We've seen various classes of languages that are *closed under complement*: if a language is in the class, then so is its complement. For example, the complement of a regular language is also regular. The complement of a decidable language is also decidable. The complement of a language in **P** is also in **P** (why?). However, we do not know whether or not **NP** is closed under complement. Define **coNP** to be the set of complements of languages in **NP**. In other words, **coNP** is defined as the class of languages L such that $\overline{L}$ is in **NP**. The question of whether **NP** is closed under complement, is exactly the question of whether **NP** = **coNP**, which is a question related to the question of whether or not **P** = **NP**. In these problems, we investigate some relationships between **NP** and **coNP**.

1. We saw that **NP** is the class of languages of the form $L = \{x \mid \exists w \in \{0,1\}^{p(|x|)} \text{ s.t. } (x,w) \in L_0\}$ for some language L_0 in **P** and some polynomial p. Express any language L in **coNP** in terms of a language $L_1 \in$ **P**, similar to the above expression for languages in **NP**.

2. Suppose **NP** $\subseteq$ **coNP**. Show that then **NP** = **coNP**.

3. Show that if any **NP**-complete problem is in **coNP**, then **NP** = **coNP**.

   Hint: You may use the fact that if $L \in$ **coNP** and there is a polynomial time reduction from L' to L, then $L' \in$ **coNP**. Think about this part at home.