Designing DFAs via product construction and designing NFAs.

1. Describe a DFA that accepts the following language over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

   All strings in which the number of 0s is even and the number of 1s is *not* divisible by 3.

2. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

   For example, the string 1100 is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

3. Design an NFA for the language $(01)^+ + (010)^+$.

Work on these later:

Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. You may find it easier to describe these DFAs formally than to draw pictures.

4. All strings w such that $\binom{|w|}{2} \bmod 6 = 4$. *[Hint: Maintain both* $\binom{|w|}{2} \bmod 6$ *and* $|w| \bmod 6$.*]*

*5. All strings w such that $F_{\#(\mathtt{10},w)} \bmod 10 = 4$, where $\#(\mathtt{10}, w)$ denotes the number of times 10 appears as a substring of w, and F_n is the nth Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$