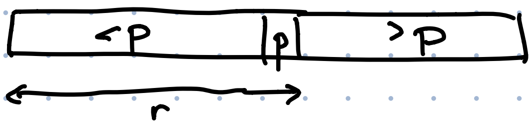
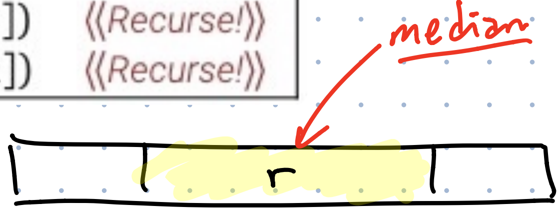
```
QuickSort(A[1..n]):
  if (n > 1)
      Choose a pivot element A[p]
      r ← Partition(A, p)
      QuickSort(A[1..r − 1])      ⟨⟨Recurse!⟩⟩
      QuickSort(A[r + 1..n])      ⟨⟨Recurse!⟩⟩
```

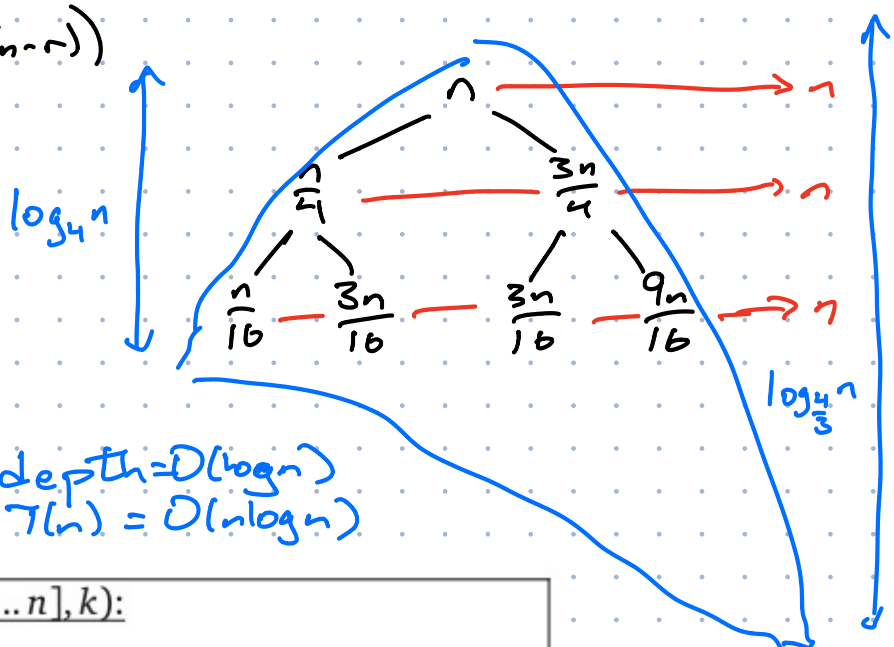




$T(n) \le O(n) + T(\frac{n}{4}) + T(\frac{3n}{4})$

$T(n) \le O(n) + \max_r (T(r-1) + T(n-r))$

$= O(n^2)$



depth $= O(\log n)$

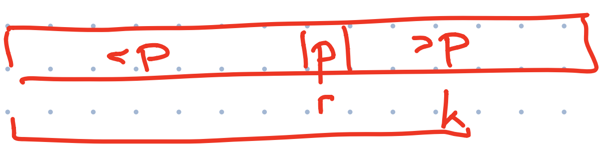
$T(n) = O(n \log n)$

Find the kth smallest element of input array

```
QuickSelect(A[1..n], k):
  if n = 1
      return A[1]
  else
      Choose a pivot element A[p]
      r ← Partition(A[1..n], p)

      if k < r
          return QuickSelect(A[1..r − 1], k)
      else if k > r
          return QuickSelect(A[r + 1..n], k − r)
      else // if r = k
          return A[r]
```

Figure 1.12. Quickselect, or one-armed quicksort



$T(n) \le O(n) + \max_r (\max\{T(r-1), T(n-r)\})$

$= O(n^2)$

Magic: if $\frac{n}{4} < r < \frac{3n}{4}$: $\quad T(n) \le O(n) + T\left(\frac{3n}{4}\right) = O(n)$

$$\sum_{i \ge 0} a^i = \frac{1}{1-a} \quad \text{if } |a| < 1$$

$\boxed{n}$

$\frac{3n}{4}$

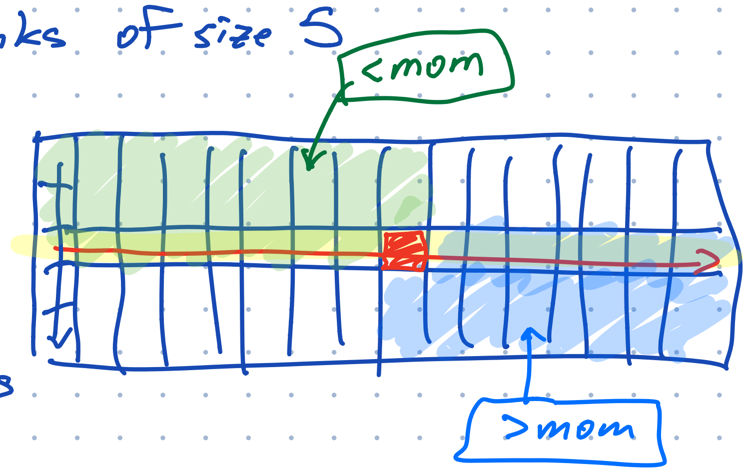
$\frac{9n}{16}$

$\vdots$

Descending geometric series

Blum Floyd Pratt Rivest Tarjan

split $A[1..n]$ into $\frac{n}{5}$ chunks of size 5



① Find the median of each chunk in $O(1)$ time

$\Uparrow$

$O(n)$ time

② Find median of those medians recursively!

③ Use mom as pivot in Quickselect

MomSelect($A[1..n], k$):
  if $n \le 25$ ⟨⟨or whatever⟩⟩
    use brute force
  else
    $m \leftarrow \lceil n/5 \rceil$
    for $i \leftarrow 1$ to m
      $M[i] \leftarrow$ MedianOfFive($A[5i-4..5i]$) ⟨⟨Brute force!⟩⟩
    $mom \leftarrow$ MomSelect($M[1..m], \lfloor m/2 \rfloor$) ⟨⟨Recursion!⟩⟩

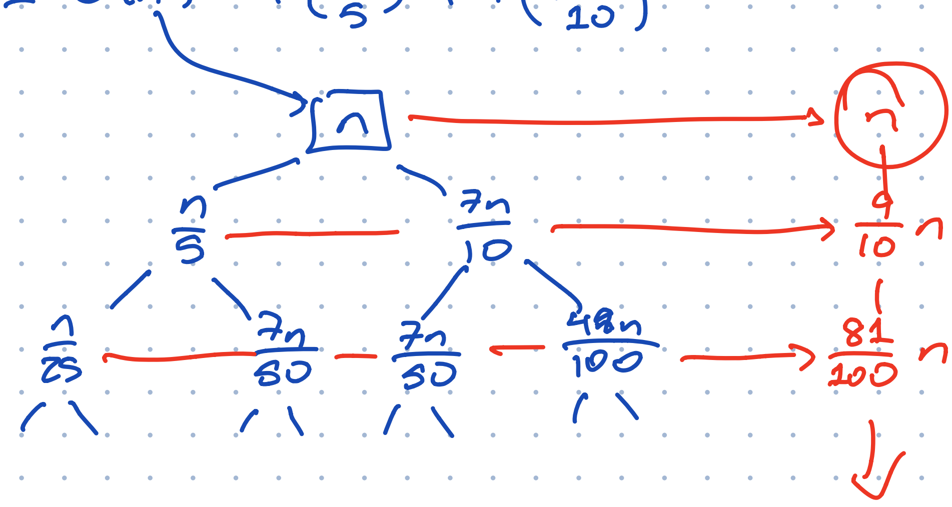
    $r \leftarrow$ Partition($A[1..n], mom$)

    if $k < r$
      return MomSelect($A[1..r-1], k$) ⟨⟨Recursion!⟩⟩
    else if $k > r$
      return MomSelect($A[r+1..n], k-r$) ⟨⟨Recursion!⟩⟩
    else
      return mom

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + O(n) + \max_r \{T(r-1), T(n-r)\}$$

$$\leq O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$$

$O(n)$

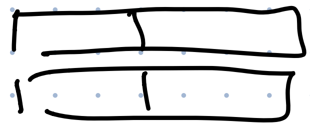


$$\begin{array}{r} 123 \\ 456 \\ \hline 738 \\ 615 \\ 492 \\ \hline 56088 \end{array}$$

$\leftarrow$ 2 n digit #s

multiply in $O(n^2)$ time

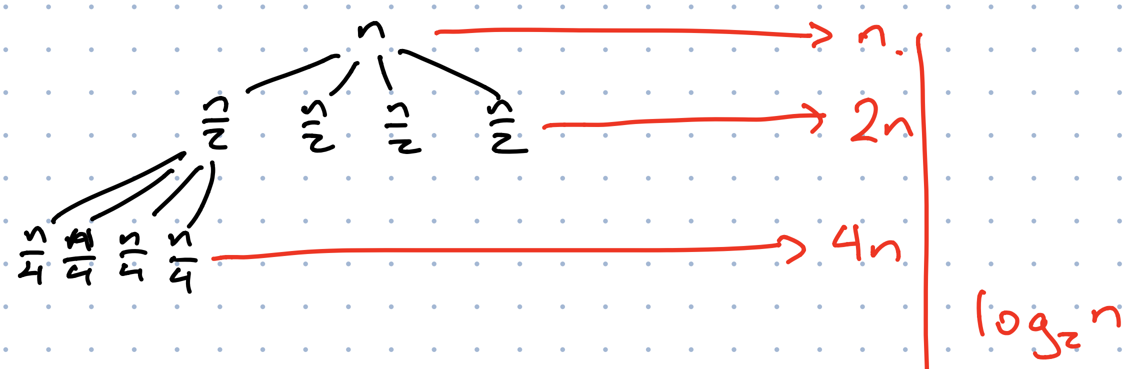
"n^2 conjecture"



$x = a + b \cdot 10^{n/2}$ $\qquad$ $y = c + d \cdot 10^{n/2}$

```
SplitMultiply(x, y, n):
    if n = 1
        return x · y
    else
        m ← ⌈n/2⌉
        a ← ⌊x/10^m⌋;  b ← x mod 10^m        ⟨⟨x = 10^m a + b⟩⟩
        c ← ⌊y/10^m⌋;  d ← y mod 10^m        ⟨⟨y = 10^m c + d⟩⟩
        e ← SplitMultiply(a, c, m)
        f ← SplitMultiply(b, d, m)
        g ← SplitMultiply(b, c, m)
        h ← SplitMultiply(a, d, m)
        return 10^(2m) e + 10^m (g + h) + f
```

$$T(n) = 4T\left(\frac{n}{2}\right) + O(n)$$



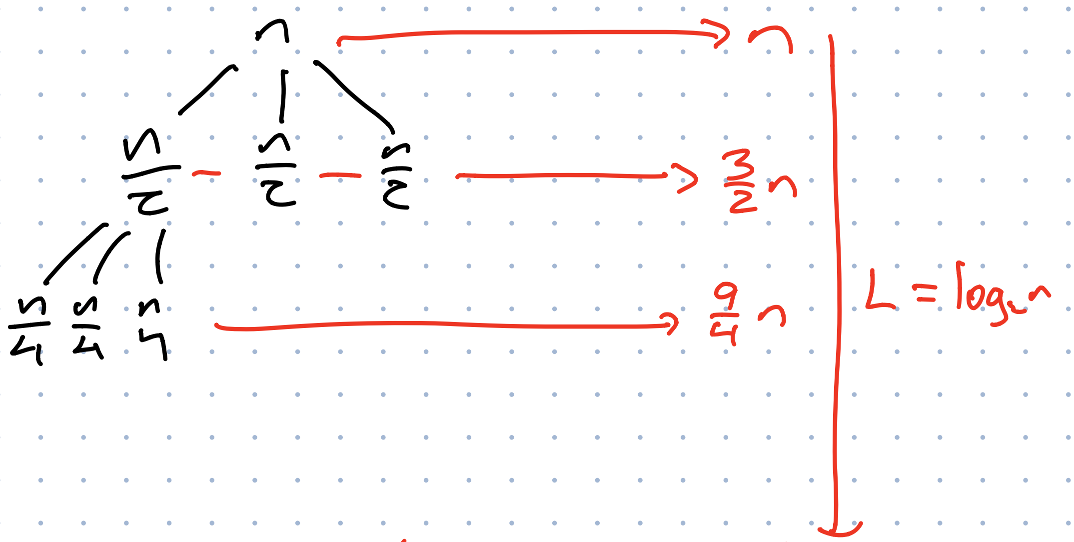
$$\underline{n^2} = 2^{\log_2 n} \cdot n = \boxed{2^L \cdot n}$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array} = \boxed{ac}\boxed{ad+bc}\boxed{bd}$$

$$ac + bd - (a-b)(c-d) = bc + ad$$

```
FastMultiply(x, y, n):
  if n = 1
      return x · y
  else
      m ← ⌈n/2⌉
      a ← ⌊x/10^m⌋;  b ← x mod 10^m        ⟨⟨x = 10^m a + b⟩⟩
      c ← ⌊y/10^m⌋;  d ← y mod 10^m        ⟨⟨y = 10^m c + d⟩⟩
      e ← FastMultiply(a, c, m)
      f ← FastMultiply(b, d, m)
      g ← FastMultiply(a − b, c − d, m)
      return 10^{2m} e + 10^m (e + f − g) + f
```

$$T(n) = 3T\left(\tfrac{n}{2}\right) + O(n)$$



$$O(n^{1.78}) = n^{\log_2 3/2} \cdot n = \left(\tfrac{3}{2}\right)^{\log_2 n} \cdot n = \left(\tfrac{3}{2}\right)^{L} \cdot n$$

Gauss
↓
FFTs

$O(n \log n \cdot \text{noise})$ (noise: $\log \log n$)

Strassen

$$\rightarrow O(n \log n) \leftarrow$$