Design Turing machines $M=(Q,\Sigma,\Gamma,\delta,\mathsf{start},\mathsf{accept},\mathsf{reject})$ for each of the following tasks, either by listing the states Q, the tape alphabet Γ , and the transition function δ (in a table), or by drawing the corresponding labeled graph.

Version: 1.0

Each of these machines uses the input alphabet $\Sigma = \{1, \#\}$; the tape alphabet Γ can be any superset of $\{1, \#, \square, \triangleright\}$ where \square is the blank symbol and \triangleright is a special symbol marking the left end of the tape. Each machine should reject any input not in the form specified below.

- 1 On input 1^n , for any non-negative integer n, write $1^n \# 1^n$ on the tape and accept.
- On input $\#^n 1^m$, for any non-negative integers m and n, write 1^m on the tape and accept. In other words, delete all the #s and shift the 1s to the start of the tape.
- 3 On input $\#1^n$, for any non-negative integer n, write $\#1^{2n}$ on the tape and accept. (**Hint:** Modify the Turing machine from problem 1.)
- 4 On input 1^n , for any non-negative integer n, write 1^{2^n} on the tape and accept. (**Hint:** Use the three previous Turing machines as subroutines.)