“CS 374” Fall 2015 — Homework 2

Due Tuesday, September 15, 2015 at 10am

• • • Some important course policies • • •

- **You may work in groups of up to three people.** However, each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the names and NetIDs of each person contributing.
- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use. See the academic integrity policies on the course web site for more details.
- **Submit your pdf solutions in Moodle.** See instructions on the course website and submit a separate pdf for each problem. Ideally, your solutions should be typeset in LaTex. If you hand write your homework make sure that the pdf scan is easy to read. Illegible scans will receive no points.
- **Avoid the Three Deadly Sins!** There are a few dangerous writing (and thinking) habits that will trigger an automatic zero on any homework or exam problem. Yes, we are completely serious.
  - Give complete solutions, not just examples.
  - Declare all your variables.
  - Never use weak induction.
- Unlike previous editions of this and other theory courses we are not using the “I don’t know” policy.

See the course web site for more information.

If you have any questions about these policies,
please don’t hesitate to ask in class, in office hours, or on Piazza.

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1. (a) Draw an NFA with alphabet $\{0,1\}$, that accepts the language $\{w \mid w$ contains exactly one maximal substring of 0s of odd length$\}$. ($u \in \{0\}^+$ is a maximal substring of 0s of w if u occurs in w such that there is no 0 immediately to the left or right of u.) Note that the language does have strings with any number of maximal substrings of 0s of even length (but exactly one of odd length).

   (b) i. Draw an NFA for the regular expression $(100)^* + 10^*$.

   ii. Now apply the powerset construction (also called the subset construction) to your NFA, to obtain a DFA for the same language. Label the states of your DFA with names that are sets of states of your NFA. (Note that no points will be awarded for a DFA construction that is not derived from your NFA as directed.)

2. Let L be a regular language, accepted by a DFA $M = (\Sigma, Q, \delta, s, F)$. Show that $L^{\frac{1}{2}} = \{w \mid ww \in L\}$ is also a regular language by constructing an NFA $N = (\Sigma, Q', \delta', s', F')$ that accepts it. You should formally specify (using precise mathematical notation) the components of N in terms of those of M. Be sure to describe in English how your NFA works.

   [Hint: Your NFA could "guess" which state M will be in when it finishes seeing w.]

3. Suppose $N_1 = (\Sigma, Q_1, \delta_1, s_1, F_1)$ and $N_2 = (\Sigma, Q_2, \delta_2, s_2, F_2)$ are two NFAs (possibly with ε-moves) such that $F_1 = \{f_1\}$ and $F_2 = \{f_2\}$. Assume that $Q_1 \cap Q_2 = \emptyset$. Below N_{comp}, N_{star}, N_{cat} and N_{union} are purported constructions for NFAs accepting $\overline{L(N_1)}$, $L(N_1)^*$, $L(N_1)L(N_2)$ and $L(N_1) \cup L(N_2)$ respectively. You should give counter-examples in each case.

   (a) Let N_{comp} be obtained by swapping the sets of final and non-final states. Formally, let $N_{\text{comp}} = (\Sigma, Q_1, \delta_1, s_1, Q_1 - F_1)$.

   Give an example of N_1 such that $L(N_{\text{comp}}) \neq \overline{L(N_1)}$. (Describe the languages $L(N_{\text{comp}})$ and $\overline{L(N_1)}$ explicitly.)

   (b) Let N_{star} be obtained from N_1 by adding an ε-move from f_1 to s_1, and also setting the start state to be a final state. Formally, let $N_{\text{star}} = (\Sigma, Q_1, \delta, s_1, F)$, where $F = \{f_1, s_1\}$ and $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathscr{P}(Q)$ is defined as follows.

$$\delta(q,a) = \begin{cases} \delta_1(f_1, \varepsilon) \cup \{s_1\} & \text{if } (q,a) = (f_1, \varepsilon) \\ \delta_1(q,a) & \text{otherwise} \end{cases}$$

   Give an example of N_1 such that $L(N_{\text{star}}) \neq L(N_1)^*$. (Describe the languages $L(N_{\text{star}})$ and $L(N_1)^*$ explicitly.)

In the next two parts, we write replace(S, q, q') to denote the set obtained by modifying a set S by replacing q by q' if it occurs in S: i.e.,

$$\text{replace}(S, q, q') = \begin{cases} S & \text{if } q \notin S, \\ S \cup \{q'\} - \{q\} & \text{if } q \in S. \end{cases}$$

(c) Let N_{cat} be obtained by merging the only final state f_1 of N_1 with the start state s_2 of N_2 (to obtain a state denoted by f_1), and setting f_2 to be the final state. Formally, let $N_{\text{cat}} = (\Sigma, Q, \delta, s, F)$ where $Q = Q_1 \cup Q_2 - \{s_2\}$, $s = s_1$, $F = F_2$ and $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathscr{P}(Q)$ is defined as follows.

$$\delta(q, a) = \begin{cases} \delta_1(f_1, a) \cup \text{replace}(\delta_2(s_2, a), s_2, f_1) & \text{if } q = f_1 \\ \delta_1(q, a) & \text{if } q \in Q_1 - \{f_1\} \\ \text{replace}(\delta_2(q, a), s_2, f_1) & \text{if } q \in Q_2 - \{s_2\} \end{cases}$$

Give an example of N_1 and N_2 such that $L(N_{\text{cat}}) \neq L(N_1)L(N_2)$. (Describe the languages $L(N_{\text{cat}})$ and $L(N_1)L(N_2)$ explicitly.)

(d) Let N_{union} be obtained by merging the start states of N_1 and N_2 into a single state (denoted by s_1), and setting f_1 and f_2 to be the final states. Formally, let $N_{\text{union}} = (\Sigma, Q, \delta, s, F)$, where $Q = Q_1 \cup Q_2 - \{s_2\}$, $s = s_1$, $F = F_1 \cup F_2$ and $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathscr{P}(Q)$ is defined as follows.

$$\delta(q, a) = \begin{cases} \delta_1(s_1, a) \cup \text{replace}(\delta_2(s_2, a), s_2, s_1) & \text{if } q = s_1 \\ \delta_1(q, a) & \text{if } q \in Q_1 - \{s_1\} \\ \text{replace}(\delta_2(q, a), s_2, s_1) & \text{if } q \in Q_2 - \{s_2\} \end{cases}$$

Give an example of N_1 and N_2 such that $L(N_{\text{union}}) \neq L(N_1) \cup L(N_2)$. (Describe the languages $L(N_{\text{union}})$ and $L(N_1) \cup L(N_2)$ explicitly.)