$\alpha = -2.04$ //