Graphs

Lecture17

Two techniques for algorithm design

- We have seen recursion techniques so far
- Next few weeks, we will see graph algorithms
- E.g. Dijkstra…

Two techniques of algorithm design

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- Next few weeks, we will see graph algorithms
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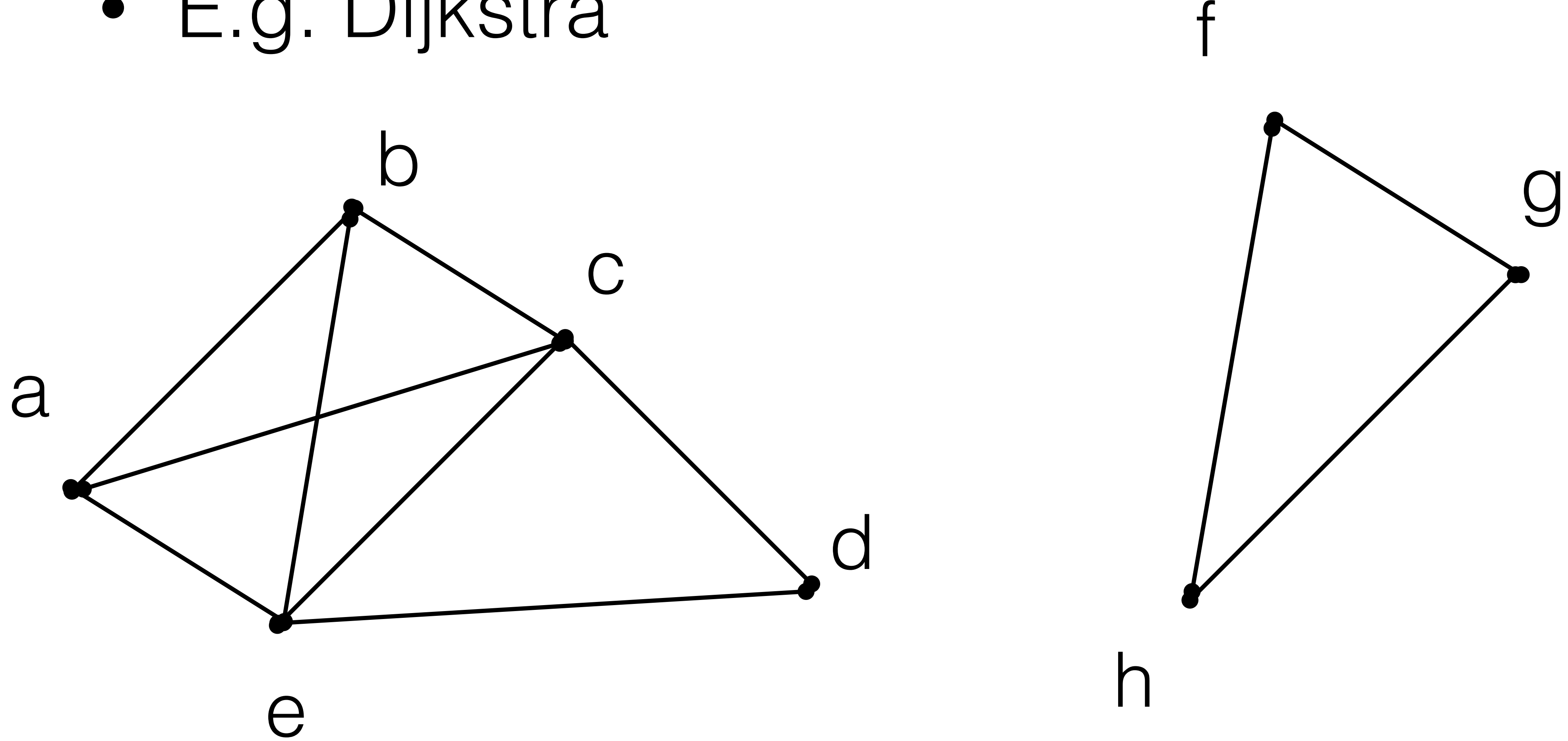
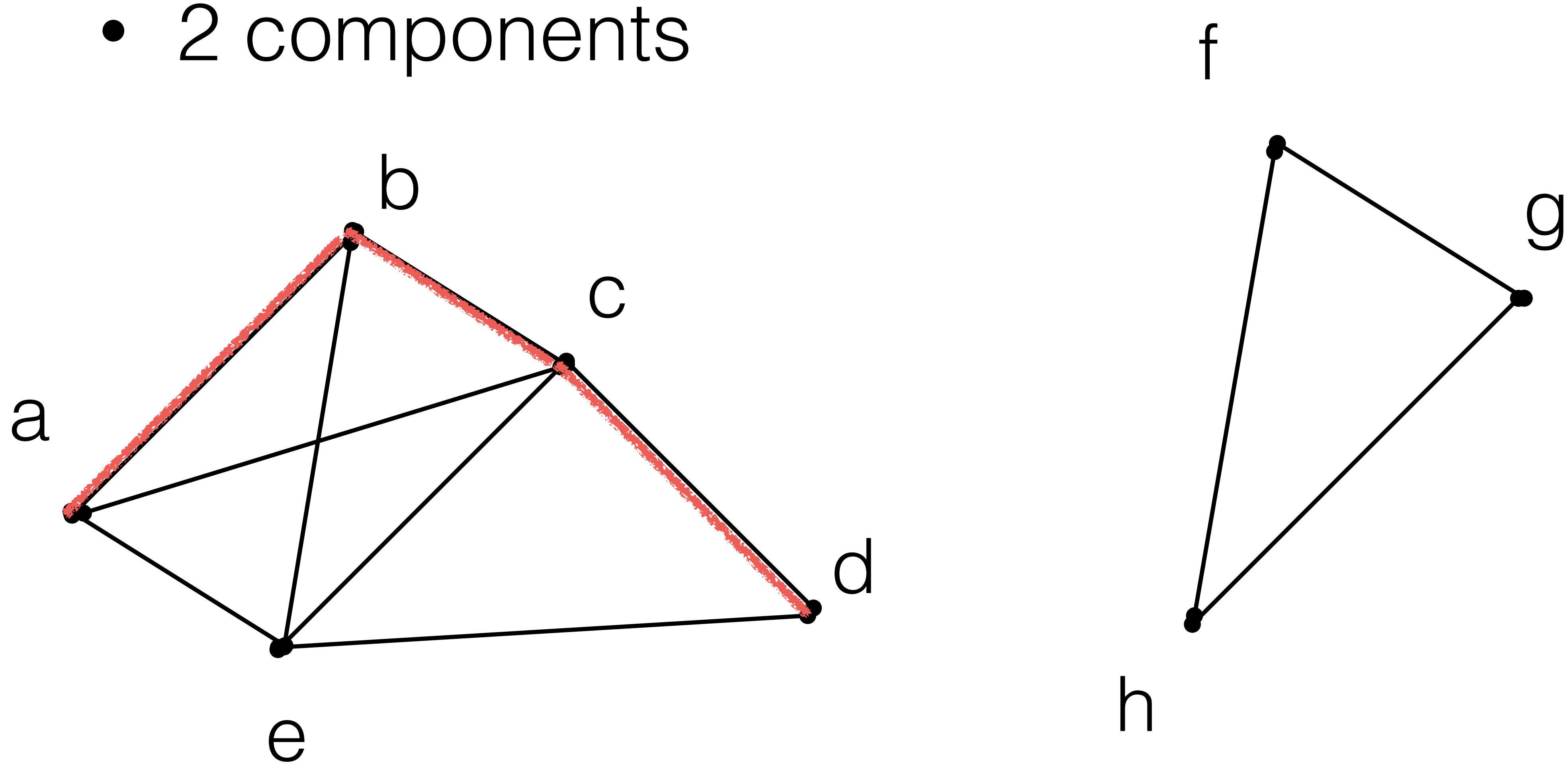


Graphs

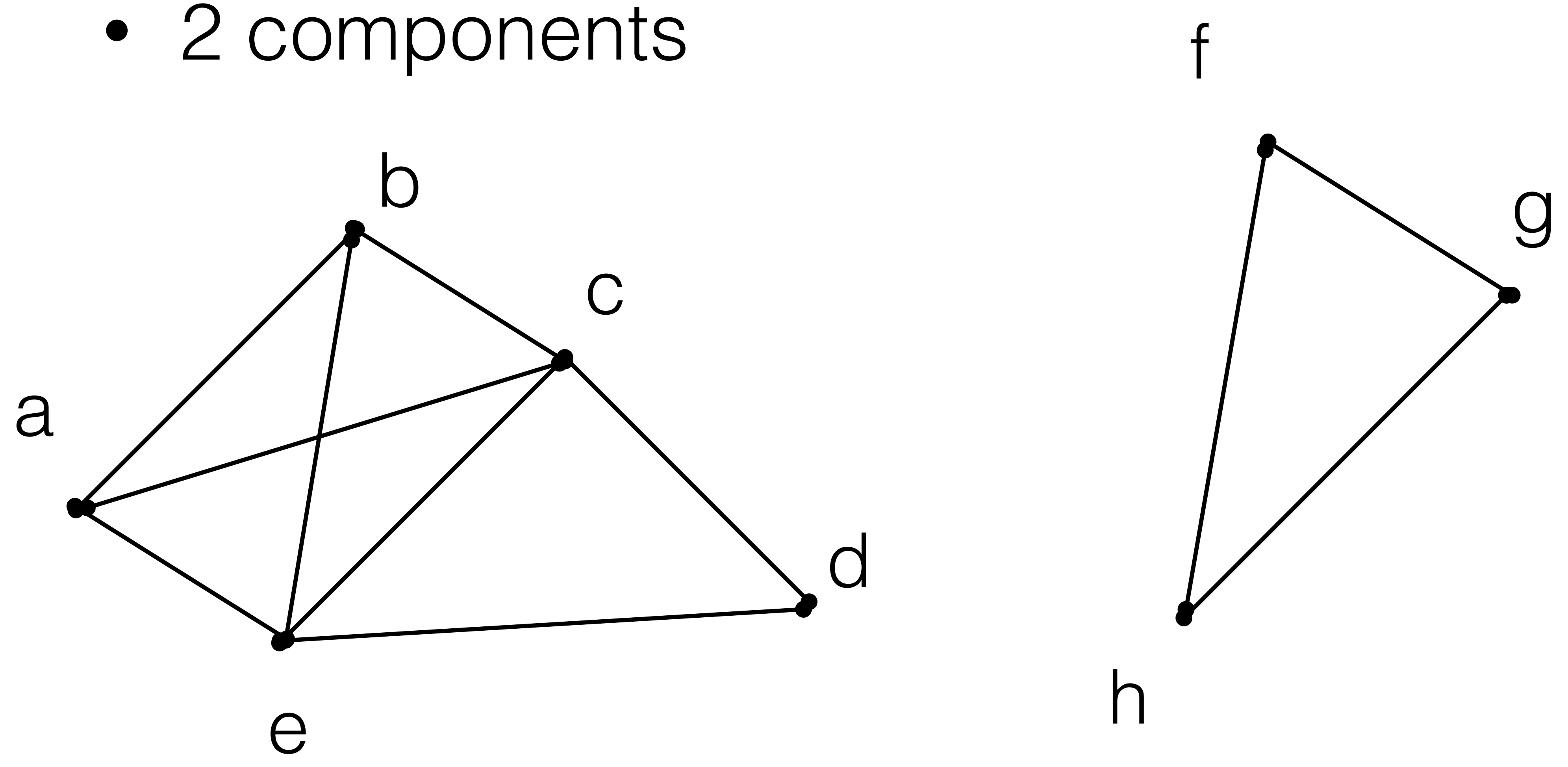
- 8 vertices
- 10 edges
- 2 components



Graph Representation

- 8 vertices
- 10 edges
- 2 components

Just a representation of the graph!



Graph Definition

- A (simple) graph is
  - Non empty finite set V, called vertex set
  - Set E of pairs of vertices, called edges.
  - Undirected (u,v)={u,v}
  - Directed (u,v) = u → v

Graph Representation

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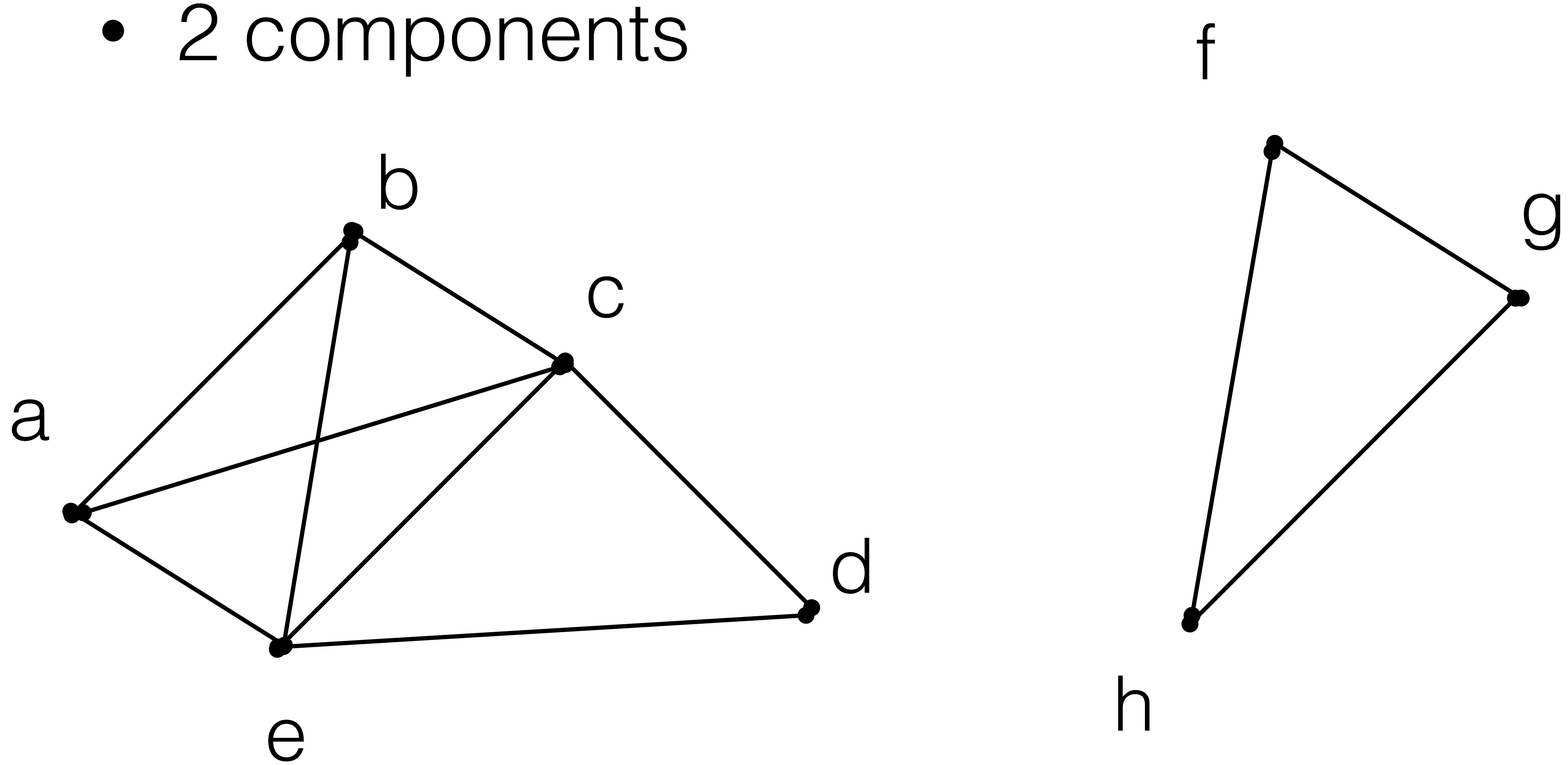

Graph Representation

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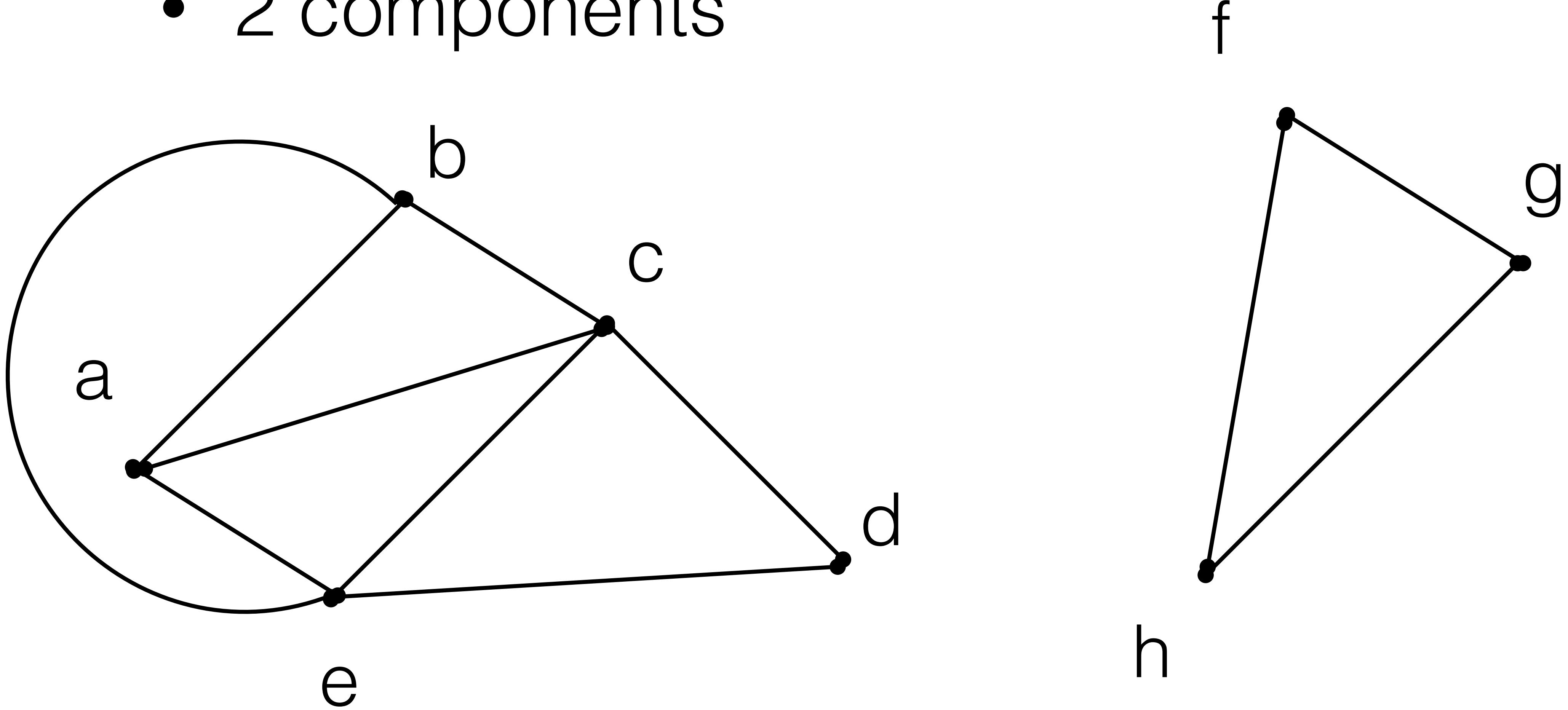
Planar Graph! independent of the representation



Graph Representation

- 8 vertices
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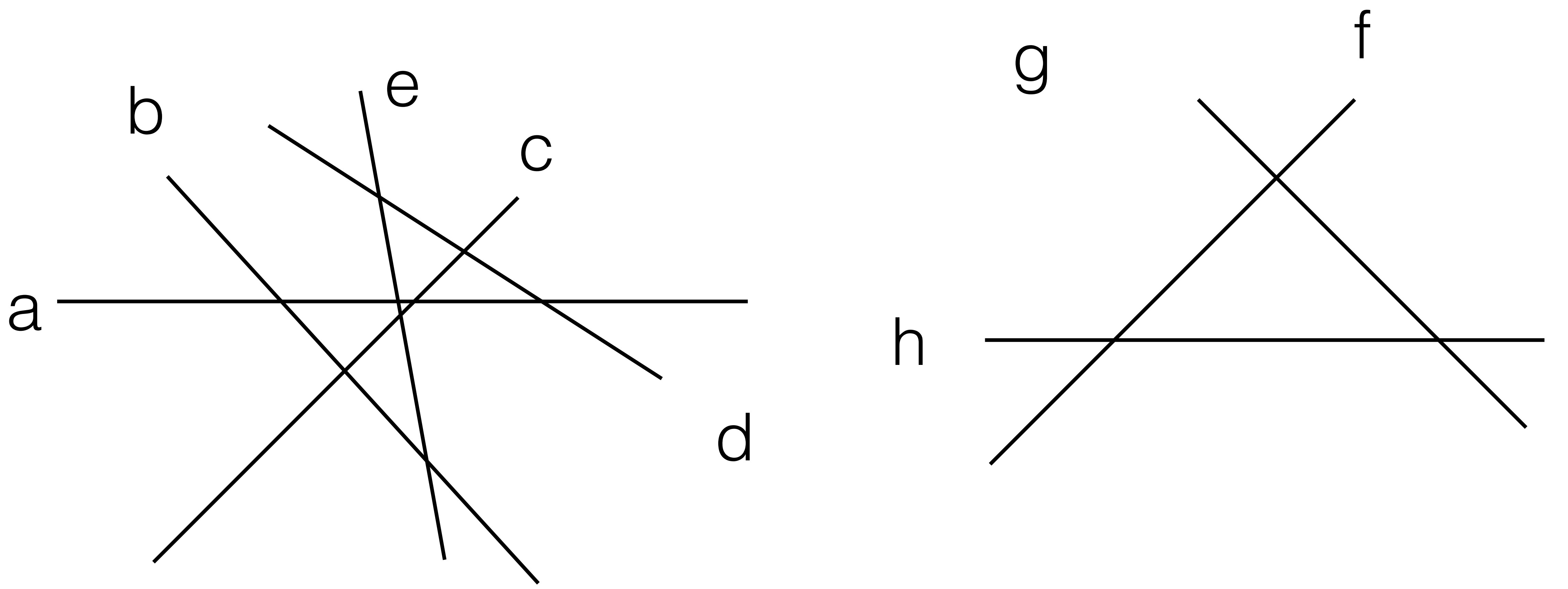
Another representation:
Intersection graph



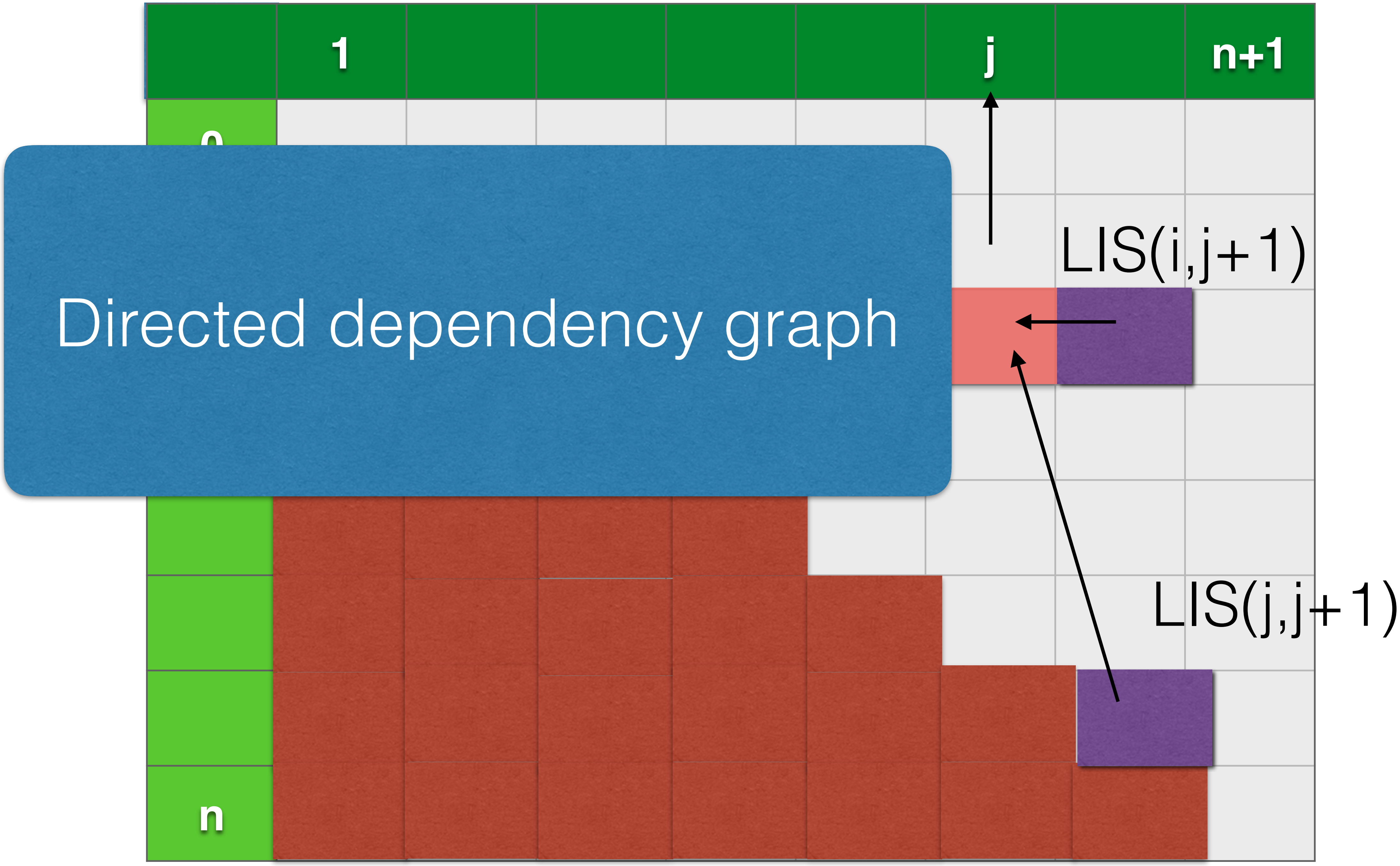
Graph Representation

- 8 vertices
- 10 edges
- 2 components

Another representation: Intervals

For i<j

$$LIS(i,j) = \begin{cases} 0 & \text{if } j > n \\ LIS(i, j+1) & \text{if } A[i] \geq A[j] \\ \max\{LIS(i,j+1),\ 1 + LIS(j, j+1)\} & \text{otherwise} \end{cases}$$



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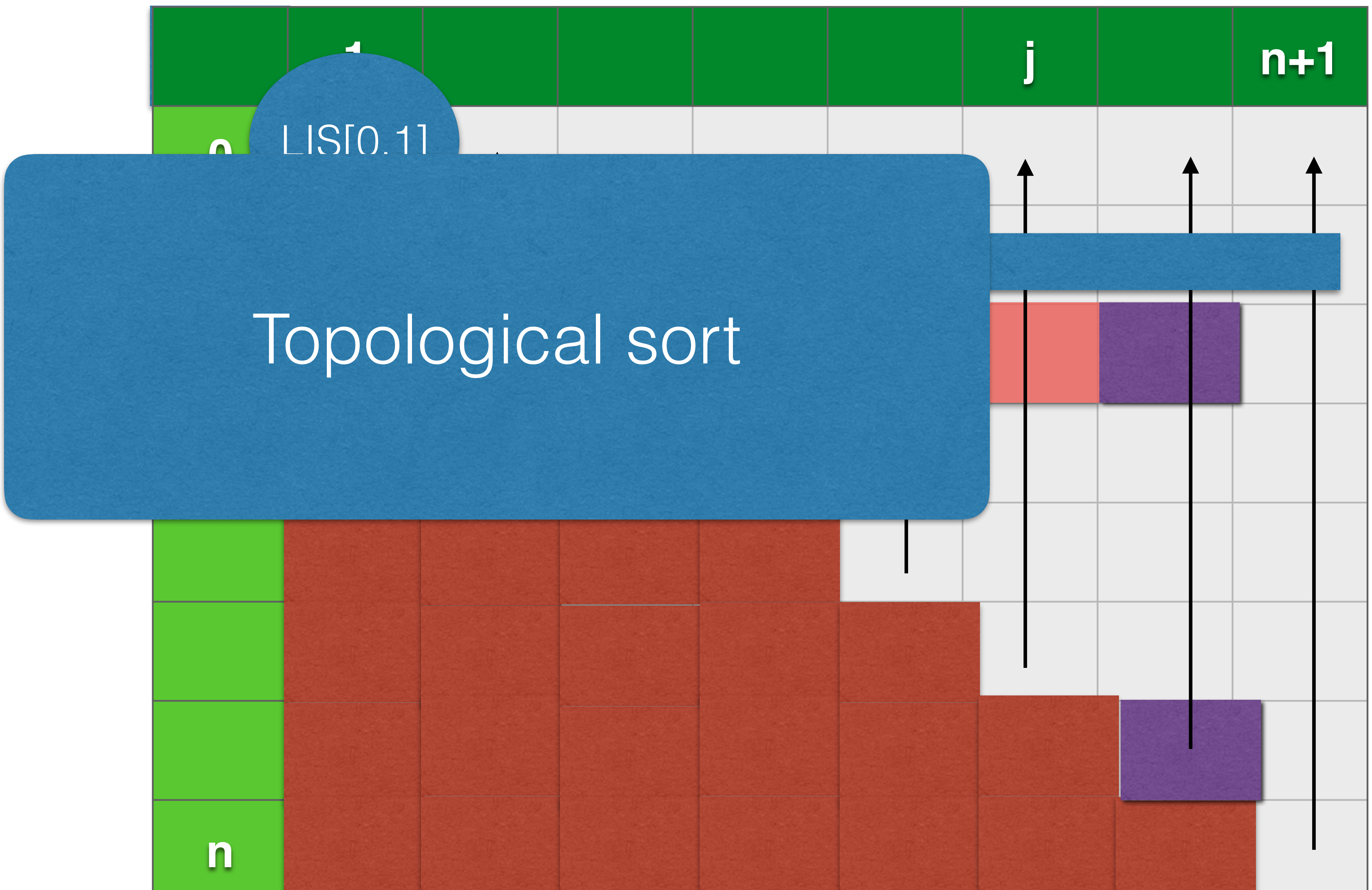
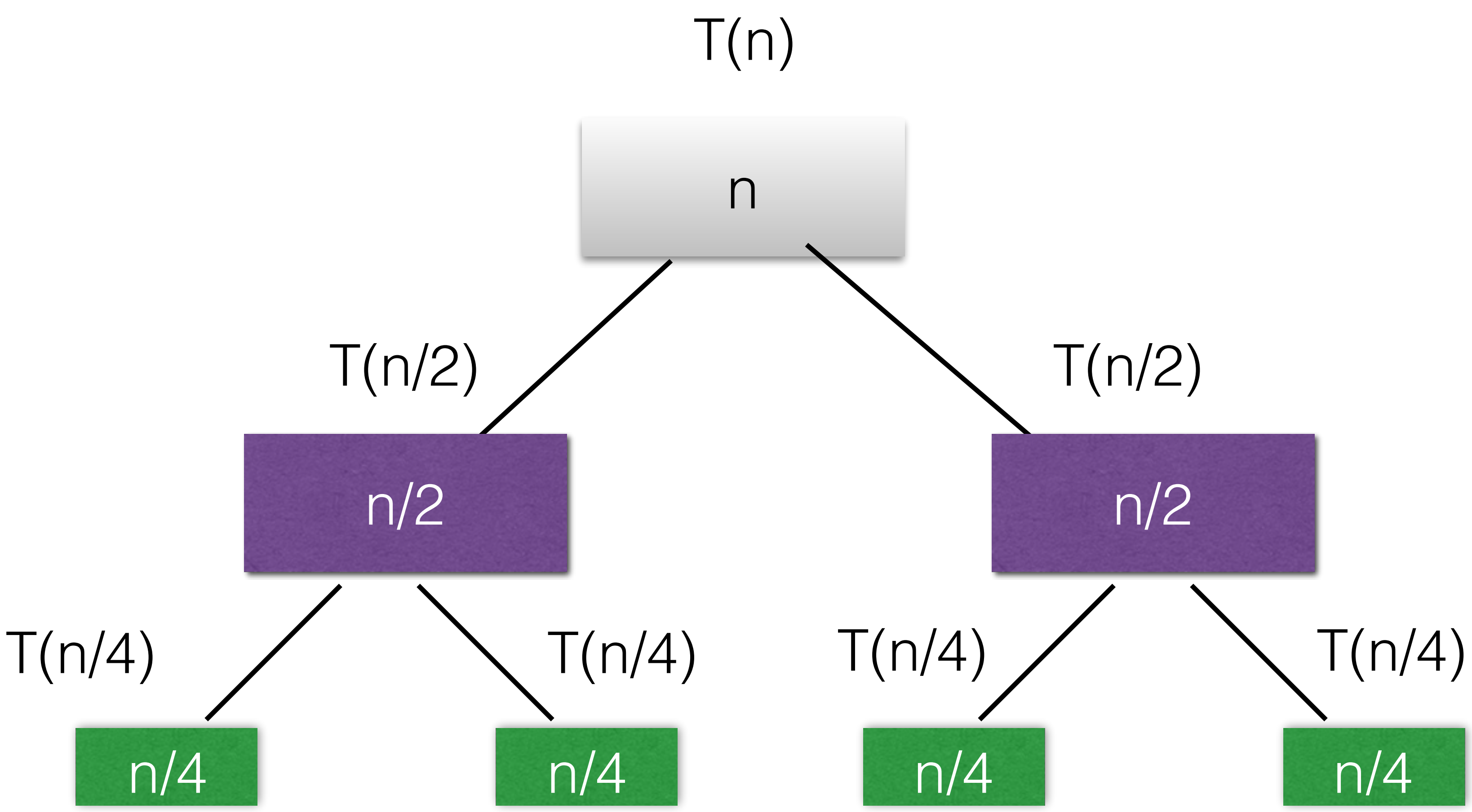
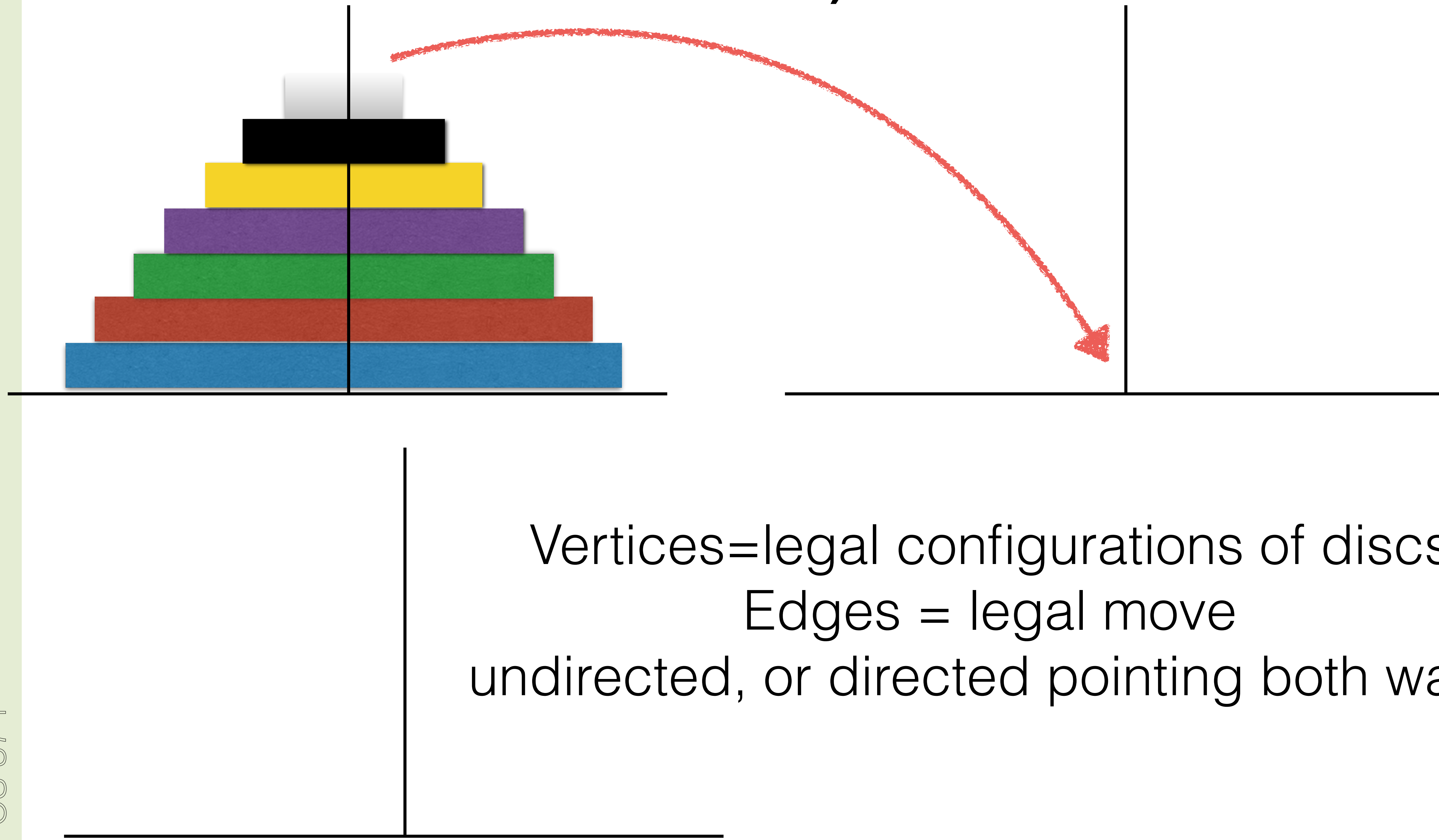


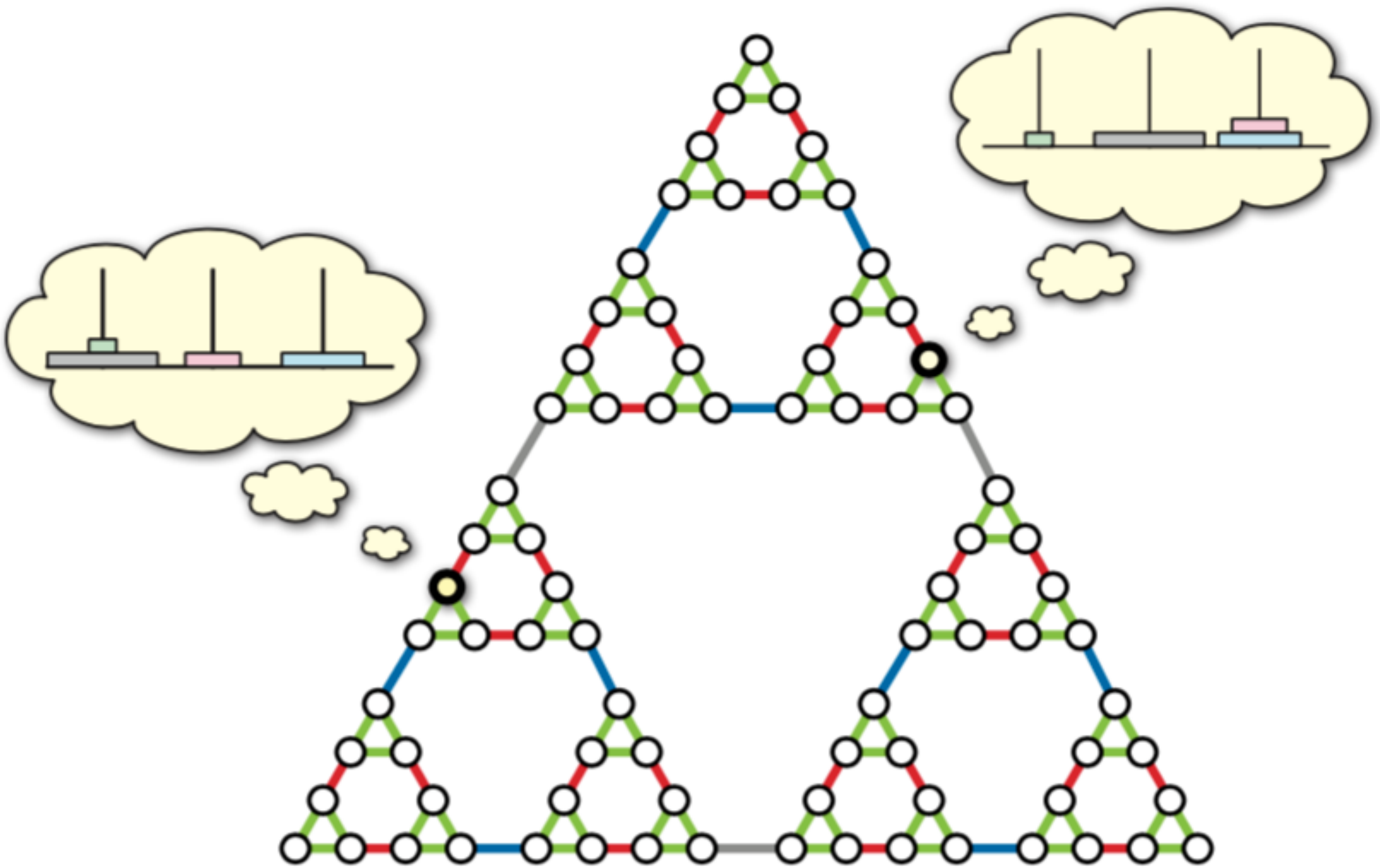
Recursion Tree (of Mergesort)



Configuration graph (Tower of Hanoi)

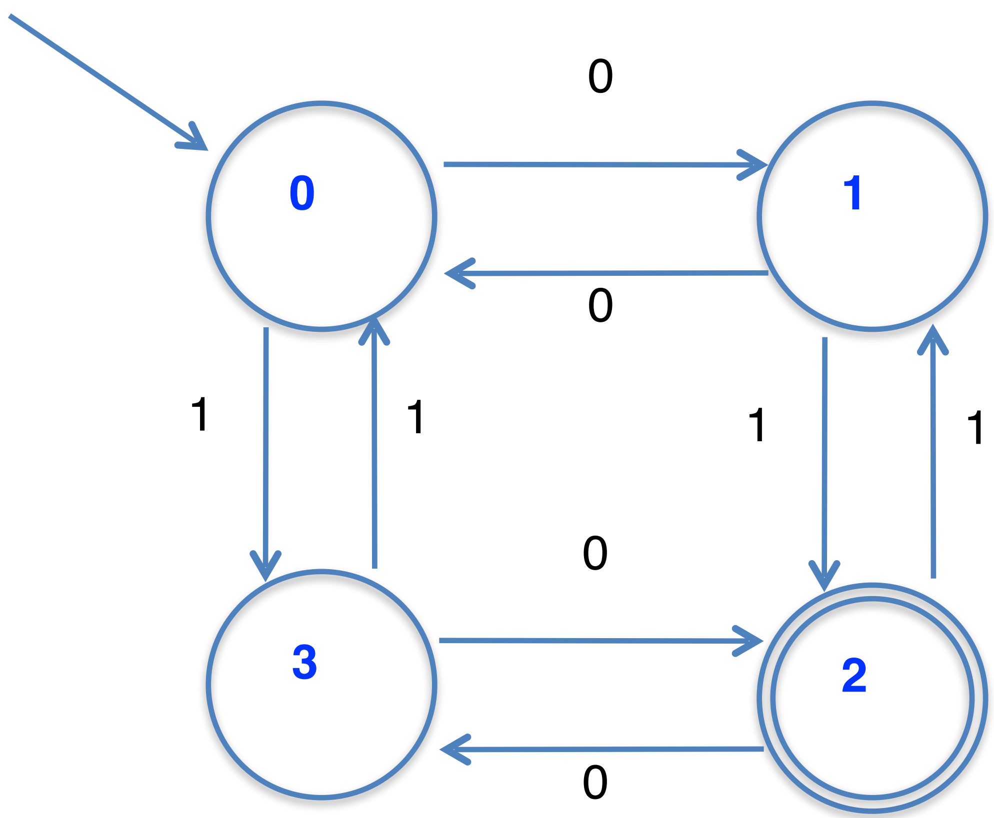


Vertices=legal configurations of discs
Edges = legal move
undirected, or directed pointing both ways



The configuration graph of the 4-disk Tower of Hanoi.

DFA as graph

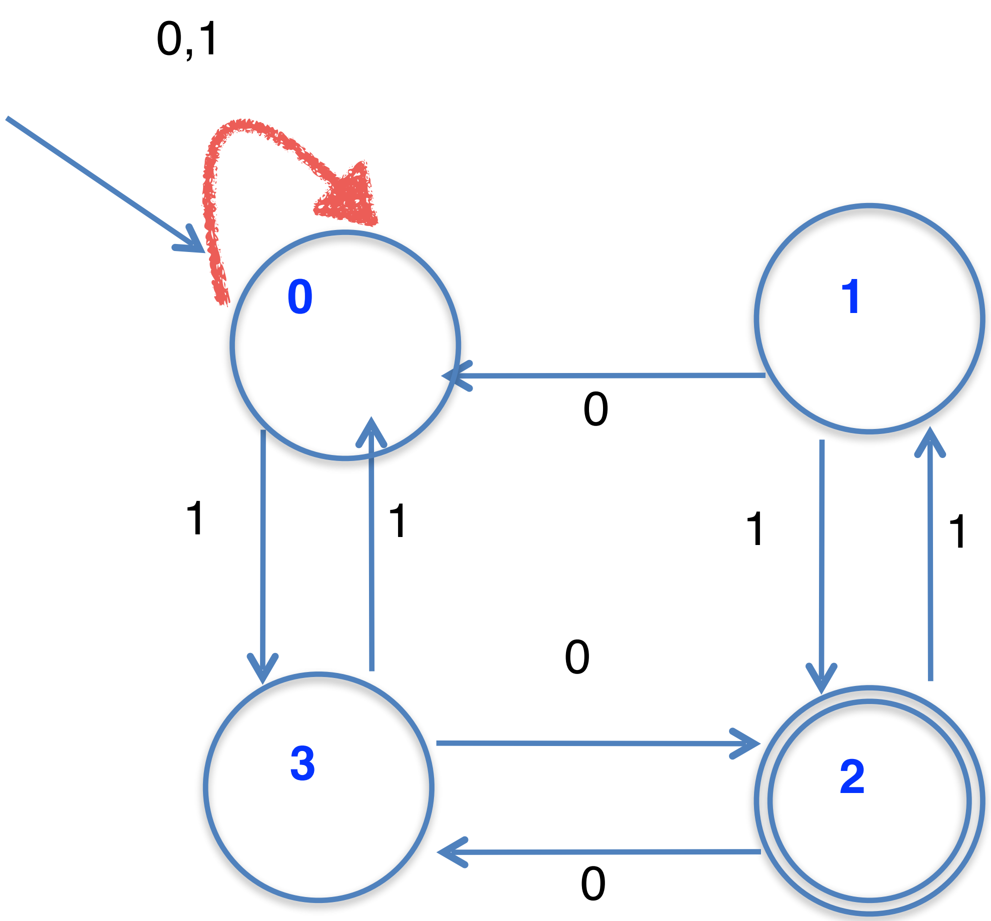


Labeled graph,
With conditions

q	$\delta[q, 0]$	$\delta[q, 1]$	$A[q]$
0	0	1	TRUE
1	2	3	FALSE
2	4	0	FALSE
3	1	2	FALSE
4	3	4	FALSE

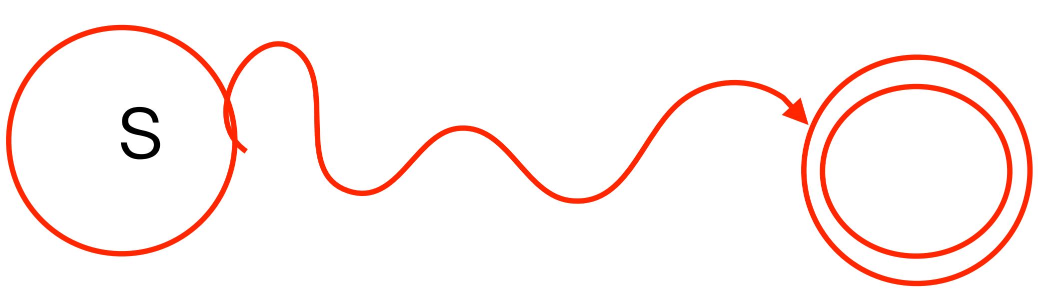
lookup table from transition function is data structure

NFA as graph

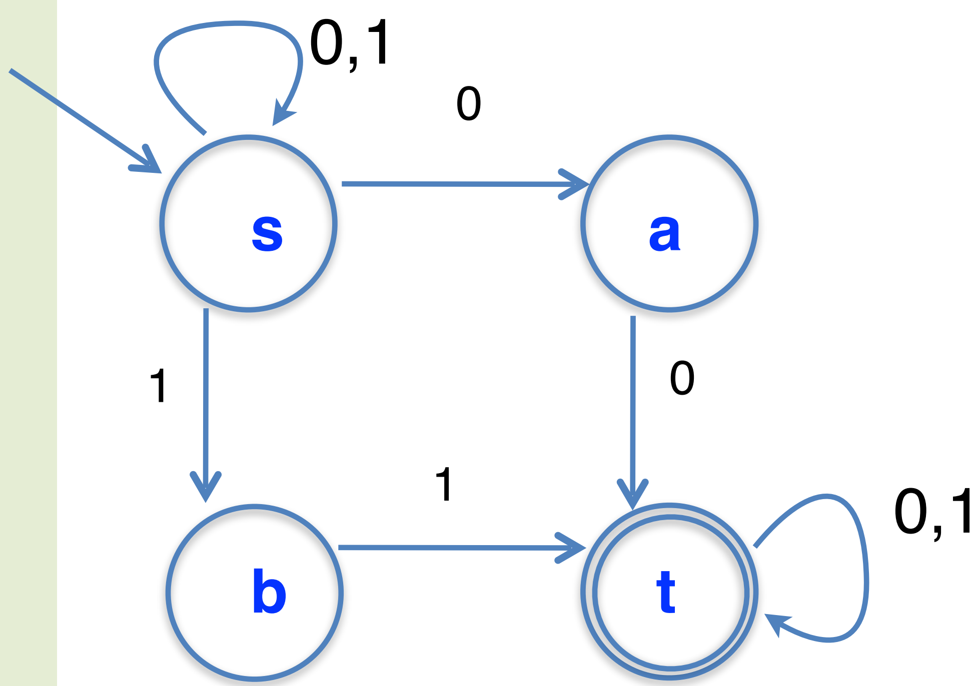


Labeled graph,
Without conditions

How to determine if NFA accepts anything?
Can s reach an accepting state?
DFS, reachability



NFA to DFA (subset construction)



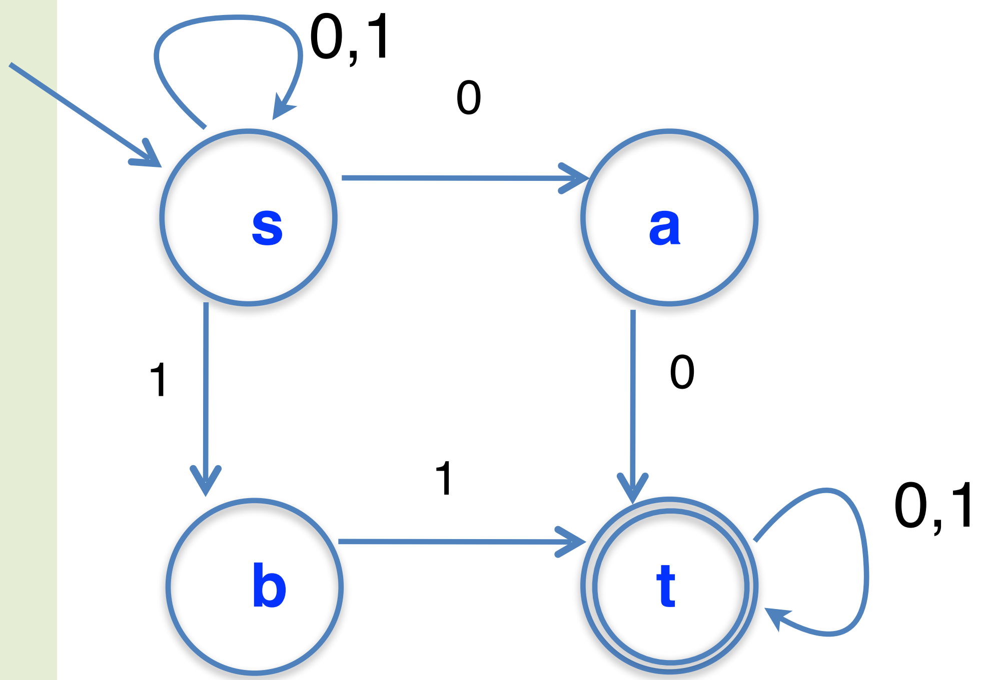
Some times the graph given is not the right graph!

V= 2^Q

E ={A → B | for all u in A there is

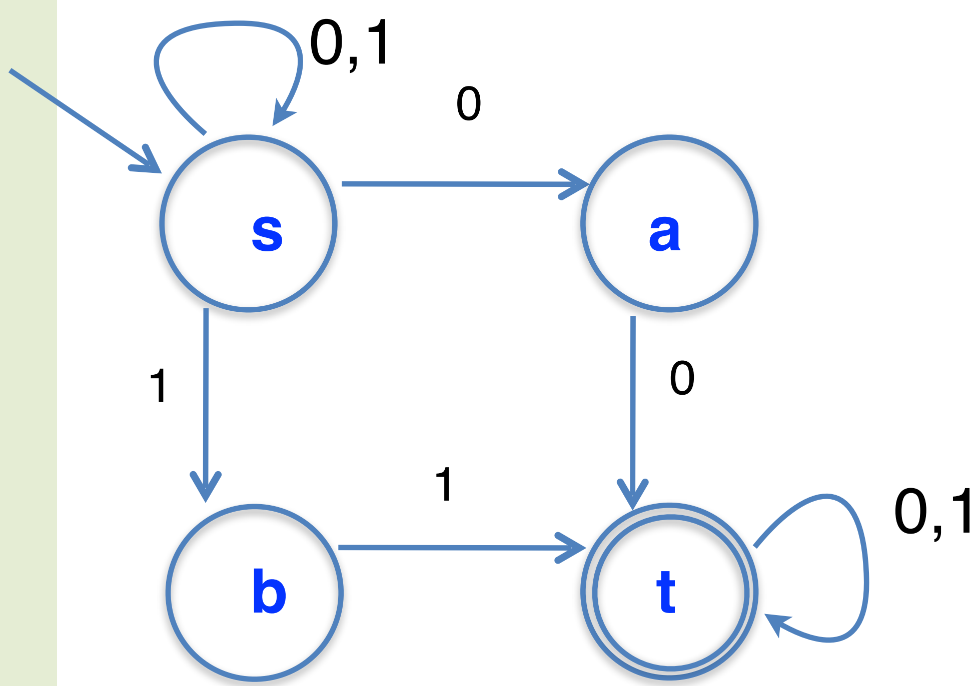
v in B such that u →v}

This is a 16 node graph!



P	ε	$\delta'(P,0)$	$\delta'(P,1)$	$q' \in A'$
s	s	as	bs	No
as	as	ats	bs	No
bs	bs	as	bts	No
ats	ats	ats	bts	Yes
bts	bts	ats	bts	Yes

NFA to DFA (subset construction)



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$V = 2^Q$

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This is a 16 node graph!

Incremental subset construction was running BFS in the DFA, though we were only explicitly given the NFA graph

Graph Boilerplate

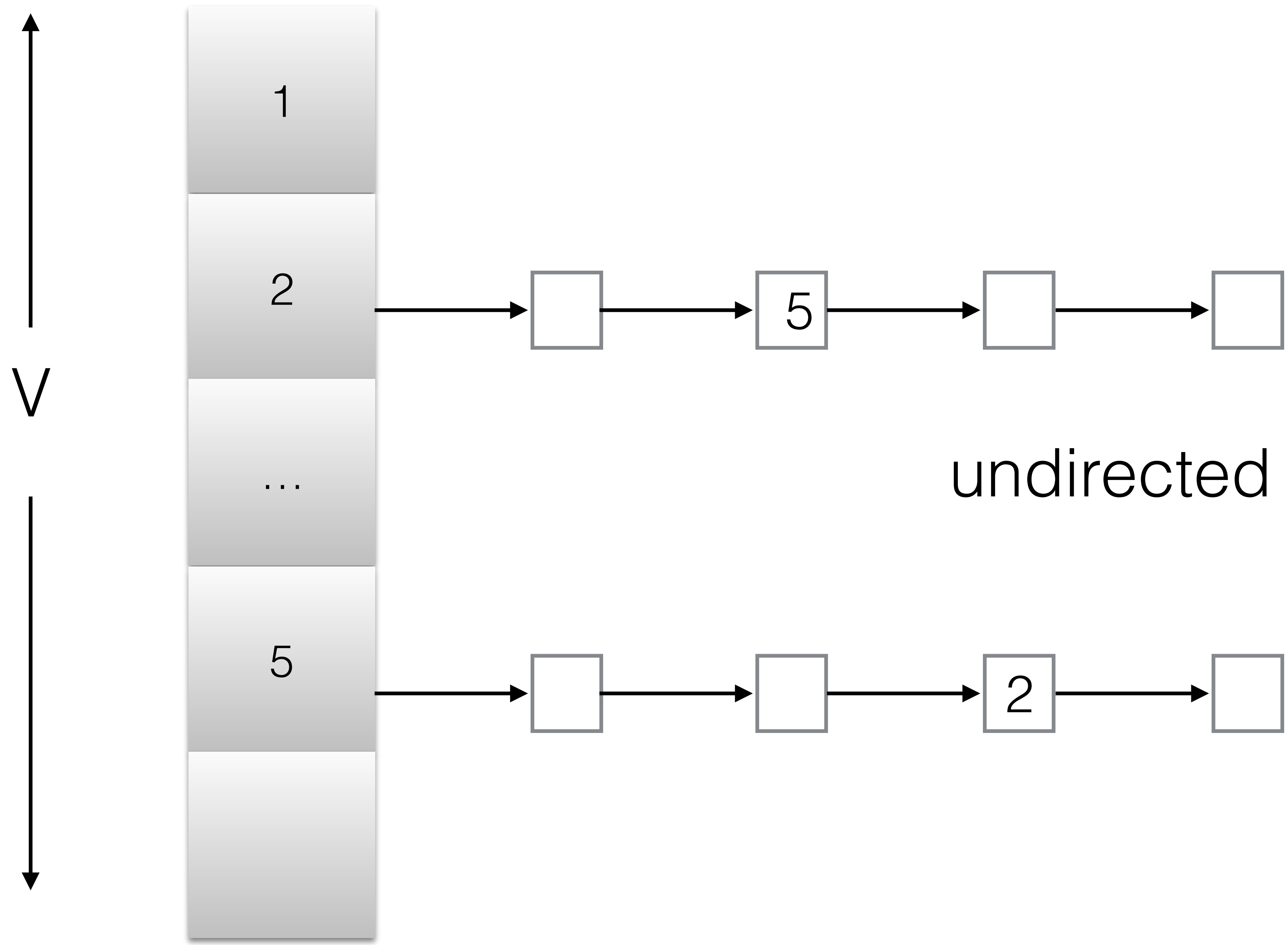
- When I design algorithm on a graph:
- V =?
- E=?
- Problem
- Algorithm
- Running time in terms of original input

Graph Algorithms

- "Given a graph G(V,E), do …"
- What does that mean?
- How to represent a graph ? (string is represented by array etc..)
- Two standard representations

Adjacency Lists

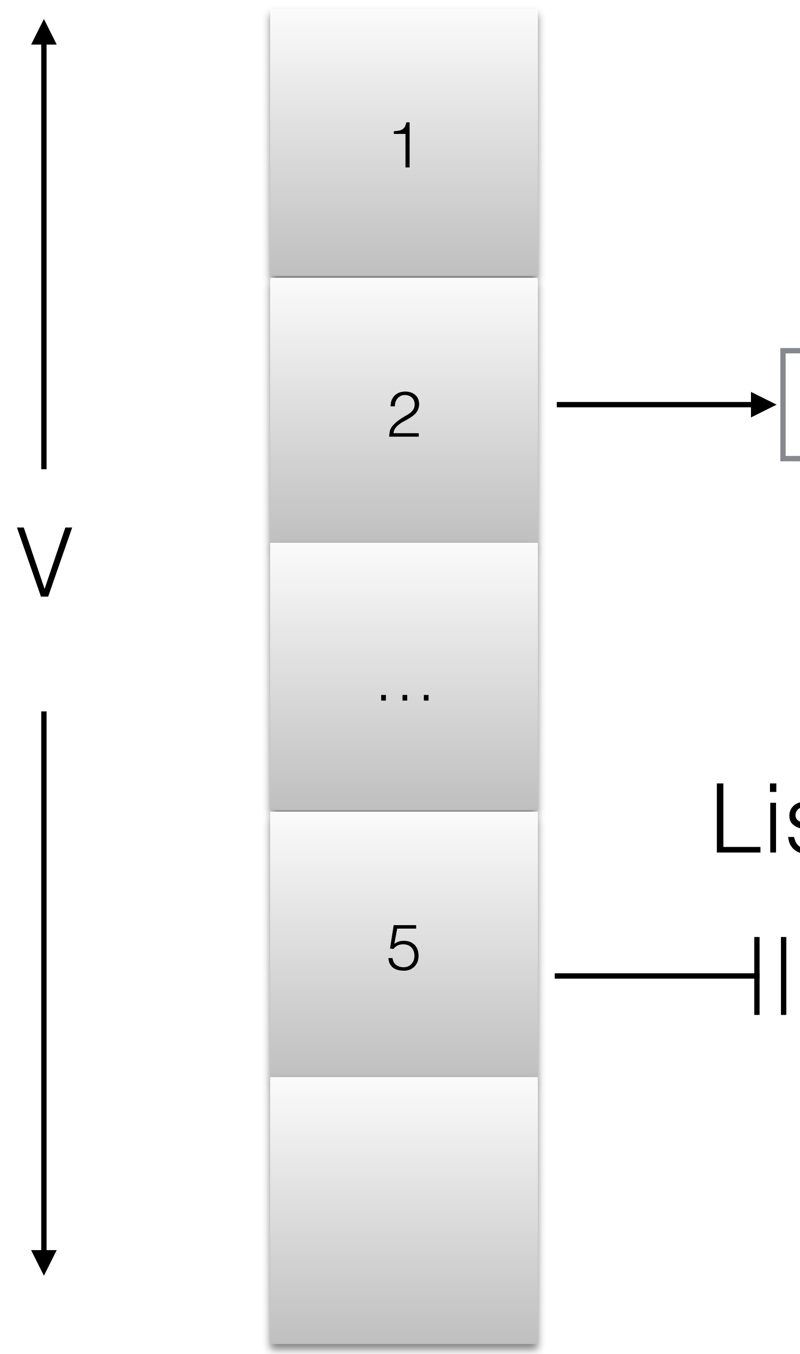
- Adjacency List (= Array of lists)

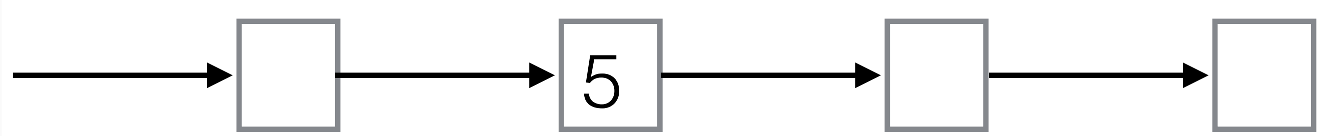


undirected graph

Adjacency Lists

- Adjacency List (= Array of lists)





O(|V|+|E|) space

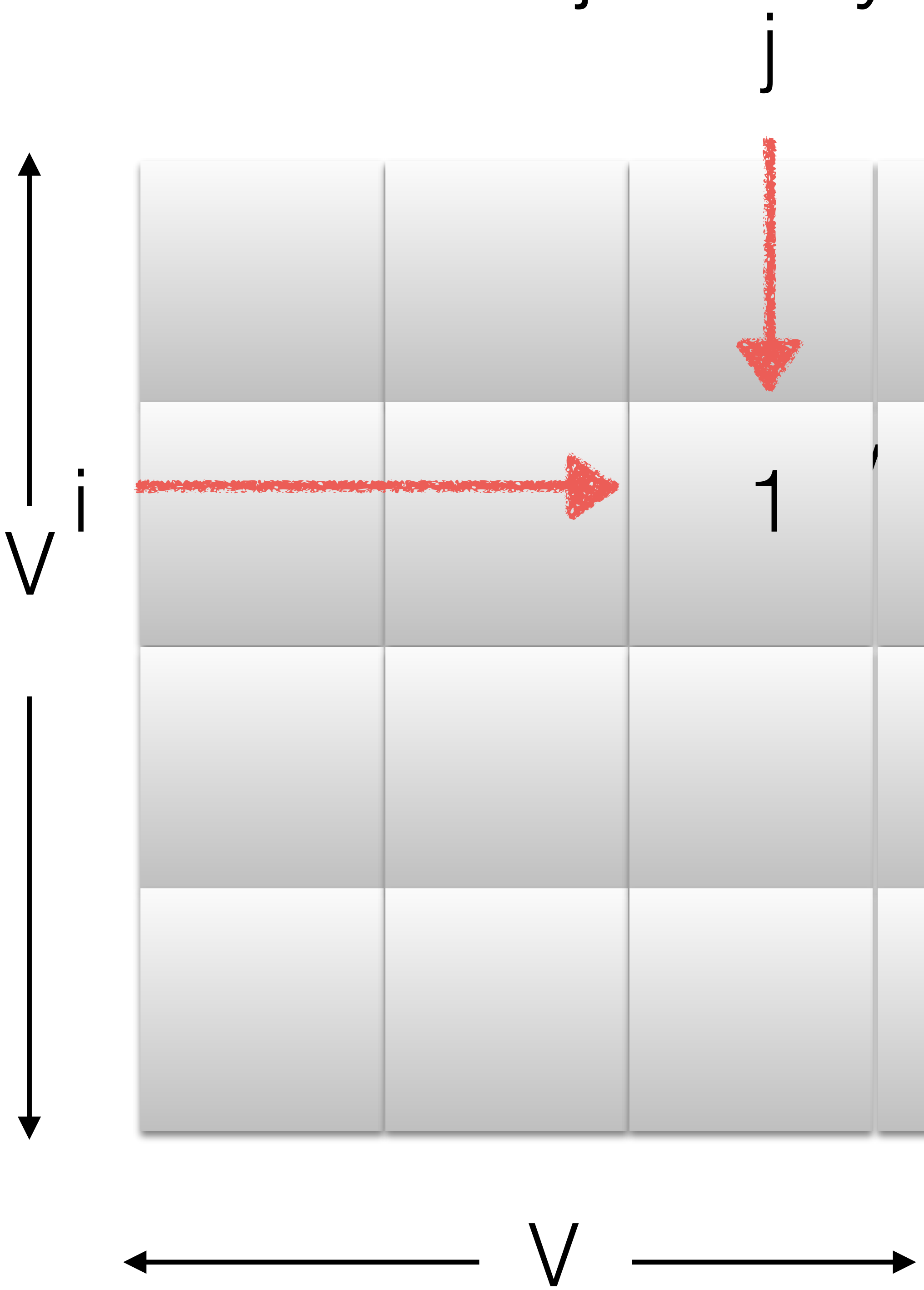
Why Adjacecny, lists?
Acces each node in O(1) time
List edges at each node in O(1) time each
Insert edge

Hard: is (u,v) in E?
More efficient data structure?
Why use linked lists?

Adjacency Matrix

- Adjacency matrix

Why use those at all?



A(i,j)=1 if (i,j) edge
0 otherwise

O(1) time to decide if (u,v) edge
Always O(n^2) space!

O(V) time to list all the neighbors of a vertex u even though there are only constant number of edges!

	Adjacency matrix	Standard adjacency list (linked lists)	Adjacency list (hash tables)
Space	$\Theta(V^2)$	$\Theta(V+E)$	$\Theta(V+E)$
Time to test if $uv \in E$	$O(1)$	$O(1+\min\{\deg(u), \deg(v)\}) = O(V)$	$O(1)$
Time to test if $u \to v \in E$	$O(1)$	$O(1+\deg(u)) = O(V)$	$O(1)$
Time to list the neighbors of v	$O(V)$	$O(1+\deg(v))$	$O(1+\deg(v))$
Time to list all edges	$\Theta(V^2)$	$\Theta(V+E)$	$\Theta(V+E)$
Time to add edge uv	$O(1)$	$O(1)$	$O(1)^*$
Time to delete edge uv	$O(1)$	$O(\deg(u)+\deg(v)) = O(V)$	$O(1)^*$

How to traverse a graph?

- Traversal in general: e.g. you have a data structure with pointers and you want to print it out once

- How to traverse a graph?

```
RecursiveDFS(v):
    if v is unmarked
        mark v
        for each edge vw
            RecursiveDFS(w)
```

How to traverse a graph?

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RecursiveDFS(v):
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```

O(|V|+|E|) time

```
IterativeDFS(s):
  Push(s)
  while the stack is not empty
    v ← Pop
    if v is unmarked
      mark v
      for each edge vw
        Push(w)
```

How to traverse a graph?

```
TRAVERSE(s):
    put s into the bag
    while the bag is not empty
        take v from the bag
        if v is unmarked
            mark v
            for each edge vw
                put w into the bag
```

stack =LIFO (DFS)
Queue = FIFO (BFS)
Priority Queue = lightest out
Random, etc

Whatever First Search

TRAVERSE(s):
  put $(\emptyset, s)$ in bag
  while the bag is not empty
    take (p, v) from the bag $(\star)$
    if v is unmarked
      mark v
      $parent(v) \leftarrow p$
      for each edge vw $(\dagger)$
        put (v, w) into the bag $(\star\star)$

stack =LIFO (DFS)
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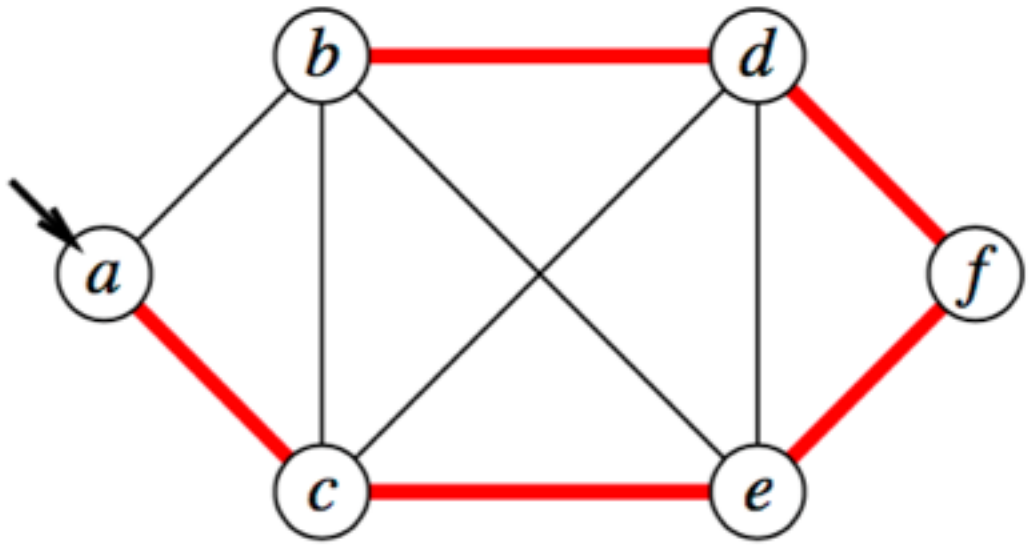
Whatever First Search

Traverse(s) marks every vertex in a connected graph exactly once, and the set of pairs (v, parent(v)) with parent(v) not empty, defines a spanning tree of the graph

BFS tree has shortest paths from s!

DFS tree



BFS tree