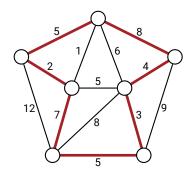
Proving that a problem *X* is NP-hard requires several steps:

- Choose a problem *Y* that you already know is NP-hard (because we told you so in class).
- Describe an algorithm to solve *Y*, using an algorithm for *X* as a subroutine. Typically this algorithm has the following form: Given an instance of *Y*, transform it into an instance of *X*, and then call the magic black-box algorithm for *X*.
- *Prove* that your algorithm is correct. This always requires two separate steps, which are usually of the following form:
  - Prove that your algorithm transforms "good" instances of Y into "good" instances of X.
  - Prove that your algorithm transforms "bad" instances of Y into "bad" instances of X.
    Equivalently: Prove that if your transformation produces a "good" instance of X, then it was given a "good" instance of Y.
- Argue that your algorithm for Y runs in polynomial time. (This is usually trivial.)
- 1. Recall the following kColor problem: Given an undirected graph G, can its vertices be colored with k colors, so that every edge touches vertices with two different colors?
  - (a) Describe a direct polynomial-time reduction from 3Color to 4Color.
  - (b) Prove that kColor problem is NP-hard for any  $k \ge 3$ .
- 2. A *Hamiltonian cycle* in a graph *G* is a cycle that goes through every vertex of *G* exactly once. Deciding whether an arbitrary graph contains a Hamiltonian cycle is NP-hard.

A *tonian cycle* in a graph *G* is a cycle that goes through at least *half* of the vertices of *G*. Prove that deciding whether a graph contains a tonian cycle is NP-hard.

## To think about later:

3. Let *G* be an undirected graph with weighted edges. A Hamiltonian cycle in *G* is *heavy* if the total weight of edges in the cycle is at least half of the total weight of all edges in *G*. Prove that deciding whether a graph contains a heavy Hamiltonian cycle is NP-hard.



A heavy Hamiltonian cycle. The cycle has total weight 34; the graph has total weight 67.