Give regular expressions for each of the following languages over the alphabet $\{0, 1\}$.

1 All strings containing the substring 000.

2 All strings *not* containing the substring 000.

3 All strings in which every run of 0s has length at least 3.

4 All strings in which 1 does not appear after a substring 000.

5 All strings containing at least three 0s.

6 Every string except 000. (**Hint:** Don't try to be clever.)

Work on these later:

7 All strings w such that *in every prefix of* w, the number of 0s and 1s differ by at most 1. *Solution:* Equivalently, strings that alternate between 0s and 1s: $(01 + 10)^*(\varepsilon + 0 + 1)$

8 (**Hard.**) All strings containing at least two 0s and at least one 1.

Solution: There are three possibilities for how such a string can begin:

- Start with 00, then any number of 0s, then 1, then anything.
- Start with 01, then any number of 1s, then 0, then anything.
- Start with 1, then a substring with exactly two 0s, then anything.

All together: $000^*1(0 + 1)^* \; + \; 011^*0(0 + 1)^* \; + \; 11^*01^*0(0 + 1)^*$

9 (**Hard.**) All strings w such that *in every prefix of* w, the number of 0s and 1s differ by at most 2.

10 (**Really hard.**) All strings in which the substring 000 appears an even number of times.

(For example, 0001000 and 0000 are in this language, but 00000 is not.)