"CS 374" Fall 2015 — Homework 3

Due Tuesday, September 22, 2015 at 10am

••• Some important course policies •••

- **You may work in groups of up to three people.** However, each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the names and NetIDs of each person contributing.
- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use. See the academic integrity policies on the course web site for more details.
- **Submit your pdf solutions in Moodle.** See instructions on the course website and submit a separate pdf for each problem. Ideally, your solutions should be typeset in LaTex. If you hand write your homework make sure that the pdf scan is easy to read. Illegible scans will receive no points.
- **Avoid the Three Deadly Sins!** There are a few dangerous writing (and thinking) habits that will trigger an automatic zero on any homework or exam problem. Yes, we are completely serious.
  - Give complete solutions, not just examples.
  - Declare all your variables.
  - Never use weak induction.
- Unlike previous editions of this and other theory courses we are not using the "I don't know" policy.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. Proving non-regularity.

   (a) For each of the following languages over the alphabet $\{0,1\}$, use the fooling-set method to show that it is not regular.

      i. $\{0^n 1^n x x \mid 0 \le n \le 3, x \in \{0,1\}^+\}$.

      ii. $\{0^i 1^j \mid \gcd(i,j) = 1\}$.

   (b) In each of the following problems, show that L_1 is not regular using the fact that L is not regular, and relying on closure properties of regular languages. If necessary, your proof may use the fact that certain other languages are *regular* (but it should *not* rely on any language other than L being non-regular). Do not directly use the fooling-set argument.

      i. Given $L = \{a^i b^j \mid i > j \ge 0\}$ is not regular, show that $L_1 = \{a^i b^j c^k \mid i > j > k \ge 0\}$ is not regular.

      ii. Given $L = \{0^p \mid p \text{ is prime}\}$ is not regular, show that $L_1 = L \cup \{0^{2^n} \mid n \ge 0\}$ is not regular.

2. Give CFGs for the following languages, and clearly explain how they work and the role of each nonterminal. Grammars can be very difficult to understand, and if the grader does not understand how your construction is intended to generate the language, then you will receive 0 points.

   (a) $\{a^i b^j c^k \mid i + k < j\}$

   (b) $\{w \mid w \in \{0,1\}^*$ and w is *not* a palindrome$\}$

3. (a) Give a CFG for the language $L = \{w \mid w \in \{0,1\}^*$ and w has twice as many 0s as 1s$\}$.

   (Hint: *For a string u define $\Delta(u) = \#_0(u) - 2\#_1(u)$. Introduce intermediate variables that derive strings with $\Delta(u) = 1$ and $\Delta(u) = -1$ and use them to define a variable that derives L.*)

   (Further Hint: *Let u_i denote the prefix of u of length i. If $\Delta(u) = 1$, what can you say about the smallest i for which $\Delta(u_i)$ becomes 1? How does u split up at that position? If $\Delta(u) = -1$, what can you say about the smallest i such that $\Delta(u_i)$ becomes -1?*)

   (b) Prove the correctness of your grammar. You need to show, using mathematical induction, that $L \subseteq L(G)$ and $L(G) \subseteq L$, where G is your grammar.