CS/ECE 374 A ✧ Fall 2019

"Homework" 11

"Due" Monday, December 9, 2019

This homework is *not* for submission. However, undecidability questions are in scope for the final exam, so we still strongly recommend treating at least those questions as regular homework. Solutions will be released next Monday.

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1. Let $\langle M\rangle$ denote the encoding of a Turing machine M (or if you prefer, the Python source code for the executable code M). Recall that w^R denotes the reversal of string w. Prove that the following language is undecidable.

   $$\textsc{SelfRevAccept} := \left\{\langle M\rangle \;\middle|\; M \text{ accepts the string } \langle M\rangle^R\right\}$$

   Note that Rice's theorem does *not* apply to this language.

2. Let M be a Turing machine, let w be an arbitrary input string, and let s be an integer. We say that ***M accepts w in space s*** if, given w as input, M accesses only the first s (or fewer) cells on its tape and eventually accepts.

   (a) Prove that the following language is undecidable:

   $$\textsc{SomeSquareSpace} = \left\{\langle M\rangle \;\middle|\; M \text{ accepts at least one string } w \text{ in space } |w|^2\right\}$$

   [Hint: The only thing you need to know about Turing machines for this problem is that they consume a resource called "space".]

   *(b) Sketch a Turing machine/algorithm that correctly decides the following language:

   $$\textsc{SquareSpace} = \left\{\langle M, w\rangle \;\middle|\; M \text{ accepts } w \text{ in space } |w|^2\right\}$$

   [Hint: This question is only for people who really want to get down in the Turing-machine weeds. Nothing like this will appear on the final exam.]

3. Consider the following language:

   $$\textsc{Picky} = \left\{\langle M\rangle \;\middle|\; \begin{array}{c} M \text{ accepts at least one input string} \\ \text{and } M \text{ rejects at least one input string} \end{array}\right\}$$

   (a) Prove that $\textsc{Picky}$ is undecidable.

   (b) Sketch a Turing machine/algorithm that *accepts* $\textsc{Picky}$.