> **Write your answers in the separate answer booklet.**
>
> Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is **always** true and "No" otherwise. Each correct answer is worth $+1$ point; each incorrect answer is worth $-\frac{1}{2}$ point; checking "I don't know" is worth $+\frac{1}{4}$ point; and flipping a coin is (on average) worth $+\frac{1}{4}$ point. You do **not** need to prove your answer is correct.

    Read each statement *very* carefully. Some of these are deliberately subtle.

    (a) If $2 + 2 = 5$, then zero is odd.

    (b) Language L is regular if and only if there is a DFA that accepts every string in L.

    (c) Two languages L and L' are regular if and only if $L \cup L'$ is regular.

    (d) For every language L, if L^* is empty, then L is empty.

    (e) Every regular language is recognized by a DFA with exactly one accepting state.

    (f) If L has a fooling set of size 374, then L is regular.

    (g) The language $\left\{ 0^{374n} \mid n \geq 374 \right\}$ is regular.

    (h) The language $\left\{ 0^{37n} 1^{4n} \mid n \geq 374 \right\}$ is regular.

    (i) The language $\left\{ 0^{3n} 1^{74n} \mid n \leq 374 \right\}$ is regular.

    (j) Every language is either regular or context-free.

2. For any string $w \in \{0, 1\}^*$, let $slash(w)$ be the string in $\{0, 1, /\}^*$ obtained from w by inserting a new symbol $/$ between any two consecutive appearances of the same symbol. For example:

    $$slash(\varepsilon) = \varepsilon$$
    $$slash(10101) = 10101$$
    $$slash(001010111) = 0/010101/1/1$$

    For any language $L \subseteq \{0, 1\}^*$, let $slash(L) = \{slash(w) \mid w \in L\}$.

    (a) Draw or describe a DFA that accepts the language $slash(\{0, 1\}^*)$.

    (b) Give a regular expression for the language $slash(\{0, 1\}^*)$.

    (c) **Prove** that for any regular language L, the language $slash(L)$ is also regular.

    (You do not need to justify your answers to parts (a) and (b).)

3. Let L be the language $\left\{ 0^a 1^b 0^c \mid 2a = b + c \right\}$.

    (a) **Prove** that L is not a regular language.

    (b) Describe a context-free grammar for L. (You do not need to justify your answer.)

4. For each of the following languages L, give a regular expression that represents L **and** draw or describe a DFA that recognizes L. You do not need to justify your answers.

    (a) All strings in $\{0, 1\}^*$ that do not contain either 100 or 011 as a substring

    (b) All strings in $\{0, 1, 2\}^*$ that do not contain either 01 or 12 or 20 as a substring

5. For any string $w \in \{0, 1\}^*$, let *stupefy*(w) denote the string obtained from w by deleting the first 1 (if any) and replacing each remaining 1 with a 0. For example:

$$stupefy(\varepsilon) = \varepsilon$$
$$stupefy(000) = 000$$
$$stupefy(00100) = 0000$$
$$stupefy(111111) = 00000$$
$$stupefy(0100001101) = 000000000$$

Let L be an arbitrary regular language.

    (a) **Prove** that the language $\{stupefy(w) \mid w \in L\}$ is regular.

    (b) **Prove** that the language $\{w \in \{0, 1\}^* \mid stupefy(w) \in L\}$ is regular.