CS/ECE 374 A ✧ Fall 2019

Fake Midterm 1

September 30, 2019

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- ***Don't panic!***
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your real name, your university NetID, your Gradescope name, and your Gradescope email address in the boxes above. However, if you are using your real name and your university email address on Gradescope, you do ***not*** need to write everything twice. **We will not scan this page into Gradescope.**
- Please also print **only the name you are using on Gradescope** at the top of every page of the answer booklet, except this cover page. These are the pages we will scan into Gradescope.
- Please do not write outside the black boxes on each page; these indicate the area of the page that the scanner can actually see.
- If you run out of space for an answer, feel free to use the scratch pages at the back of the answer booklet, but **please clearly indicate where we should look**.
- Proofs are required for full credit if and only if we explicitly ask for them, using the word ***prove*** in bold italics.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper.

CS/ECE 374 A ✧ Fall 2019	Gradescope name:
Fake Midterm 1 Problem 1	Jeff E

For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point.

Yes No IDK — Every integer in the empty set is prime.

Yes No IDK — The language $\{0^m 1^n \mid m + n \le \cancel{374}\ 5\}$ is regular.

Yes No IDK — The language $\{0^m 1^n \mid m - n \le \cancel{374}\ 5\}$ is regular.

Yes No IDK — For all languages L, the language L^* is regular.

$L = \{0^n 1^n \mid n \geq 0\}$

Yes No IDK — For all languages L, the language L^* is infinite.

$L = \varnothing \rightarrow L^* = \{\varepsilon\}$

Yes No IDK — For all languages $L \subset \Sigma^*$, if L ~~can be represented by a regular expression~~ is regular, then $\Sigma^* \setminus L$ is ~~recognized by a DFA~~ regular.

Yes No IDK — For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

$L = \{0^n 1^n \mid n \geq 1\}$ $L' = \{1^n 0^n \mid n \geq 1\}$

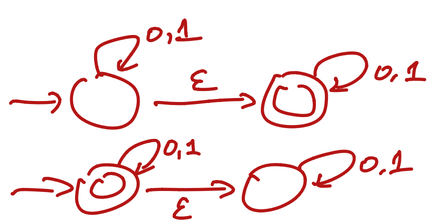
Yes No IDK — Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **DFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

Yes No IDK — Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **NFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

Yes No IDK — For all context-free language L, the language L^* is also context-free.

$S \rightarrow \varepsilon \mid LS$

$L \rightarrow$ - - -



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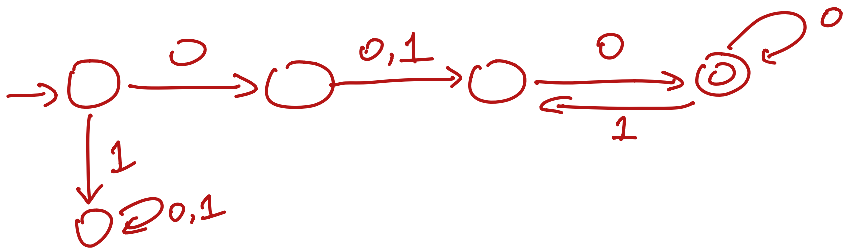
Fake Midterm 1 Problem 2	JeffE

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.*** Both of these languages contain the string 00110100000110100.

000001011000

1. $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

$0(0+1)^+0$



2. $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$

Let $F = 1^*$

Let x, y be arbitrary strings in F

$x = 1^n \quad y = 1^m$ for some $n \neq m$

Let $z = 01^n = 0x$

Then $xz = 1^n 0 1^n \in L$

$yz = 1^m 0 1^n \notin L$ because $m \neq n$

So F is fooling set, F is infinite $\square$

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Fake Midterm 1 Problem 3	Jeff E

The *parity* of a bit-string w is `0` if w has an even number of `1`s, and `1` if w has an odd number of `1`s. For example:

$$parity(\varepsilon) = \texttt{0} \qquad parity(\texttt{0010100}) = \texttt{0} \qquad parity(\texttt{00101110100}) = \texttt{1}$$

(a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do ***not*** refer to # or other functions defined in class.)

$$parity(w) = \begin{cases} 0 & w = \varepsilon \\ a \oplus parity(x) & w = ax \end{cases}$$

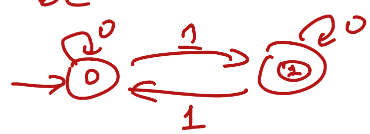
$(a + parity(x)) \bmod 2$

$\oplus$ is xor: $0 \oplus 0 = 1 \oplus 1 = 0$
$0 \oplus 1 = 1 \oplus 0 = 1$

(b) Let L be an arbitrary regular language. Prove that the language $OddParity(L) := \{w \in L \mid parity(w) = \texttt{1}\}$ is also regular.

Let M be a DFA for L, acc. states A

Let M' be



Let M'' be product of M and M'

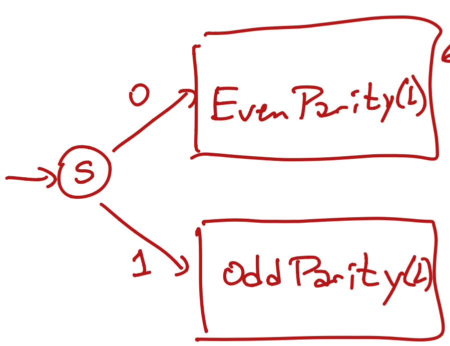
where $A'' = \{(q, q') \mid q \in A \text{ and } q' = 1\}$

$Odd = 0^*(10^*10^*)^*10^*$

$OddParity(L) = L \cap Odd.$

$$\boxed{EvenParity((0+1)\cdot L) = (0+1)\cdot L \cap 0^*(10^*10^*)}$$

(c) Let L be an arbitrary regular language. Prove that the language $AddParity(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular.



same as except $A'' = \{(q, q') \mid q \in A, q' = 0\}$

$Q' = \{s'\} \cup Q \times \Sigma \times \Sigma$

First bit

parity

$s' = s'$

$A = \{(q, a, b) \mid q \in A, a = b\}$

$\delta(s', 0) = (s, 0, 0)$

$\delta(s', 1) = (s, 1, 0)$

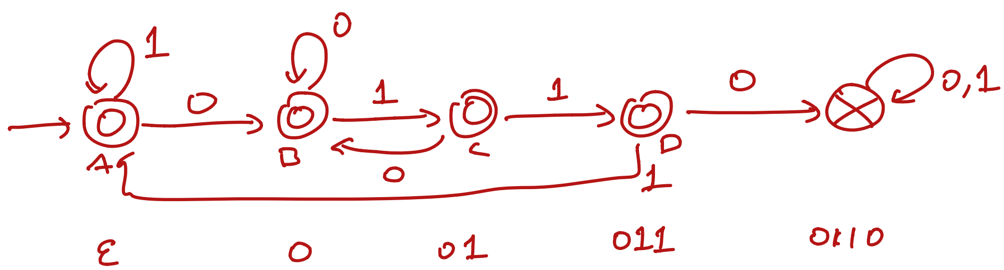
$\delta((q, a, b), c) = (\delta(q, c), a, b \oplus c)$

[Hint: Yes, you have enough room.]

CS/ECE 374 A ✧ Fall 2019 **Fake Midterm 1 Problem 4**	Gradescope name: Jeff E

For each of the following languages L, give a regular expression that represents L ***and*** describe a DFA that recognizes L. You do ***not*** need to prove that your answers are correct.

(a) All strings in $(0+1)^*$ that do not contain the substring 0110.



$A \to A$: 1 or 0 $B \to B$ 1 1 1

$(0+10)^*$

$\boxed{(1+0(0+10)^*111)^*(\varepsilon+0(0+10)^*(\varepsilon+1+1\cancel{0}1))}$

$\boxed{1^*0^*(0^+(1+1111^*))^*0^*1^*}$

(b) All strings in 0^*10^* whose length is a multiple of 3.

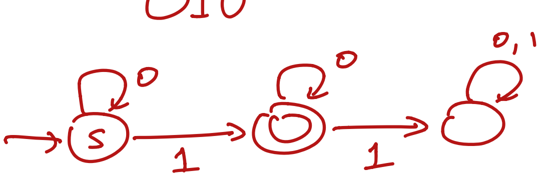
$(000)^*1(000)^*00$

$+\ (000)^*01\cancel{0}0(000)^*$

$+\ (000)^*001\cancel{0}(000)^*$

prod. con.

0^*10^* mult 3



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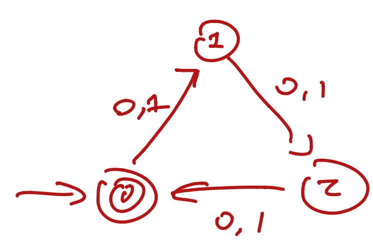

Accept iff <u>both</u> accept

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Fake Midterm 1 Problem 5	Jeff E

Consider the language L of all strings in $\{0,1\}^*$ in which the number of 0s is even, the number of 1s is divisible by 3, and the total number of symbols is divisible by 5. For example, the strings 01011 and 0000000000 are in L, but the strings 01001 and 10101010 are not.

Formally describe a DFA $M = (Q, s, A, \delta)$ over the alphabet $\Sigma = \{0,1\}$ that recognizes L. ***Do not attempt to <u>draw</u> the DFA. Do not use the phrase "product construction".*** Instead, formally and explicitly specify each of the the components Q, s, A, and δ.

$$Q = \underbrace{\{0,1\}}_{\#0} \times \underbrace{\{0,1,2\}}_{\#1} \times \underbrace{\{0,1,2,3,4\}}_{\text{len}}$$

$$s = (0,0,0)$$

$$A = \{(0,0,0)\}$$

$$\delta((p,q,r), 0) = (\,!p\,,\ q\,,\ r+1 \bmod 5)$$

$$\delta((p,q,r), 1) = (\,p\,,\ q+1 \bmod 3,\ r+1 \bmod 5)$$