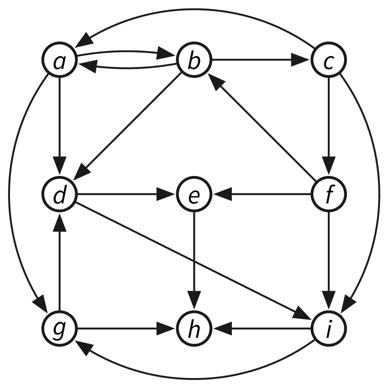
Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. ***Clearly*** indicate the following structures in the directed graph below, or write NONE if the indicated structure does not exist. Don't be subtle; to indicate a collection of edges, draw a heavy black line along the entire length of each edge.



(a) A depth-first search tree rooted at vertex a.

(b) A breadth-first tree rooted at vertex a.

(c) The strong components of G. (Circle each strong component.)

(d) Draw the strong-component graph of G.

2. As the days get shorter in winter, Eggsy Hutmacher is increasingly worried about his walk home from work. The city has recently been invaded by the notorious Antimilliner gang, whose members hang out on dark street corners and steal hats from unwary passers-by, and a gentleman is simply *not* seen out in public without a hat. The city council is slowly installing street lamps at intersections to deter the Antimilliners, whose uncovered faces can be easily identified in the light. Eggsy keeps k extra hats in his briefcase in case of theft or other millinery emergencies.

   Eggsy has a map of the city in the form of an undirected graph G, whose vertices represent intersections and whose edges represent streets between them. A subset of the vertices are marked to indicate that the corresponding intersections are lit. Every edge e has a non-negative length $\ell(e)$. The graph has two special nodes s and t, which represent Eggsy's work and home, respectively.

   Describe an algorithm that computes the shortest path in G from s to t that visits at most k unlit vertices.

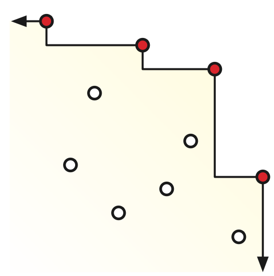
3. An undirected graph $G = (V, E)$ is ***bipartite*** if each of its vertices can be colored either black or white, so that every edge in E has one white endpoint and one black endpoint. Describe and analyze an algorithm to determine, given an undirected graph G as input, whether G is bipartite. *[Hint: Every tree is bipartite.]*

4. Satya is in charge of establishing a new testing center for the Standardized Awesomeness Test (SAT), and found an old conference hall that is perfect. The conference hall has n rooms of various sizes along a single long hallway, numbered in order from 1 through n. Satya knows exactly how many students fit into each room, and he wants to use a subset of the rooms to host as many students as possible for testing.

   Unfortunately, there have been several incidents of students cheating at other testing centers by tapping secret codes through walls. To prevent this type of cheating, Satya can use two adjacent rooms only if he demolishes the wall between them. The city's chief architect has determined that demolishing the walls on both sides of the same room would threaten the building's structural integrity. For this reason, Satya can never host students in three consecutive rooms.

   Describe an efficient algorithm that computes the largest number of students that Satya can host for testing without using three consecutive rooms. The input to your algorithm is an array $S[1..n]$, where each $S[i]$ is the (non-negative integer) number of students that can fit in room i.

5. Suppose you are given a set P of n points in the plane. A point $p \in P$ is *maximal* in P if no other point in P is both above and to the right of P. Intuitively, the maximal points define a "staircase" with all the other points of P below it.



A set of ten points, four of which are maximal.

   Describe and analyze an algorithm to compute the number of maximal points in P in $O(n \log n)$ time. For example, given the ten points shown above, your algorithm should return the integer 4. The input to your algorithm is a pair of arrays $X[1..n]$ and $Y[1..n]$ containing the x- and y-coordinates of the points in P.