

- Suppose that you have just finished computing the array $dist[1..V, 1..V]$ of shortest-path distances between **all** pairs of vertices in an edge-weighted directed graph G . Unfortunately, you discover that you incorrectly entered the weight of a single edge $u \rightarrow v$, so all that precious CPU time was wasted. Or was it? Maybe your distances are correct after all!

In each of the following problems, let $w(u \rightarrow v)$ denote the weight that you used in your distance computation, and let $w'(u \rightarrow v)$ denote the correct weight of $u \rightarrow v$.

- Suppose $w(u \rightarrow v) > w'(u \rightarrow v)$; that is, the weight you used for $u \rightarrow v$ was *larger* than its true weight. Describe an algorithm that repairs the distance array in $O(V^2)$ time under this assumption. [Hint: For every pair of vertices x and y , either $u \rightarrow v$ is on the shortest path from x to y or it isn't.]
 - Maybe even that was too much work. Describe an algorithm that determines whether your original distance array is actually correct in $O(1)$ time, again assuming that $w(u \rightarrow v) > w'(u \rightarrow v)$. [Hint: Either $u \rightarrow v$ is the shortest path from u to v or it isn't.]
 - To think about later:** Describe an algorithm that determines in $O(VE)$ time whether your distance array is actually correct, even if $w(u \rightarrow v) < w'(u \rightarrow v)$.
 - To think about later:** Argue that when $w(u \rightarrow v) < w'(u \rightarrow v)$, repairing the distance array *requires* recomputing shortest paths from scratch, at least in the worst case.
- You—yes, *you*—can cause a major economic collapse with the power of graph algorithms!¹ The *arbitrage* business is a money-making scheme that takes advantage of differences in currency exchange. In particular, suppose that 1 US dollar buys 120 Japanese yen; 1 yen buys 0.01 euros; and 1 euro buys 1.2 US dollars. Then, a trader starting with \$1 can convert their money from dollars to yen, then from yen to euros, and finally from euros back to dollars, ending with \$1.44! The cycle of currencies $\$ \rightarrow \text{¥} \rightarrow \text{€} \rightarrow \$$ is called an **arbitrage cycle**. Of course, finding and exploiting arbitrage cycles before the prices are corrected requires extremely fast algorithms.

Suppose n different currencies are traded in your currency market. You are given the matrix $R[1..n]$ of exchange rates between every pair of currencies; for each i and j , one unit of currency i can be traded for $R[i, j]$ units of currency j . (Do *not* assume that $R[i, j] \cdot R[j, i] = 1$.)

- Describe an algorithm that returns an array $V[1..n]$, where $V[i]$ is the maximum amount of currency i that you can obtain by trading, starting with one unit of currency 1, assuming there are no arbitrage cycles.
- Describe an algorithm to determine whether the given matrix of currency exchange rates creates an arbitrage cycle.
- *To think about later:** Modify your algorithm from part (b) to actually return an arbitrage cycle, if such a cycle exists.

¹No, you can't.