HW 3

Due on **Wednesday, September 27, 2017 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Fall 2017 Version: **1.13**

Submission instructions as in previous homeworks.

1 (100 PTS.) Irregularities.

1.A. (25 PTS.) Prove that the following language is not regular by providing a fooling set. You need to prove an infinite fooling set and also prove that it is a valid fooling set. The language is

$$L = \left\{ 0^k w \overline{w} 1^k \;\middle|\; 0 \le k \le 3, w \in \{0,1\}^+ \right\},$$

where $\overline{w}$ is the complement bit-wise not operator. Formally, for $w = w_1 w_2 \dots w_m \in \{0,1\}^*$, we define $\overline{w} = \overline{w_1}\,\overline{w_2} \dots \overline{w_m}$, for $\overline{0} = 1$ and $\overline{1} = 0$.

1.B. (25 PTS.) Same as (A) for the following language. Recall that a run in a string is a maximal non-empty substring of identical symbols. Let L be the set of all strings in $\{0,1\}^*$ that do not contain any two distinct runs of 0s of equal length. As an examples, L:

- contains any string of the form $1^*0^*1^*$.
- contains the strings 011001111 and 000000100100011100010, and
- does not contain the strings 010, 00110110011 and 00001110000.

1.C. (25 PTS.) Suppose you are given two languages L, L' where L is not regular, L' is regular, and $L \cap L'$ is regular. Prove that $L \cup L'$ is not regular.

Also, provide a counter-example for the following claim (it can be interpreted as an "inverse" of the above):

Claim: Consider two languages L and L'. If L is not regular, L' is regular, and $L \cup L'$ is regular, then $L \cap L'$ is regular.

1.D. (25 PTS.) (Hard[1]) Same as (A) for $L = \left\{ 0^{\lceil n \lg n \rceil} \;\middle|\; n \ge 3 \right\}$, where $\lg n = \log_2 n$.

2 (100 PTS.) Grammar.

Describe a context free grammar for the following languages. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

2.A. (50 PTS.) $\left\{ a^i b^j c^k d^\ell e^t \;\middle|\; i, j, k, \ell, t \ge 0 \text{ and } i + j + k + \ell = t \right\}$.

2.B. (50 PTS.) (Harder.) $L = \{ w \in \{0,1\}^* \mid \text{ there is a prefix } x \text{ of } w \text{ s.t. } \#_1(x) > \#_0(x) \}$.

3 (100 PTS.) As easy as a,b,c.

Let $L = \left\{ 0^i 1^j 2^k \;\middle|\; j = i + k \right\}$.

3.A. (40 PTS.) Prove that L is context free by describing a grammar for L.

3.B. (60 PTS.) Prove that your grammar is correct. (One way to do it – show that $L \subseteq L(G)$ and $L(G) \subseteq L$, where G is your grammar from the previous part. This is not the only way.)

[1]Don't feel bad if you can not do this part. No hints would be given for this part. We expect most solutions to be IDK for this one.