CS 374 A

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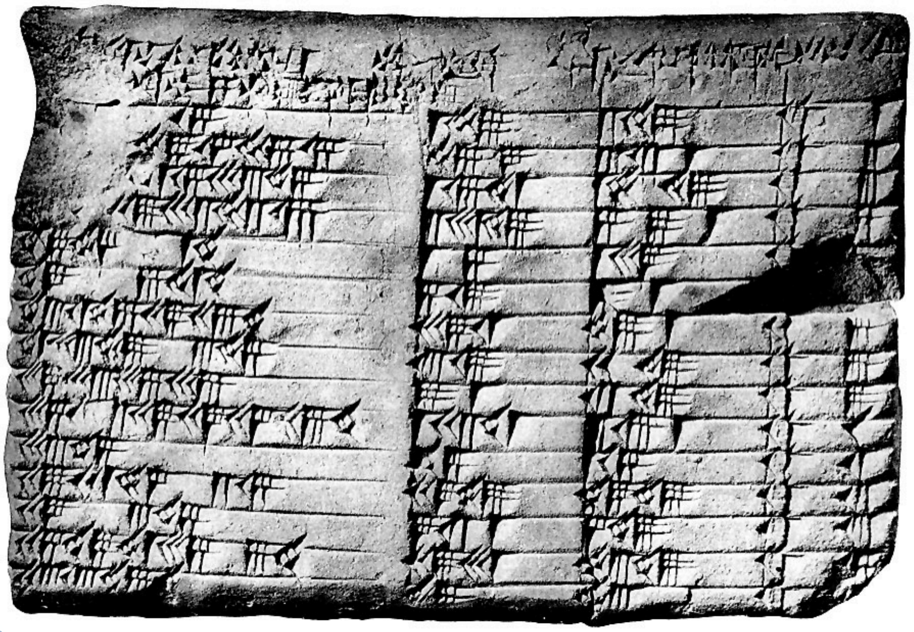
Guidance → Practice → Feedback → Guidance

IDK

Deadly sins

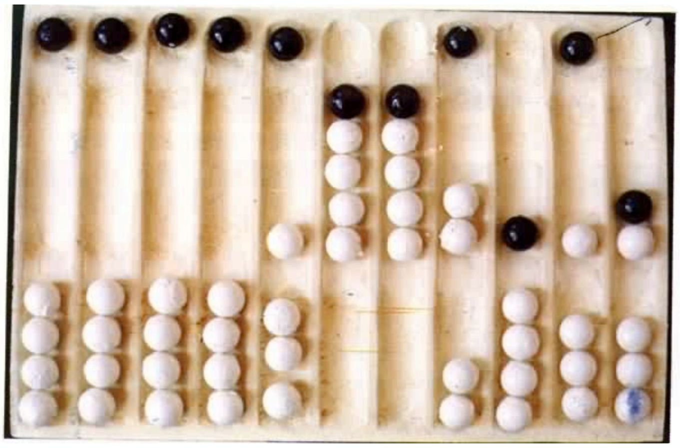
- No proofs by example
- Declare your variables
- NO WEAK INDUCTION

Cheating ~ Don't!

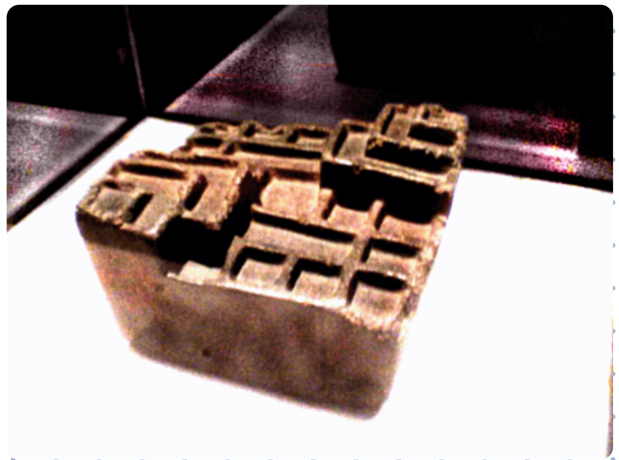


Plimpton 322 (Mesopotamia, 1800 BCE)

Rhind Papyrus (Egypt, 1550 BCE)



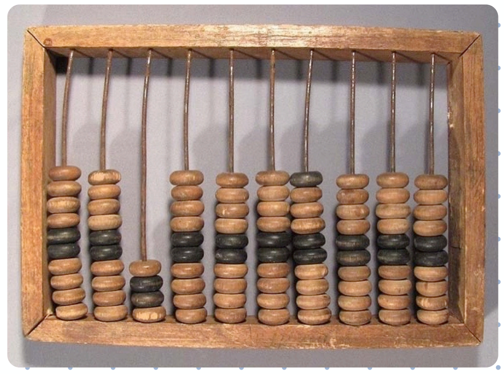
Roman abacus



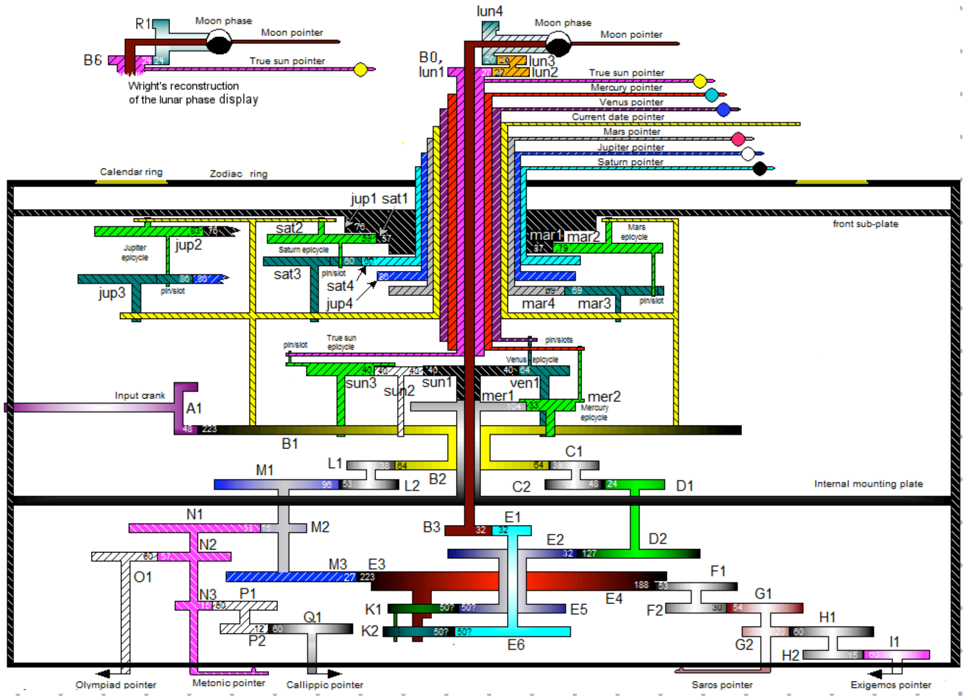
Incan Yupana

Suan Pan

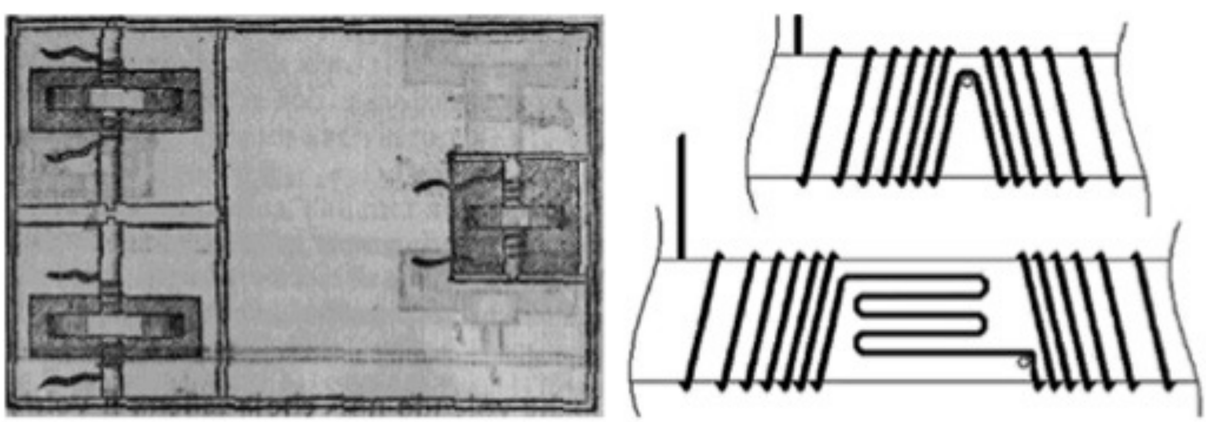
Soroban



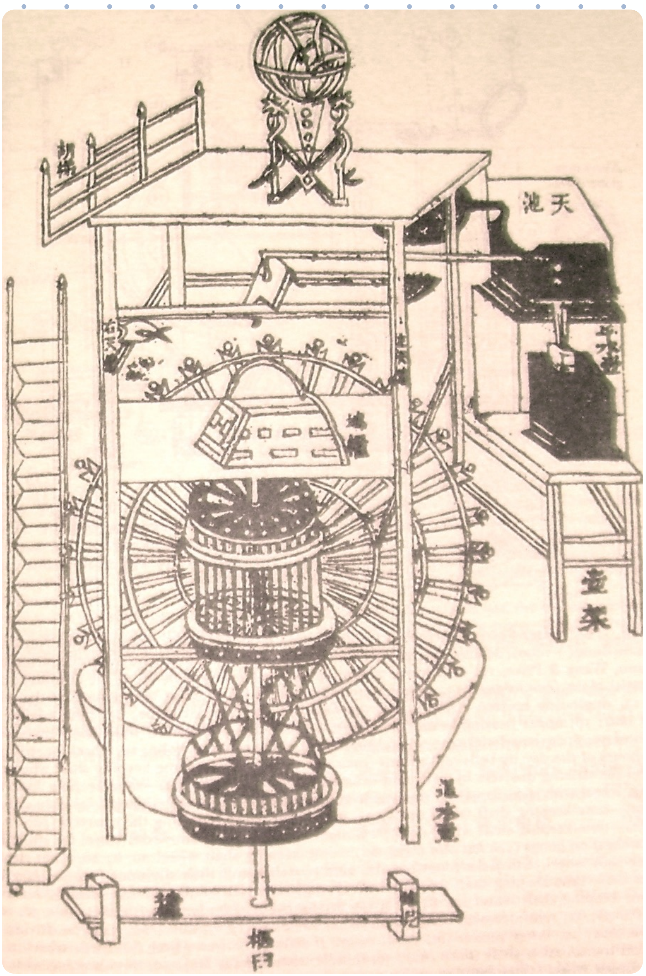
Shoty



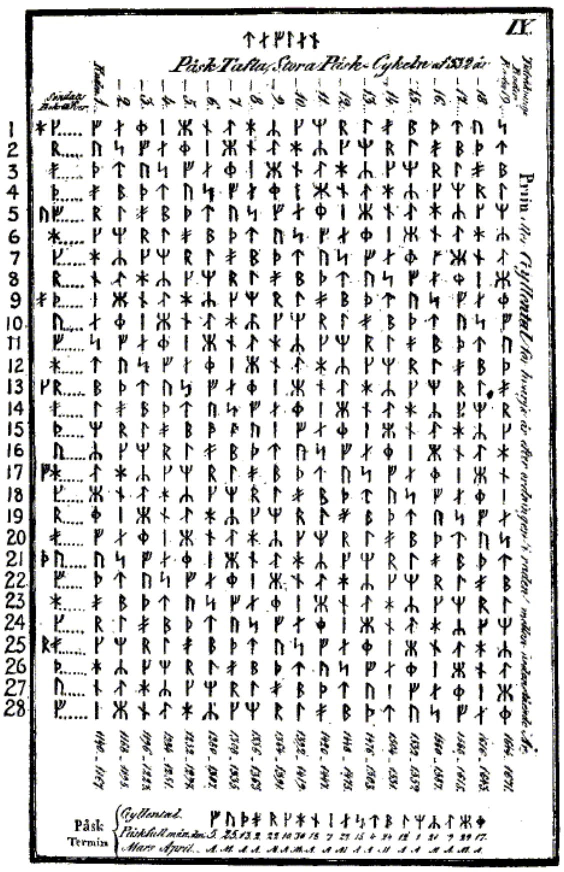
Antikythera Mechanism



Heron of Alexandria's programmable cart



Su Song's "cosmic engine" (1088CE)



Computus tables (12th C. Sweden)

Strasbourg astronomical clock (16th C.)

Automatic carillon (15th C.)

Utrecht Domtoren
automatic carillon (1975)

Leibniz 1684
(also Newton 1687)

DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(c) = 0$$

MEAN VALUE THEOREM

If f is differentiable on the interval (a, b) and continuous at the end points there exists a c in (a, b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

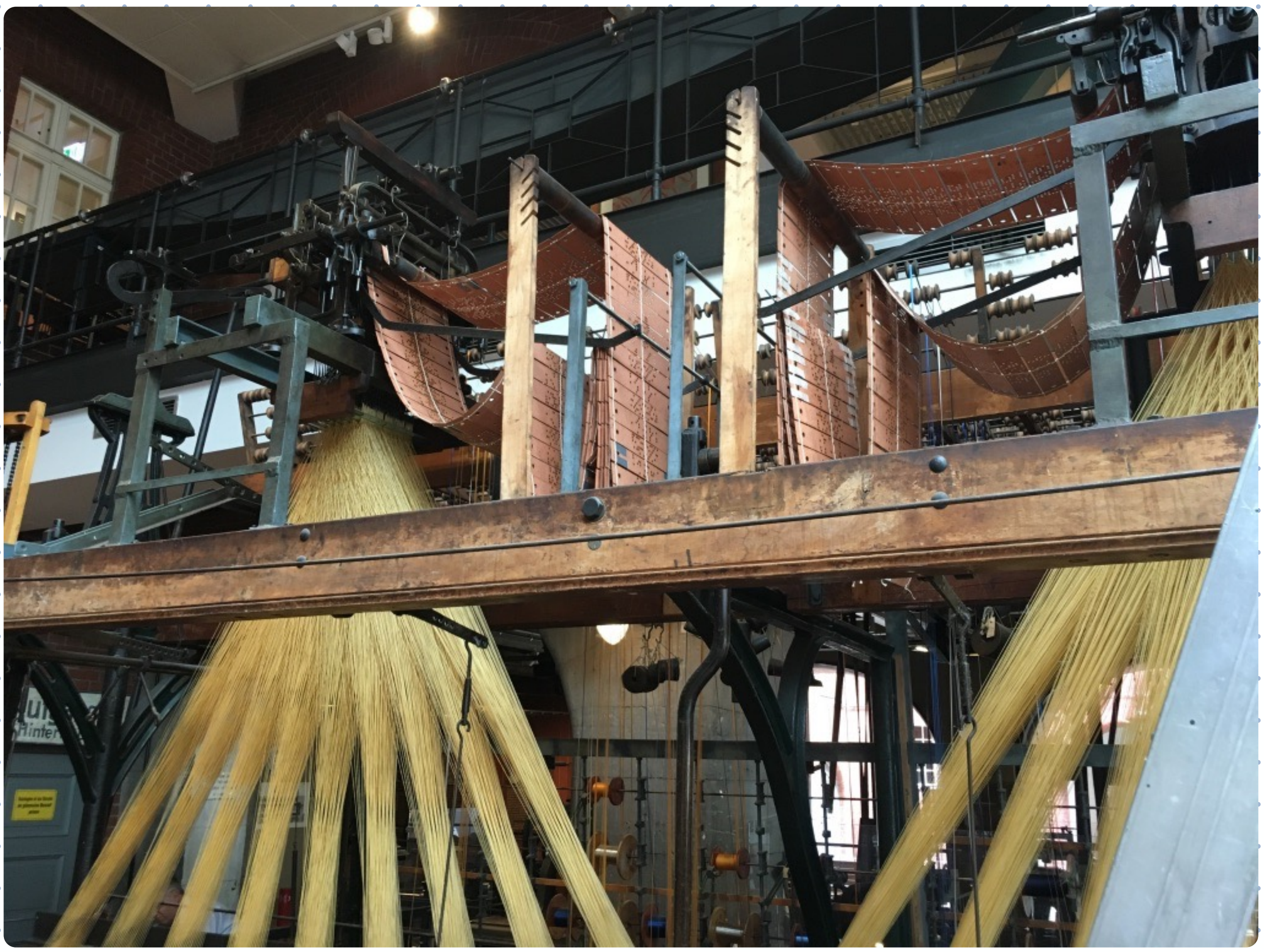
$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

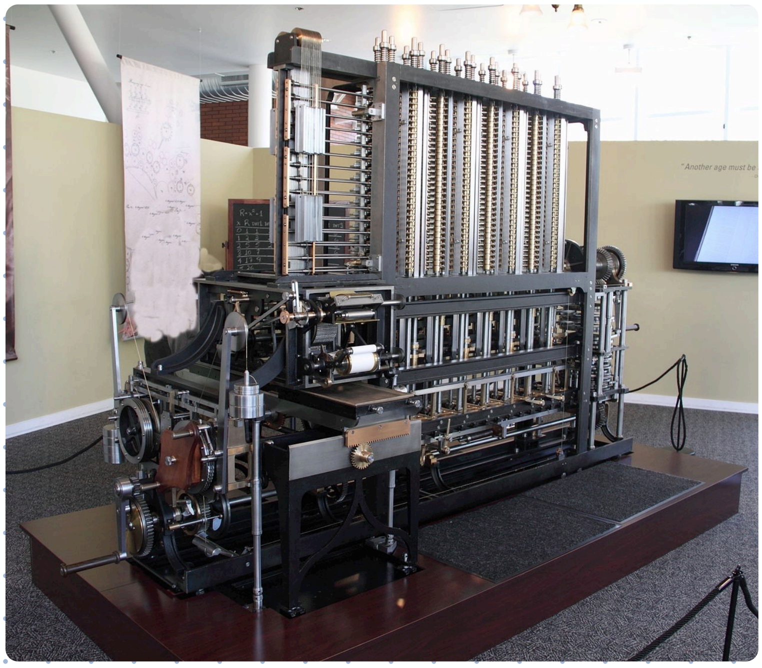
$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$$

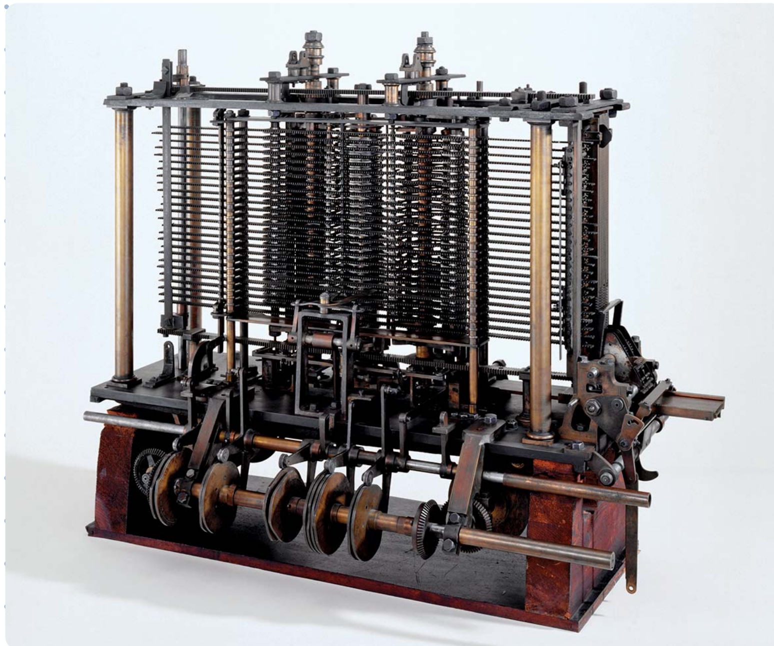
dinatæ reſpondenti curvæ VV) erit dy æqu. dv. Jam *Additio & Subtractio:* ſi ſit z -y † vv † x æqu. *v*, erit dz--y † vv † x ſeu d*v*, æqu. dz--dy † dvv † dx. *Multiplicatio*, dx*v* æqu. xd*v* † *v*dx, ſeu poſito y æqu. x*v*, fiet dy æqu. xd*v* † *v*dx. In arbitrio enim eſt vel formulam,



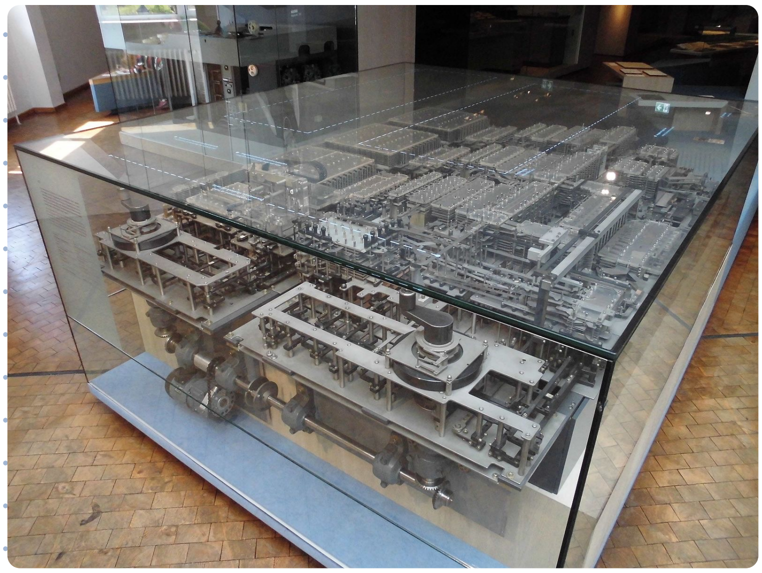
Jacquard loom (1804)



Babbage difference engine (designed 1822)



Babbage analytical engine (partially designed by 1877)



Zuse Z1 (1936–38)

One of Zuse's mechanical relays

```
COLLATZ(n):
  if n = 1
    return TRUE
  else if n is even
    n ← n/2
  else
    n ← 3n + 1
```

Sequences

↓

Strings

Let Σ be any finite set $\{0, 1\}$

A string is either

- empty ε
- $a \cdot x$ for some symbol $a \in \Sigma$ and some string x

STRING = S · TRING

= S · (T · RING) = ...

Length $|w|$ of a string w is...

$$|w| = \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = a \cdot x \end{cases}$$

Concatenation $w \bullet z$

$$w \bullet z = \begin{cases} z & \text{if } w = \varepsilon \\ a \cdot (x \bullet z) & \text{if } w = ax \end{cases}$$

$\underbrace{H}_{a}\underbrace{EAD}_{x} \bullet \underbrace{ACHE}_{z}$

Theorem: $|w \circ z| = |w| + |z|$ for all strings w and z

Proof: Let w and z be arbitrary strings.

IH: Assume $|x \circ z| = |x| + |z|$ for all strings x shorter than w.

There are two cases:

- $w = \varepsilon \Rightarrow$

$$\begin{aligned} |w \circ z| &= |\varepsilon \circ z| && w = \varepsilon \\ &= |z| && \text{by def } \circ \\ &= |\varepsilon| + |z| && \text{by def } |\,| \\ &= |w| + |z| && \text{because } w = \varepsilon \end{aligned}$$

- $w = ax \Rightarrow$

$$\begin{aligned} |w \circ z| &= |ax \circ z| && w = ax \\ &= |a \cdot (x \circ z)| && \text{def } \circ \\ &= 1 + |x \circ z| && \text{by def } |\,| \\ &\Downarrow ? \\ &= 1 + |x| + |z| && \text{by IH} \\ &= |ax| + |z| && \text{def } |\,| \\ &= |w| + |z| && w = ax \end{aligned}$$

So $|w \circ z| = |w| + |z|$