Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do ***not*** need to prove your answer is correct.

   Read each statement *very* carefully. Some of these are deliberately subtle.

   (a) If the moon is made of cheese, then Jeff is the Queen of England.

   (b) The language $\{0^m 1^n \mid m, n \geq 0\}$ is not regular.

   (c) For all languages L, the language L^* is regular.

   (d) For all languages $L \subset \Sigma^*$, if L is recognized by a DFA, then $\Sigma^* \setminus L$ can be represented by a regular expression.

   (e) For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

   (f) For all languages L, if L is not regular, then L does not have a finite fooling set.

   (g) Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **DFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

   (h) Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **NFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

   (i) For all context-free languages L and L', the language $L \bullet L'$ is also context-free.

   (j) Every non-context-free language is non-regular.

2. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.***

   (a) $\left\{0^n w 0^n \,\middle|\, w \in \Sigma^+ \text{ and } n > 0\right\}$

   (b) $\left\{w 0^n w \,\middle|\, w \in \Sigma^+ \text{ and } n > 0\right\}$

   For example, both of these languages contain the string `00110100000110100`.

3. Let $L = \{0^{2i}1^{i+2j}0^j \mid i, j \geq 0\}$ and let G be the following context free-grammar:

$$S \to AB$$
$$A \to \varepsilon \mid 00A1$$
$$B \to \varepsilon \mid 11B0$$

   (a) ***Prove*** that $L(G) \subseteq L$.

   (b) ***Prove*** that $L \subseteq L(G)$.

   [Hint: What are $L(A)$ and $L(B)$? Prove it!]

   Mea Culpa. This was a bad exam problem, mostly because the proofs take far too long to write down. CFL correctness proofs will not appear on the midterm. Nevertheless, you should understand the high-level structure of the correctness proof, to help you *design* correct CFLs.

4. For any language L, let $\text{SUFFIXES}(L) := \left\{x \;\middle|\; yx \in L \text{ for some } y \in \Sigma^*\right\}$ be the language containing all suffixes of all strings in L. For example, if $L = \{010, 101, 110\}$, then $\text{SUFFIXES}(L) = \{\varepsilon, 0, 1, 01, 10, 010, 101, 110\}$.

   Prove that for any regular language L, the language $\text{SUFFIXES}(L)$ is also regular.

5. For each of the following languages L, give a regular expression that represents L ***and*** describe a DFA that recognizes L.

   (a) The set of all strings in $\{0, 1\}^*$ that do not contain the substring 0110.

   (b) The set of all strings in $\{0, 1\}^*$ that contain exactly one of the substrings 01 or 10.

   You do ***not*** need to prove that your answers are correct.