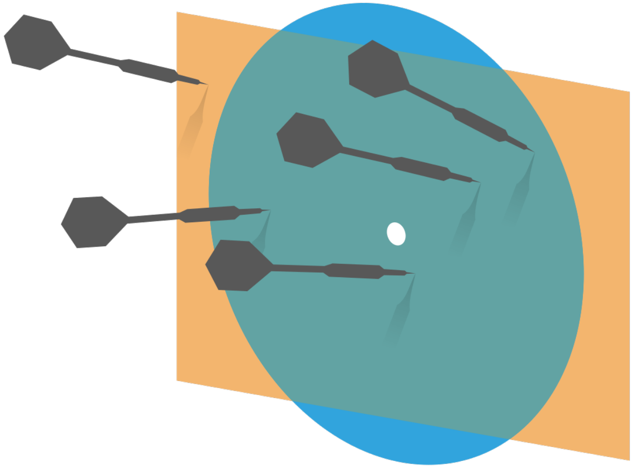
Probability and Statistics for Computer Science



Credit: wikipedia

"Unsupervised learning is arguably more typical of human and animal learning…"--- Kelvin Murphy, former professor at UBC

Hongye Liu, Teaching Assistant Prof, CS361, UIUC, 12.03.2020

Last time

- Curse of dimensions
- Unsupervised learning
- Clustering

Objectives

Q. Is k-means clustering deterministic?

A. Yes
B. No

K-means clustering example: Portugal consumers

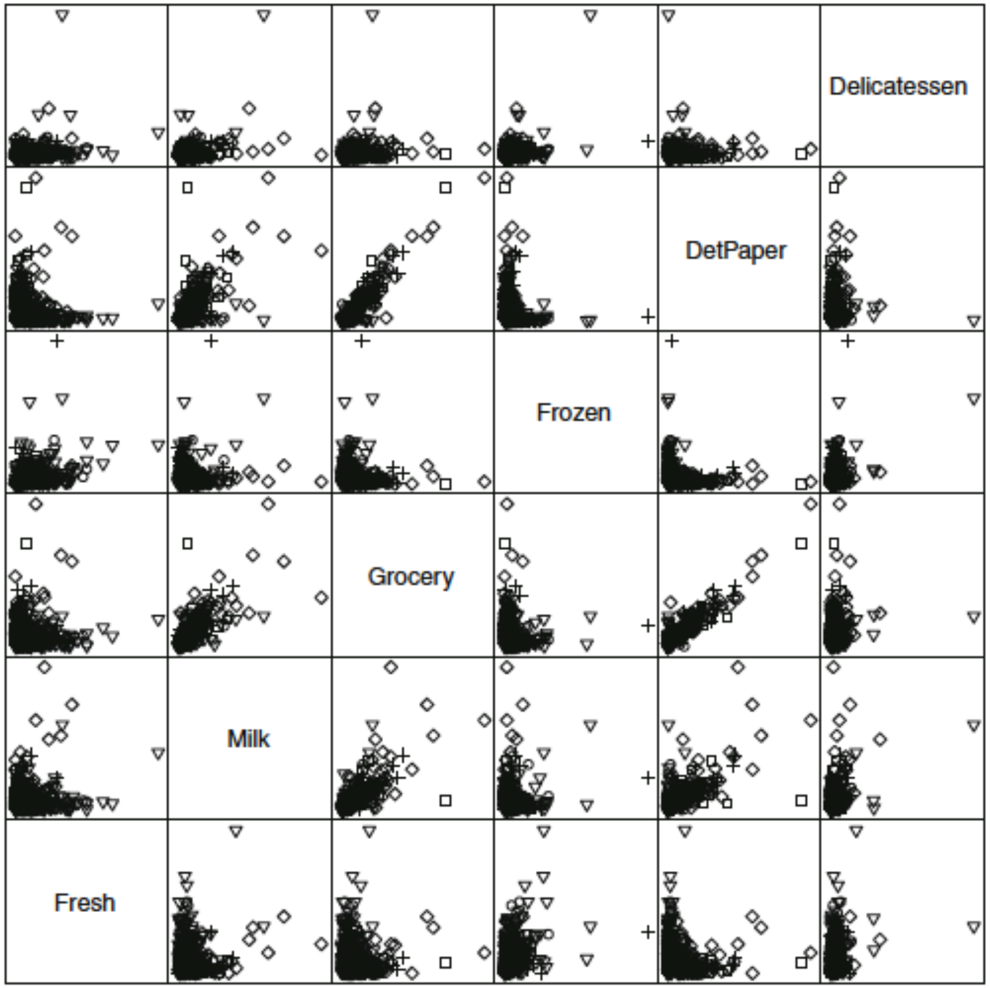
- The dataset consists of the annual grocery spending of 440 customers
- Each customer's spending is recorded in 6 features:
  - fresh food, milk, grocery, frozen, detergents/paper, delicatessen
- Each customer is labeled by: 6 labels in total
  - Channel (Channel 1 & 2) (Horeca 298, Retail 142)
  - Region (Region 1, 2 &3) (Lisbon 77, Oporto 47, Other 316)

Lisbon, Portugal

Oporto, Portugal

Visualization of the data

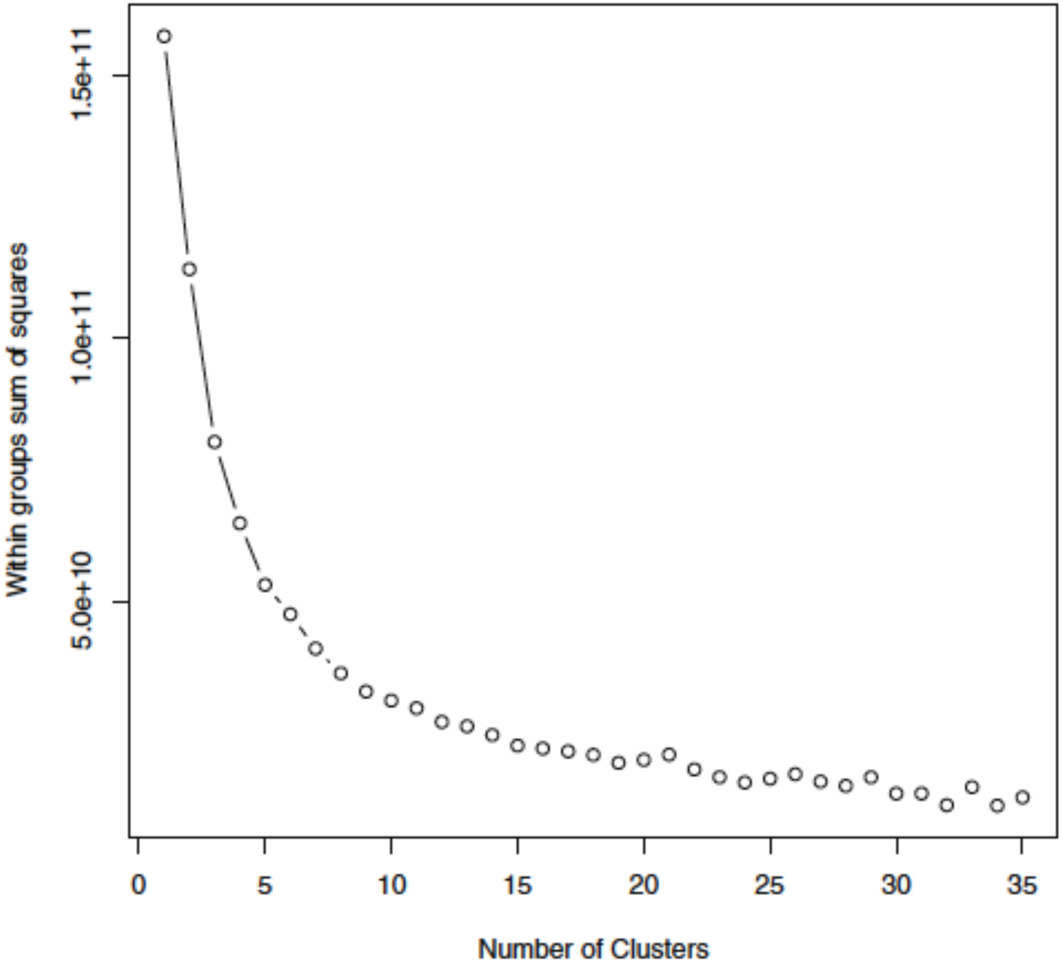
- Visualize the data with scatter plots
- We do see that some features are correlated.
- But overall we do not see significant structure or groups in the data.



Scatter Plot Matrix

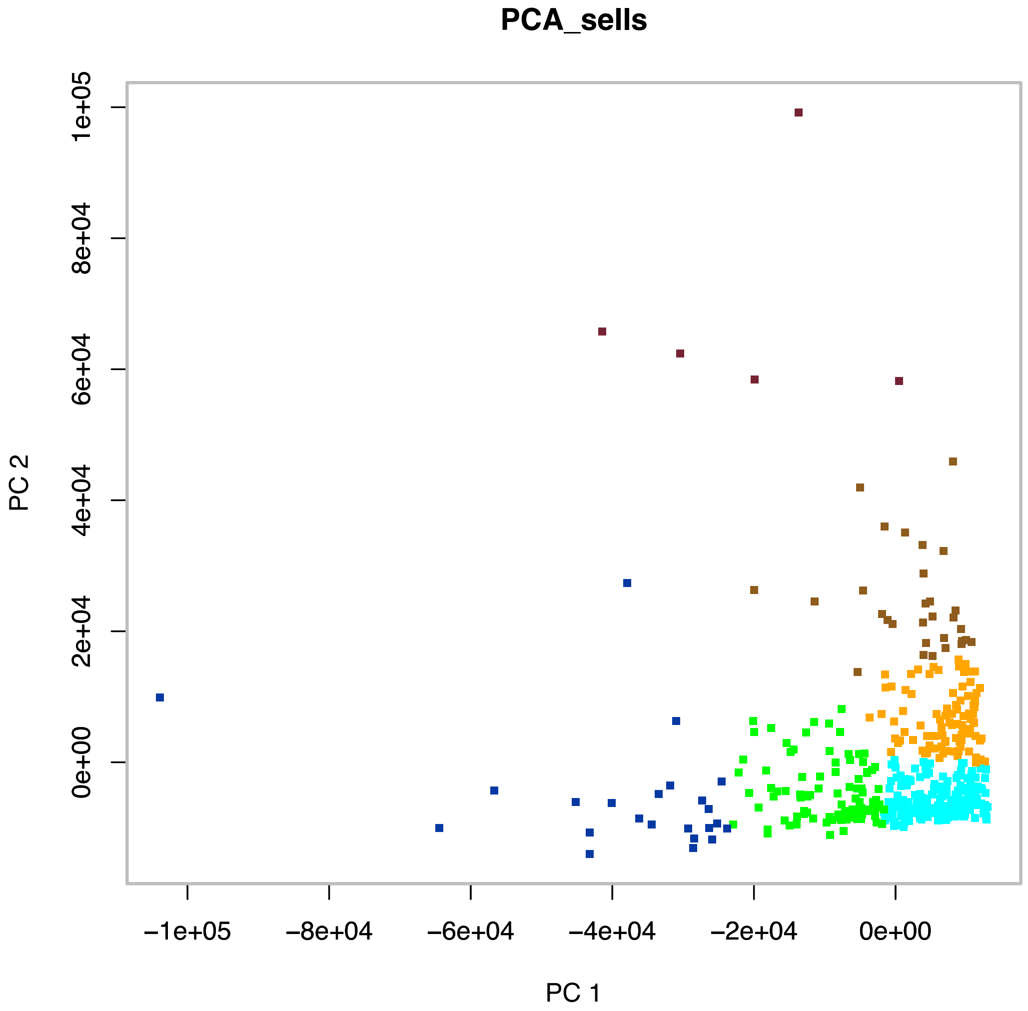
Do kmeans and choose k through the cost function

It’s good to pick a **k** around the knee:
I choose 6 for it matches the number of labels



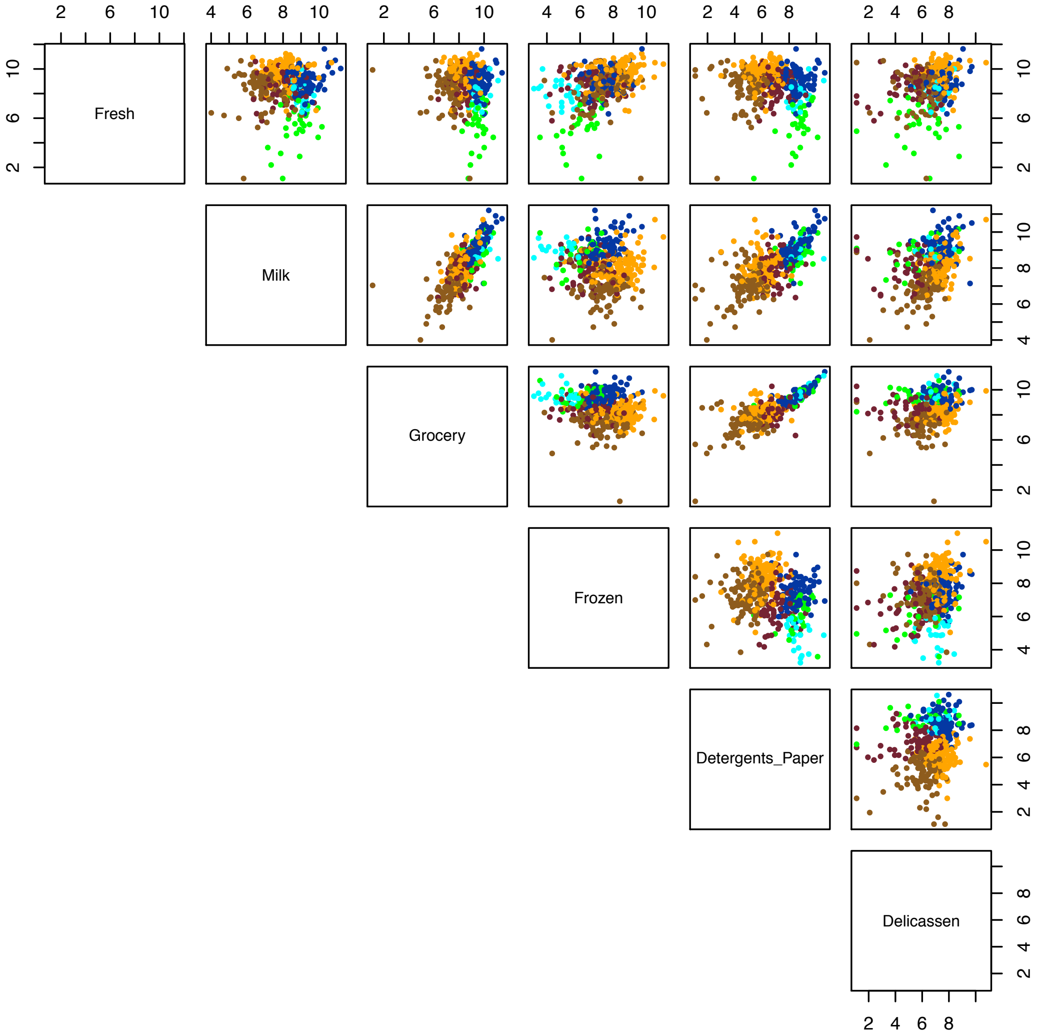
Visualization of the data (PCA)

- PCA does show some separation. **Colors are the clusters**
- Data points show large range of dynamics!



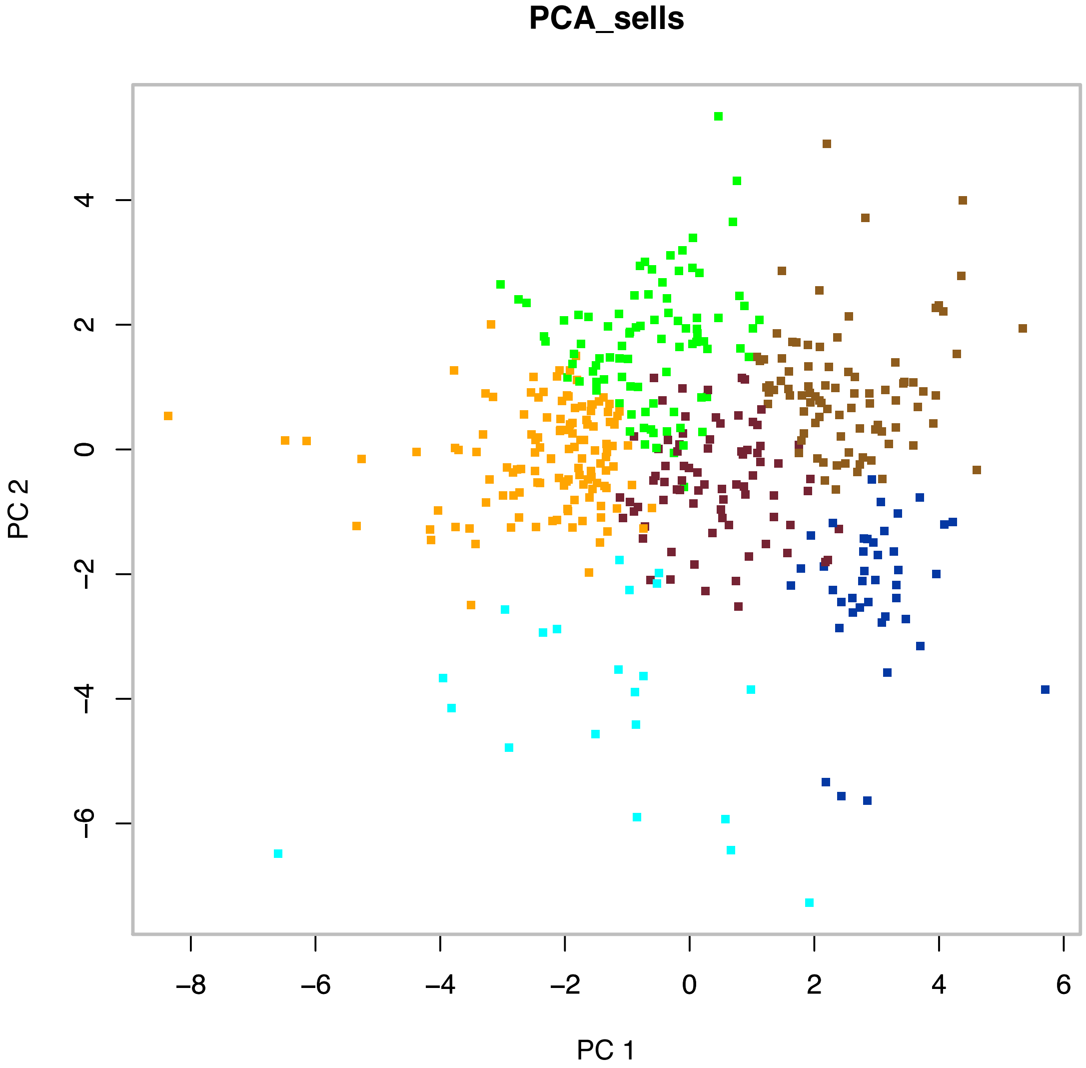
Do log transform of the data

- Log transform the data
- Do scatter plot matrix after the log transform
- Do the kmeans and color the clusters identified by k-means



PCA after log transformation: Clusters

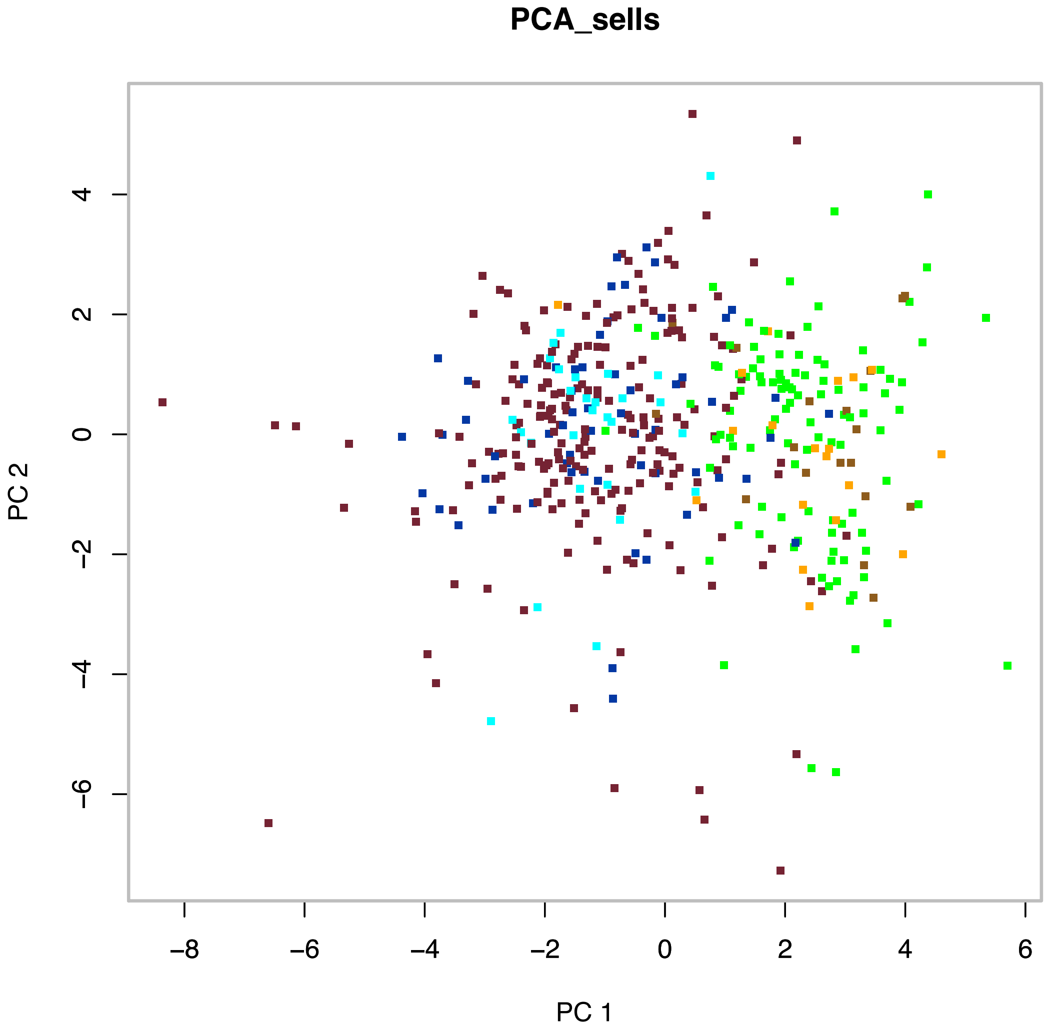
Colors show the **clusters** identified by k-means



PCA after log transformation

Colors show the **Channel-region labels**

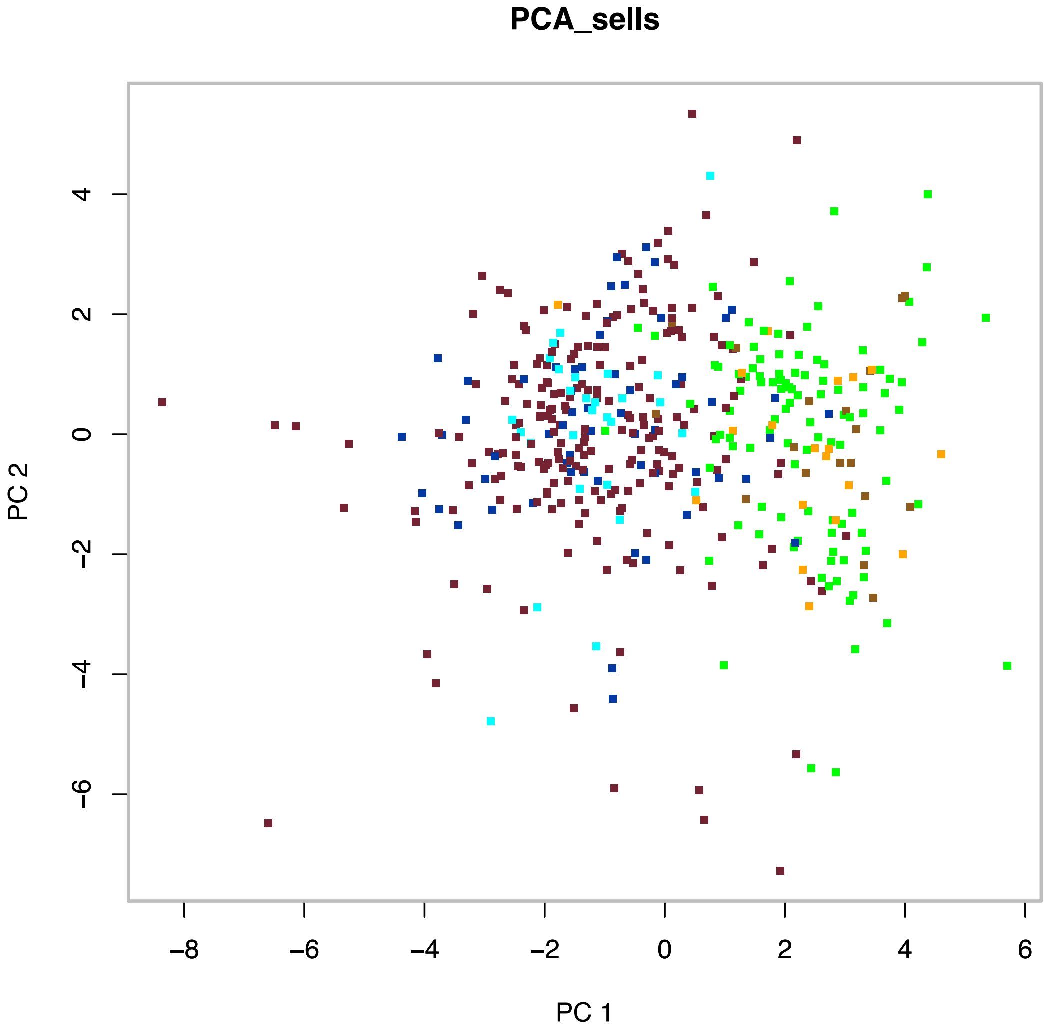
What does this tell us?



PCA after log transformation

Colors show the **Channel-region labels**

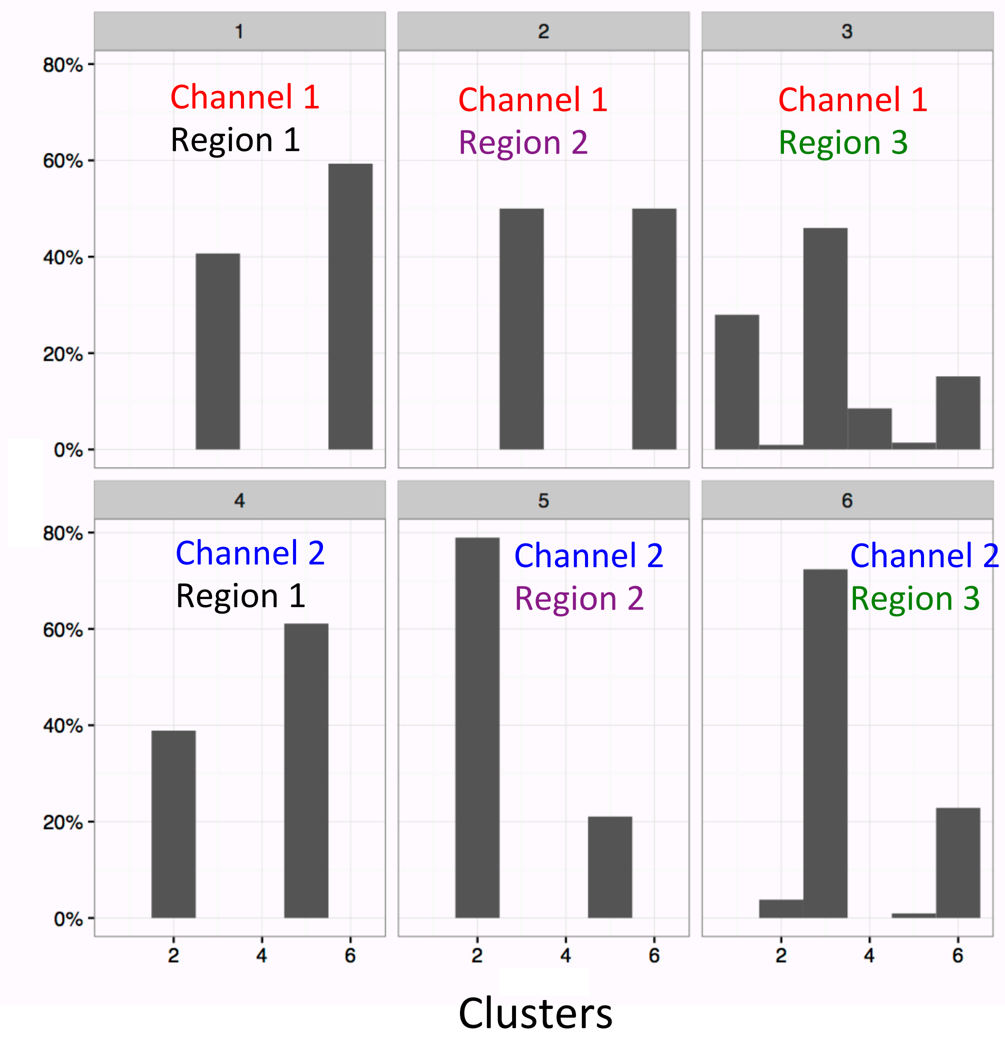
Channels differ a lot



Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the **6 cluster centers**.

- **What do you see?**

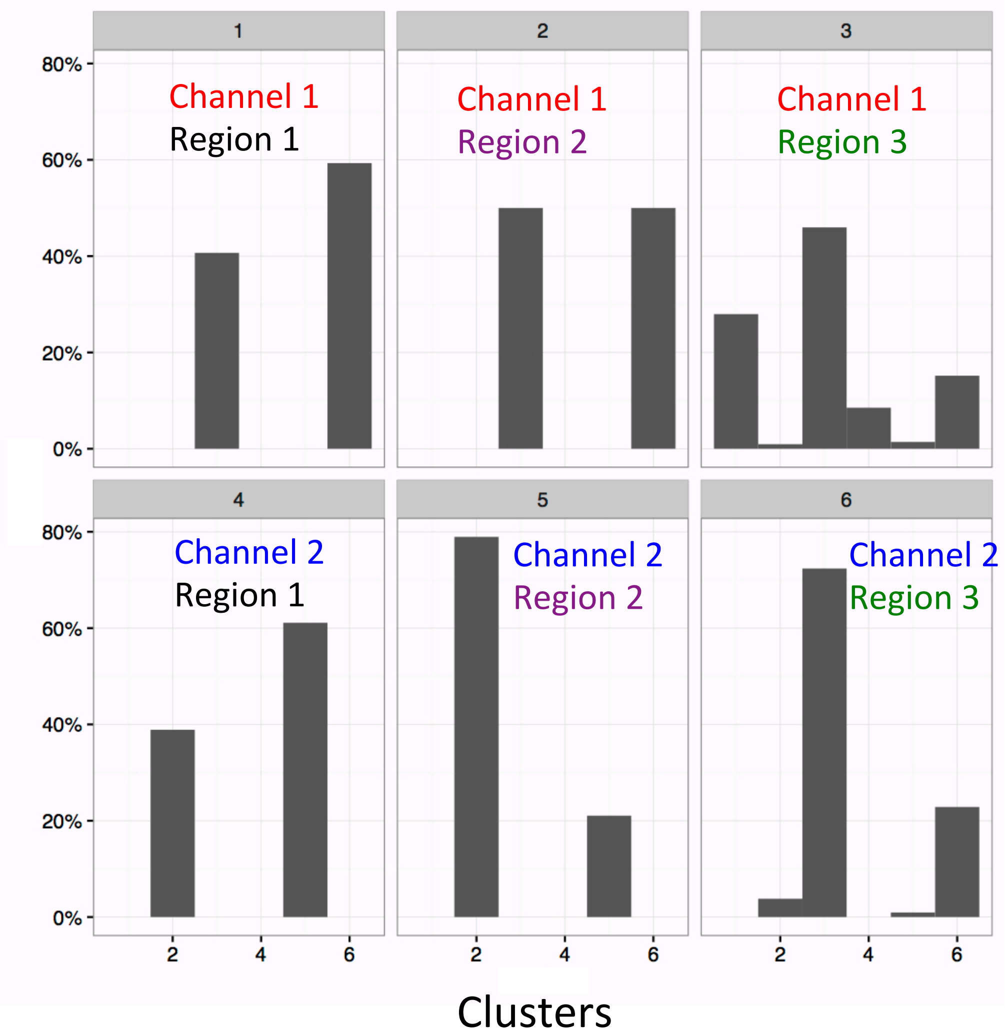


Channel1: Horeca
Channel2: Retail

Region1: Lisbon
Region2: Oporto
Region3: Other

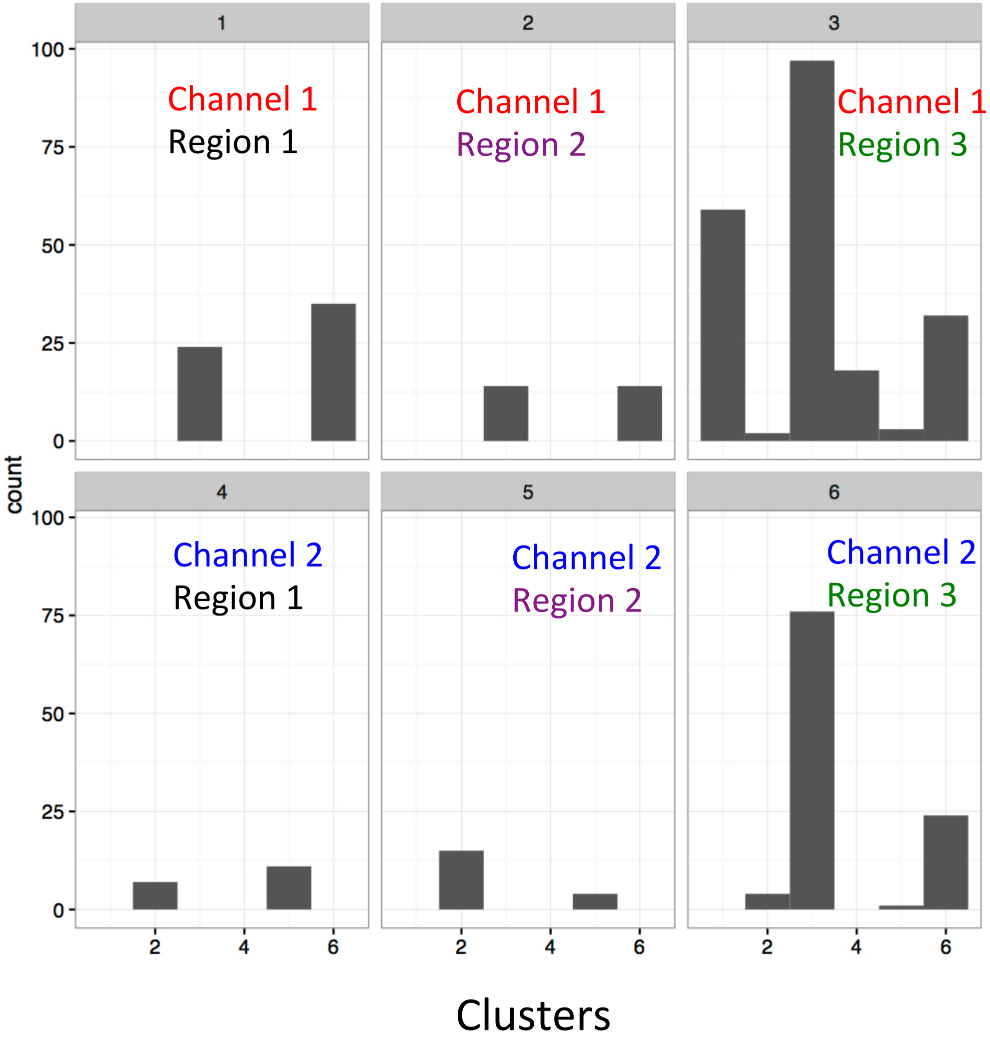
Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the 6 cluster centers.
- **Channels are significantly different!**
- **Region 3 is special**
- **Is it enough to plot the percentage?**



Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the 6 cluster centers.
- **Channels are significantly different!**
- **Region 3 is special**
- **Count matters depending on the purpose**



Q. What can we do with cluster center histograms?

A. investigate the feature patterns of data groups

B. Classify new data with the cluster center histograms.

C. Both A and B.

Markov Chain

- Motivation
- Definition of Markov model
- Graph representation – Markov chain
- Transition probability matrix
- The stationary Markov chain
- The pageRank algorithm

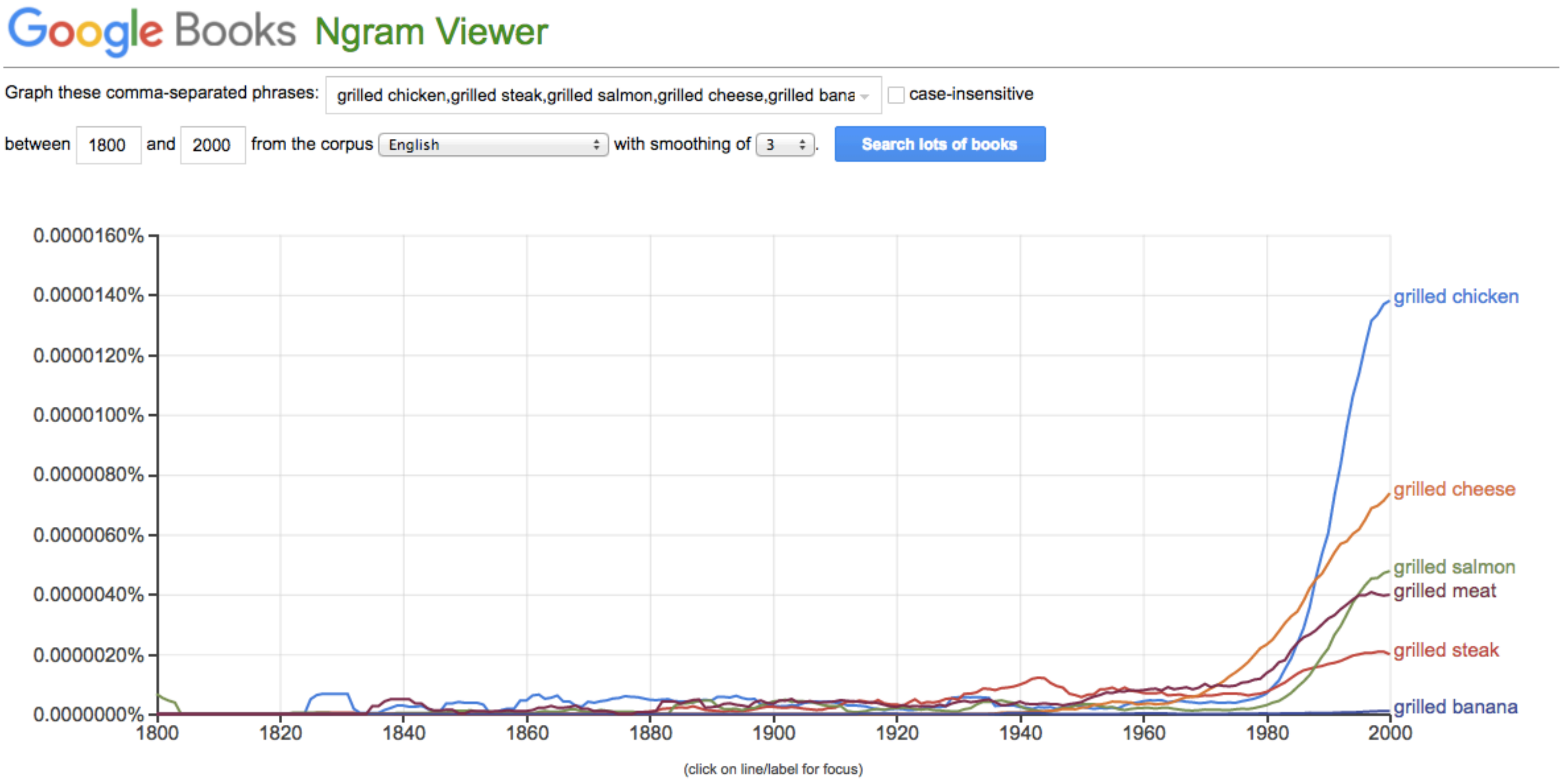
Motivation

- So far, the processes we learned such as **Bernoulli and Poisson** process are sequences of **independent** trials.
- There are a lot of real world situations where sequences of events are **Not independent** In comparison.
- Markov chain is one type of characterization of a series of **dependent** trials.

An example of dependent events in a sequence

I had a glass of wine with my grilled __________

An example of dependent events in a sequence

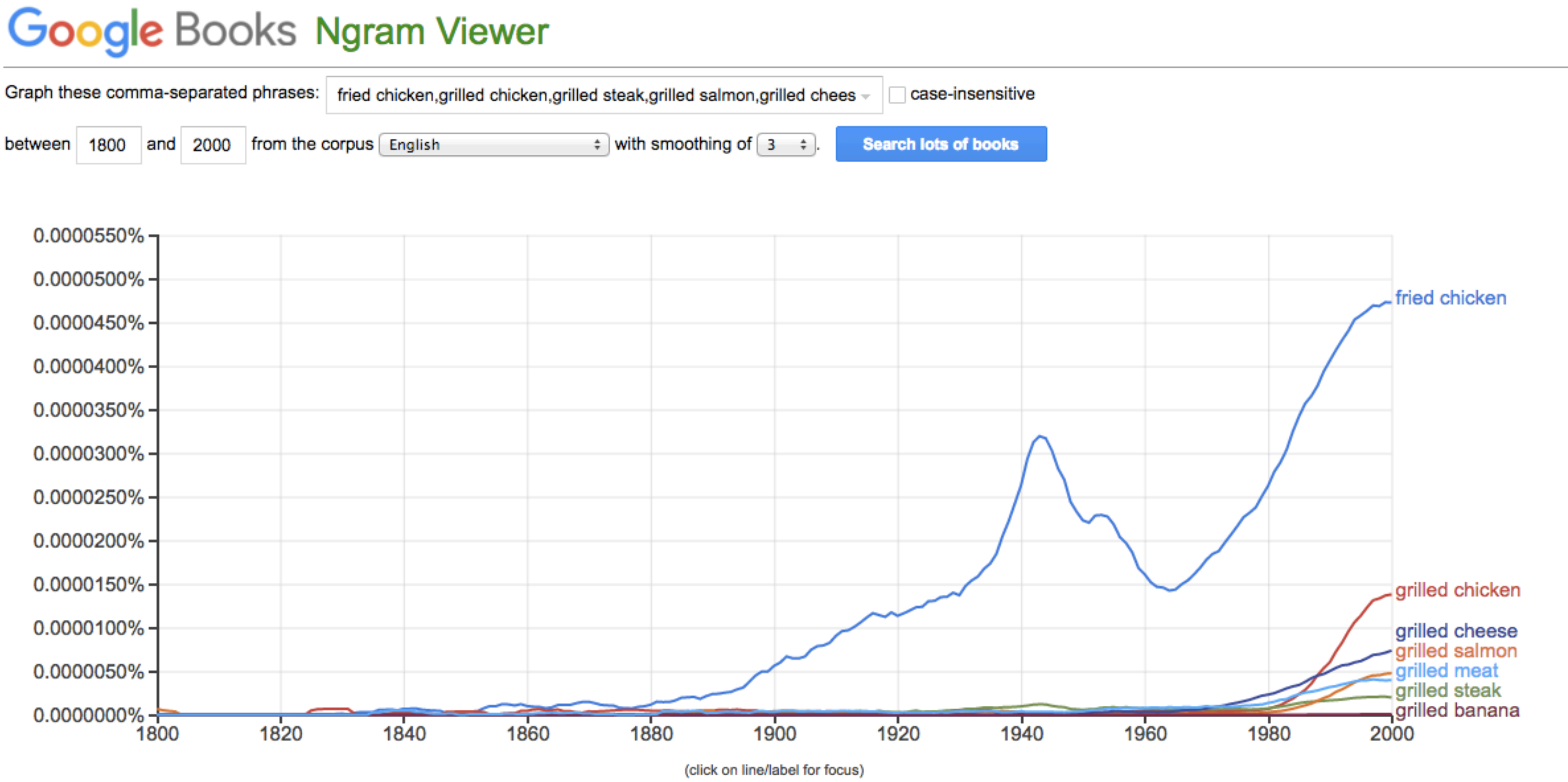


Graph these comma-separated phrases: grilled chicken,grilled steak,grilled salmon,grilled cheese,grilled bana

☐ case-insensitive

between 1800 and 2000 from the corpus English with smoothing of 3.

An example of dependent events in a sequence



Graph these comma-separated phrases: fried chicken,grilled chicken,grilled steak,grilled salmon,grilled chees

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between 1800 and 2000 from the corpus English with smoothing of 3.

Markov chain

- Markov chain is a process in which outcome of any trial in a sequence is **conditioned by the outcome of the trial immediately preceding, but not by earlier ones.**
- Such dependence is called **chain dependence**

Andrey Markov (1856-1922)

Markov chain in terms of probability

- Let X_0, X_1,… be a sequence of discrete finite-valued random variables
- The sequence is a Markov chain if the probability distribution X_t only depends on the distribution of the immediately preceding random variable X_{t-1}

$$\boxed{P(X_t|X_0...,X_{t-1}) = P(X_t|X_{t-1})}$$

- If the conditional probabilities (transition probabilities) do **NOT change with time**, it's called **constant Markov chain.**

$$P(X_t|X_{t-1}) = P(X_{t-1}|X_{t-2}) = ... = P(X_1|X_0)$$

Coin example

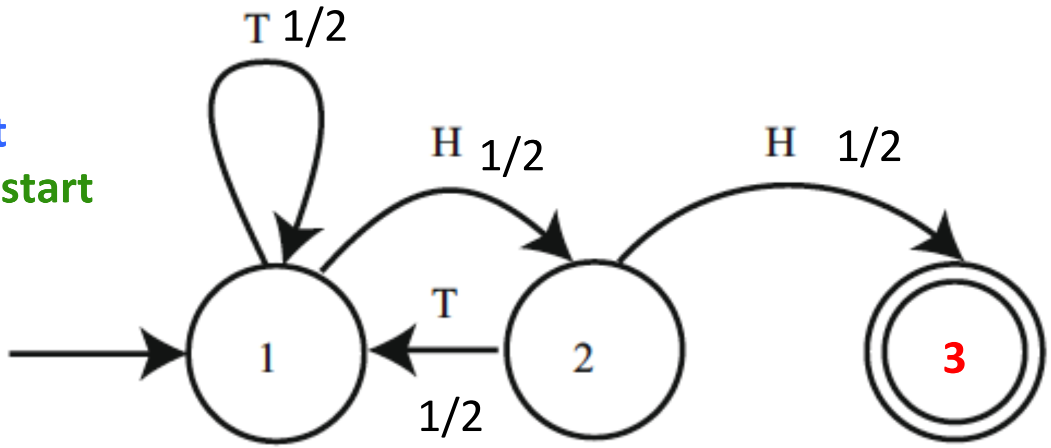
- Toss a fair coin until you see two heads in a row and then stop, what is the probability of stopping after exactly **n** flips?

- Use a state diagram, which is a **directed graph**. Circles are the states of likely outcomes. Arrow directions show the direction of transitions. Numbers over the arrows show transition probabilities.

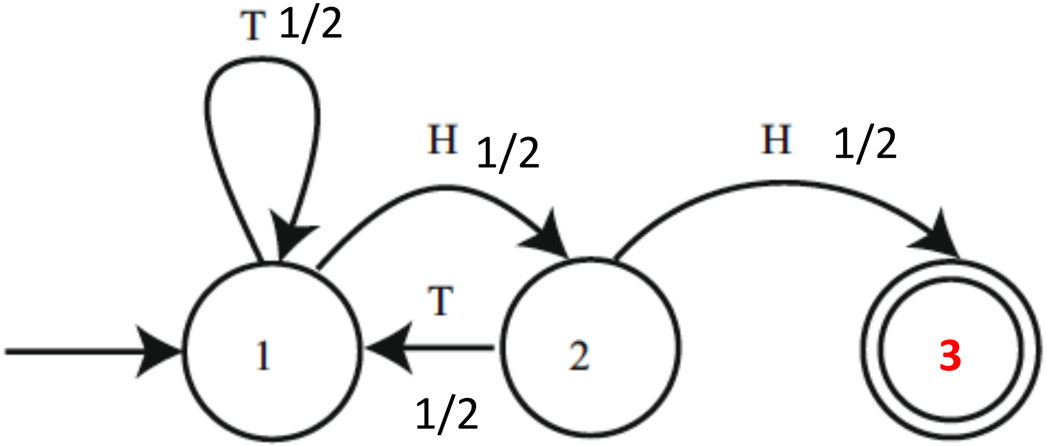
1 -> Start or just had tail/restart

2 -> had one head after start/restart

3 -> 2heads in a row/Stop



Is this a Markov chain? And why?



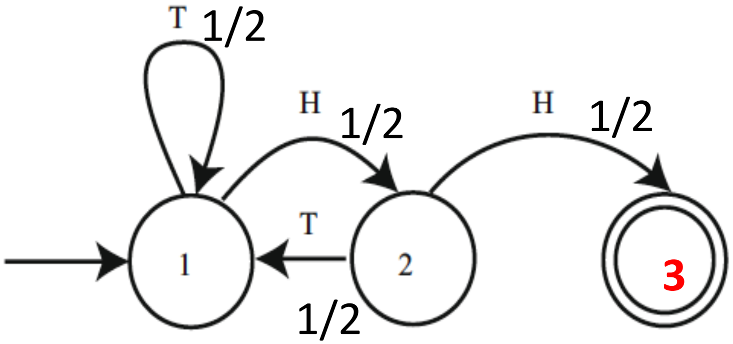
Is this a Markov chain? And why?

Yes. Because for each trial, the probability distribution of the outcomes is only conditioned on the previous trial.

The model helps form recurrence formula

- Let p_n be the probability of stopping after **n** flips

$$p_1 = 0 \quad p_2 = 1/4 \quad p_3 = 1/8 \quad p_4 = 1/8 \quad \ldots$$



The model helps form recurrence formula

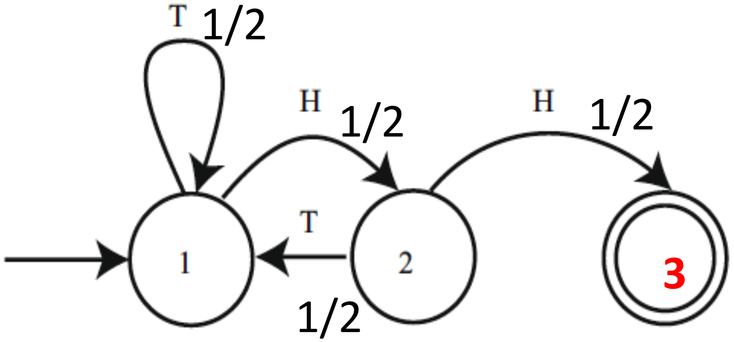
- Let p_n be the probability of stopping after **n** flips

$$p_1 = 0 \quad p_2 = 1/4 \quad p_3 = 1/8 \quad p_4 = 1/8 \quad \ldots$$

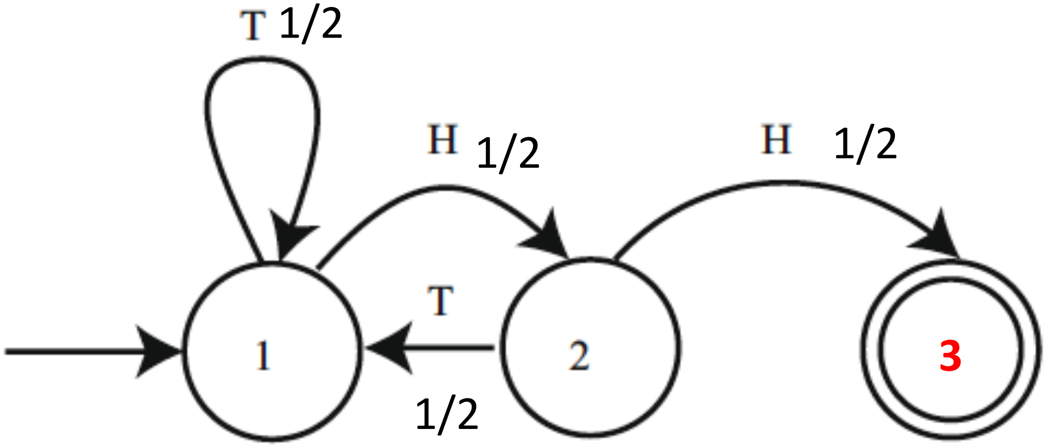
- If $n > 2$, there are two ways the sequence starts
  - Toss T and finish in n-1 tosses
  - Or toss HT and finish in n-2 tosses

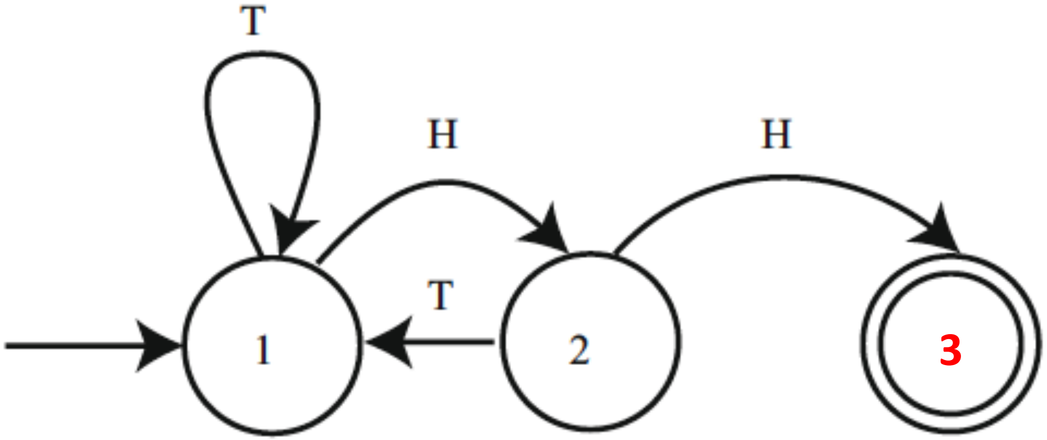
- So we can derive a recurrence relation

$$p_n = \underset{\underset{\text{P(T)}}{\uparrow}}{\frac{1}{2}} p_{n-1} + \underset{\underset{\text{P(HT)}}{\uparrow}}{\frac{1}{4}} p_{n-2}$$



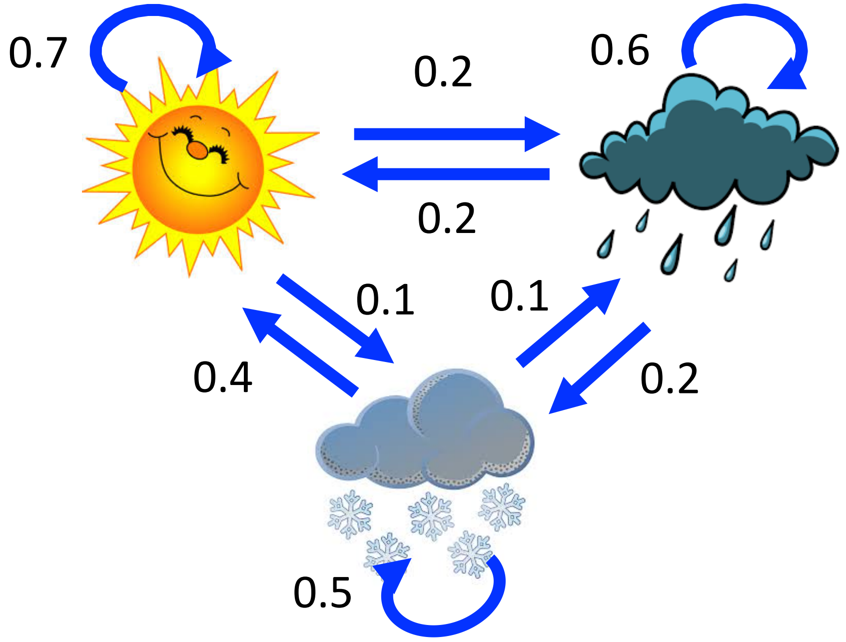
Transition probability btw states





Transition probability matrix: weather model

- Let's model daily weather as one of the three states (Sunny, Rainy, and Snowy) with Markov chain that has the transition probabilities as shown here.



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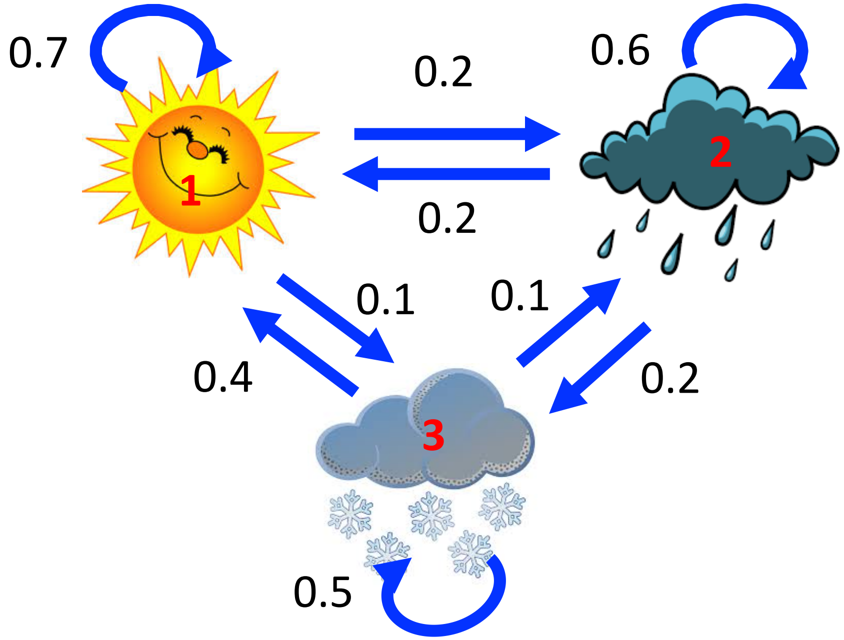

i, the current state at time point t
j, the next state at time point t+1

$$\boldsymbol{P} = \begin{array}{cl} \begin{matrix} \text{Sunny} & \text{Rainy} & \text{Snowy} \end{matrix} & \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} & \begin{matrix} \text{Sunny} \\ \text{Rainy} \\ \text{Snowy} \end{matrix} \end{array}$$

The transition probability matrix

Q: The transition probabilities for a node sum to 1

A. Yes.

B. No.

Only the row sum is 1, that is: the probabilities associated with outgoing arrows sum to 1.

Additional References

- Robert V. Hogg, Elliot A. Tanis and Dale L. Zimmerman. "Probability and Statistical Inference"
- Kelvin Murphy, "Machine learning, A Probabilistic perspective"

See you next time

See You!