1 Recall that w^R denotes the reversal of string w; for example, $TURING^R = GNIRUT$. Prove that the following language is undecidable.

$$\textsc{RevAccept} := \{\langle M\rangle \mid M \text{ accepts } \langle M\rangle^R\}$$

Note that Rice's theorem does *not* apply to this language.

2 Let M be a Turing machine, let w be an arbitrary input string, and let s be an integer. We say that ***M accepts w in space s*** if, given w as input, M accesses only the first s (or fewer) cells on its tape and eventually accepts.

2.A. Sketch a Turing machine/algorithm that correctly decides the following language:

$$\left\{\langle M, w\rangle \;\middle|\; M \text{ accepts } w \text{ in space } |w|^2\right\}$$

2.B. Prove that the following language is undecidable:

$$\left\{\langle M\rangle \;\middle|\; M \text{ accepts at least one string } w \text{ in space } |w|^2\right\}$$

3 Consider the language $\textsc{SometimesHalt} = \{\langle M\rangle \mid M \text{ halts on at least one input string}\}$. Note that $\langle M\rangle \in \textsc{SometimesHalt}$ does not imply that M *accepts* any strings; it is enough that M *halts* on (and possibly rejects) some string.

3.A. Prove that $\textsc{SometimesHalt}$ is undecidable.

3.B. Sketch a Turing machine/algorithm that *accepts* $\textsc{SometimesHalt}$.

4 For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

4.A. $L_0 = \{\langle M\rangle \mid \text{given any input string, } M \text{ eventually leaves its start state}\}$

4.B. $L_1 = \{\langle M\rangle \mid M \text{ decides } L_0\}$

4.C. $L_2 = \{\langle M\rangle \mid M \text{ decides } L_1\}$

4.D. $L_3 = \{\langle M\rangle \mid M \text{ decides } L_2\}$

4.E. $L_4 = \{\langle M\rangle \mid M \text{ decides } L_3\}$

Rubric: 10 points: 4 for part (a) + 3/2 for each other part.

Rubric:[for all undecidability proofs, out of 10 points]

Diagonalization:

1. 4 for correct wrapper Turing machine
2. 6 for self-contradiction proof (= 3 for $\Leftarrow$ + 3 for $\Rightarrow$)

Reduction:

1. 4 for correct reduction
2. 3 for "if" proof

3. 3 for "only if" proof

Rice's Theorem:

1. 4 for positive Turing machine
2. 4 for negative Turing machine
3. 2 for other details (including using the correct variant of Rice's Theorem)