HW1 due 8pm tonight

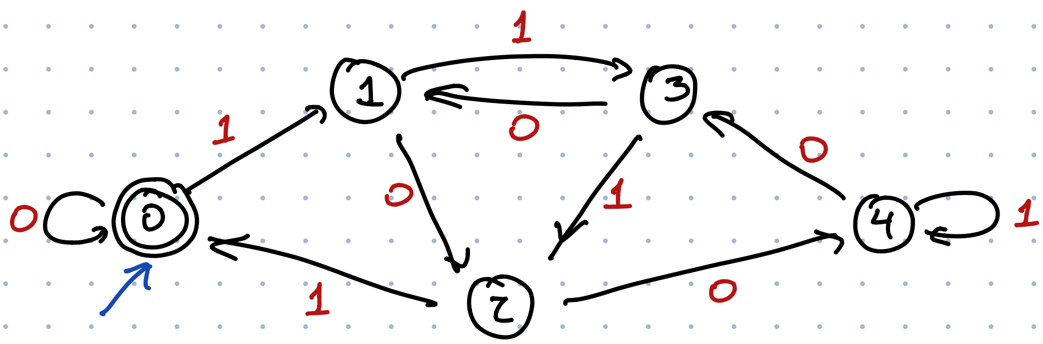
— Please upload a draft, with group names, by 5pm

HW2 due next Tue 8pm

Jeff is out of town next week (Sep 15–21)

$11001_2 = 25$

$10011_2 = 19$



Strings x and y are distinguished by string z

$xz \in L$ $\boxed{\text{xor}}$ $yz \in L$

$\Rightarrow \delta(s,x) \neq \delta(s,y)$ in <u>every</u> DFA for L

For some DFA M, if $\delta^*(s,x) = \delta^*(s,y)$ then

for all $z \in \Sigma^*$, $xz \in L \Leftrightarrow yz \in L$

because $\delta^*(s,xz) = \delta^*(s,yz)$

If there is a string z s.t. $xz \in L$ xor $yz \in L$

then for <u>every</u> DFA M

$\delta^*(s,x) \neq \delta^*(s,y)$

$L =$ binary #s div by 5

0
1
10
11
100

$x = 0$ $\quad xz = 0$ div 5!
$y = 1$ $\quad yz = 1$ not div 5!
$z = \varepsilon$

$x = 10$ $\quad xz = 101 = 5$ ✓
$y = 100$ $\quad yz = 1001 = 9$ ✗
$z = 1$

This is a fooling set for L

Every pair of elements has a dist. suffix

Every DFA for L has at least 5 states.

$L = \{0^n 1^n \mid n \geq 0\} = \{\varepsilon, 01, 0011, 000111, \ldots\}$ ← Jeff picks this

Let $F = 0^*$ ← You choose F

Claim: $\forall x, y \in F \quad x \neq y$
$\exists$ dist suffix z

Let x and y be arbitrary distinct strings in F. ← Jeff chooses x and y

Then $x = 0^n$ and $y = 0^m$ for some integers $n \neq m$

Let $z = 1^n$ ← You choose z

Then $xz = 0^n 1^n \in L$

But $yz = 0^m 1^n \notin L$ because $n \neq m$

You prove these.

So z distinguishes x and y

So F is a fooling set for L.

Because F is infinite, L cannot be regular.

$L_2 =$ palindromes $= \{w \mid w = w^R\} = \{\varepsilon, 0, 1, 00, 11, 10011011001, \ldots\}$

not 001 or 0101 or 00110 ...

Let $F = 0^*$

Let x and y be arb. distinct strings in F

So $x = 0^n$ and $y = 0^m$ where $m \neq n$

Let $z = 10^n$

Then $xz = 0^n 1 0^n$ is a palindrome

But $yz = 0^m 1 0^n$ is not because $m \neq n$

So z distinguishes x and y

So F is a fooling set for L.

Because F is infinite, L cannot be regular.

$L_3 = \{www \mid w \in \Sigma^*\} = \{\varepsilon, 000, 111, 001001001, \dots\}$

not 0000 or 01010101 or 1 or...

Let $F = 0^*$

Let x, y be arb distinct strings in F

Then $x = 0^n$ and $y = 0^m$ for some integers $n \neq m$

Let $z = 1\ 0^n 1\ 0^n 1$

Then $xz = \underbrace{0^n 1}\,\underbrace{0^n 1}\,\underbrace{0^n 1} \in L$

But $yz = \underbrace{0^m 1}\,\underbrace{0^n 1}\,\underbrace{0^n 1} \notin L$ because $n \neq m$

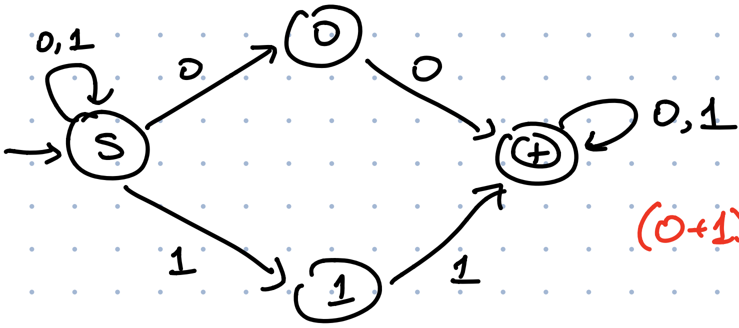
So z distinguishes x and y

So F is a fooling set for L.

Because F is infinite, L cannot be regular.

Kleene's Theorem: regular = automatic

DFA $\overset{\text{easy}}{\rightleftarrows}$ NFA $\rightleftarrows$ regex



$\delta^*(s, 10100) = \{s, 0, +\}$

$(0+1)^*(00+11)(0+1)^*$

DFA accepts $w = abc\cdots z$ iff the walk

$$s \xrightarrow{a} q_1 \xrightarrow{b} q_2 \rightarrow \cdots \xrightarrow{z} q_n \in A$$

NFA accepts $w = abc\cdots z$ iff there is a walk

$$s \xrightarrow{a} q_1 \xrightarrow{b} q_2 \rightarrow \cdots \xrightarrow{z} q_n \in A$$

NFA has the following components:

- Q – finite set of states
- $s \in Q$ start
- $A \subseteq Q$ accepting
- $\delta : Q \times \Sigma \rightarrow 2^Q$

subsets of Q

$$\delta^* : Q \times \Sigma^* \rightarrow 2^Q$$

$$\delta^*(q, w) = \begin{cases} \{q\} & w = \varepsilon \\ \bigcup_{q' \in \delta(q,a)} \delta^*(q', x) & w = ax \end{cases}$$