Rice's Theorem. *Let $\mathcal{L}$ be any set of languages that satisfies the following conditions:*

- *There is a Turing machine Y such that* $\text{Accept}(Y) \in \mathcal{L}$.
- *There is a Turing machine N such that* $\text{Accept}(N) \notin \mathcal{L}$.

The language $\text{AcceptIn}(\mathcal{L}) := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \in \mathcal{L}\right\}$ *is undecidable.*

Prove that the following languages are undecidable *using Rice's Theorem*:

1. $\text{AcceptRegular} := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \text{ is regular}\right\}$
2. $\text{AcceptIllini} := \left\{\langle M\rangle \;\middle|\; M \text{ accepts the string } \texttt{ILLINI}\right\}$
3. $\text{AcceptPalindrome} := \left\{\langle M\rangle \;\middle|\; M \text{ accepts at least one palindrome}\right\}$
4. $\text{AcceptThree} := \left\{\langle M\rangle \;\middle|\; M \text{ accepts exactly three strings}\right\}$
5. $\text{AcceptUndecidable} := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \text{ is undecidable}\right\}$

To think about later. Which of the following are undecidable? How would you prove that?

1. $\text{Accept}\{\{\varepsilon\}\} := \left\{\langle M\rangle \;\middle|\; M \text{ accepts only the string } \varepsilon\text{; that is, } \text{Accept}(M) = \{\varepsilon\}\right\}$
2. $\text{Accept}\{\varnothing\} := \left\{\langle M\rangle \;\middle|\; M \text{ does not accept any strings; that is, } \text{Accept}(M) = \varnothing\right\}$
3. $\text{Accept}\varnothing := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \text{ is not an acceptable language}\right\}$
4. $\text{Accept=Reject} := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) = \text{Reject}(M)\right\}$
5. $\text{Accept}{\neq}\text{Reject} := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \neq \text{Reject}(M)\right\}$
6. $\text{Accept}{\cup}\text{Reject} := \left\{\langle M\rangle \;\middle|\; \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^*\right\}$