1. We have n jobs $J_1, J_2, \ldots, J_n$ which we need to schedule on a machine. Each job J_i has a processing time t_i and a weight w_i. A schedule for the machine is an ordering of the jobs. Given a schedule, let C_i denote the finishing time of job J_i. For example, if job J_j is the first job in the schedule, its finishing time C_j is equal to t_j; if job J_j follows job J_i in the schedule, its finishing time C_j is equal to $C_i + t_j$. The weighted completion time of the schedule is $\sum_{i=1}^{n} w_i C_i$.

    - For the case when $w_i = 1$ for all i, *prove* that choosing the shortest job first is optimal.
    - Give an efficient algorithm that finds a schedule with minimum weighted completion time given arbitrary weights. Prove its correctness.

2. A party of n people have come to dine at a fancy restaurant and each person has ordered a different item from the menu. Let $D_1, D_2, \ldots, D_n$ be the items ordered by the diners. Since this is a fancy place, each item is prepared in a two-stage process. First, the head chef (there is only one head chef) spends a few minutes on each item to take care of the essential aspects and then hands it over to one of the many sous-chefs to finish off. Assume that there are essentially an unlimited number of sous-chefs who can work in parallel on the items once the head chef is done. Each item D_i takes h_i units of time for the head chef followed by s_i units of time for the sous-chef (the sous-chefs are all identical). The diners want all their items to be served at the same time which means that the last item to be finished defines the time when they can be served. The goal of the restaurant is to serve the diners as early as possible. Consider the following greedy algorithms that order the items according to different criteria. For each of them either describe a counter example that shows that the order does not yield an optimum solution or give a proof that the ordering yields an optimum solution for all instances.

    - Order the items in increasing order of $h_i + s_i$.
    - Order the items in decreasing order of $h_i + s_i$.
    - Order the items in increasing order of h_i.
    - Order the items in decreasing order of h_i.
    - Order the items in increasing order of s_i.
    - Order the items in decreasing order of s_i.