This is a review of context-free grammars from the lecture on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

  $S \to \varepsilon \mid S\,\mathtt{(}S\mathtt{)}$ properly nested parentheses

  Here is a different grammar for the same language:

  $S \to \varepsilon \mid \mathtt{(}S\mathtt{)} \mid SS$ properly nested parentheses

- $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a different number of 1s.

$S \to A \mid B$	$\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\}$
$A \to \mathtt{0}A \mid \mathtt{0}C$	$\{\mathtt{0}^m\mathtt{1}^n \mid m > n\}$
$B \to B\mathtt{1} \mid C\mathtt{1}$	$\{\mathtt{0}^m\mathtt{1}^n \mid m < n\}$
$C \to \varepsilon \mid \mathtt{0}C\mathtt{1}$	$\{\mathtt{0}^m\mathtt{1}^n \mid m = n\}$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1. $\{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\}$

2. $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq 2n\}$

   *[Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string **01**.]*

3. $\{\mathtt{0},\mathtt{1}\}^* \setminus \{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\}$

   [Hint: Extend the previous grammar. What's missing?]

Work on these later:

4. $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid \#(\mathtt{0},w) = 2 \cdot \#(\mathtt{1},w)\}$ — Binary strings where the number of 0s is exactly twice the number of 1s.

5. $\{\mathtt{0},\mathtt{1}\}^* \setminus \{ww \mid w \in \{\mathtt{0},\mathtt{1}\}^*\}$.

   *[Anti-hint: The language $\{ww \mid w \in \mathtt{0},\mathtt{1}^*\}$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*

6. Prove that every regular language is context free.