

A *subsequence* of a sequence (for example, an array, linked list, or string), obtained by removing zero or more elements and keeping the rest in the same sequence order. A subsequence is called a *substring* if its elements are contiguous in the original sequence. For example:

- **SUBSEQUENCE**, **UBSEQU**, and the empty string ε are all substrings of the string **SUBSEQUENCE**;
- **SBSQNC**, **UEQUE**, and **EEE** are all subsequences of **SUBSEQUENCE** but not substrings;
- **QUEUE**, **SSS**, and **FOOBAR** are not subsequences of **SUBSEQUENCE**.

Describe recursive backtracking algorithms for the following problems. *Don't worry about running times.*

1. Given an array $A[1..n]$ of integers, compute the length of a **longest increasing subsequence**. A sequence $B[1..\ell]$ is *increasing* if $B[i] > B[i - 1]$ for every index $i \geq 2$. For example, given the array

$$\langle 3, \underline{1}, \underline{4}, 1, \underline{5}, 9, 2, \underline{6}, 5, 3, \underline{5}, 8, \underline{9}, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7 \rangle$$

your algorithm should return the integer 6, because $\langle 1, 4, 5, 6, 8, 9 \rangle$ is a longest increasing subsequence (one of many).

2. Given an array $A[1..n]$ of integers, compute the length of a **longest decreasing subsequence**. A sequence $B[1..\ell]$ is *decreasing* if $B[i] < B[i - 1]$ for every index $i \geq 2$. For example, given the array

$$\langle 3, 1, 4, 1, 5, \underline{9}, 2, \underline{6}, 5, 3, \underline{5}, 8, 9, 7, 9, 3, 2, 3, 8, \underline{4}, 6, \underline{2}, 7 \rangle$$

your algorithm should return the integer 5, because $\langle 9, 6, 5, 4, 2 \rangle$ is a longest decreasing subsequence (one of many).

3. Given an array $A[1..n]$ of integers, compute the length of a **longest alternating subsequence**. A sequence $B[1..\ell]$ is *alternating* if $B[i] < B[i - 1]$ for every even index $i \geq 2$, and $B[i] > B[i - 1]$ for every odd index $i \geq 3$. For example, given the array

$$\langle \underline{3}, \underline{1}, \underline{4}, \underline{1}, \underline{5}, 9, \underline{2}, \underline{6}, \underline{5}, 3, \underline{5}, 8, 9, \underline{7}, \underline{9}, \underline{3}, 2, 3, \underline{8}, \underline{4}, \underline{6}, \underline{2}, \underline{7} \rangle$$

your algorithm should return the integer 17, because $\langle 3, 1, 4, 1, 5, 2, 6, 5, 8, 7, 9, 3, 8, 4, 6, 2, 7 \rangle$ is a longest alternating subsequence (one of many).