Drop deadline is THIS Friday, not next Friday.

Jeff OH: Wed 4pm – 5pm
Fri 3pm – 4pm

This week Thursday 2-5
Friday 9-12, 1-5

HW 6 out

Homework – use any techniques
use any subroutines
don't vomit old stuff
explain anything new.

We just...

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1. Design a recursive algorithm (backtracking)
2. Notice lots of repeated subproblems
   Choose a data structure to memoize (usually array)
3. Choose an evaluation order
   recursion → iteration
   base cases ⟹ Final solution
4. Analyze time

EDIT DISTANCE

Lewenstein	1965
Ulam	1972
Vintsyuk	1968
libCurses	1980s

FOOD
MOOD
MONOD
MONEY

Replacement FOO → FAO
Insertion FOO → FNOD
Deletion FOO → FO

F O O D
M O N E Y

Rep x y
Ins □ y
Del x □

A L G O R I T H M ← A[1..m]
A L T R U I S T I C ← B[1..n]
1 2 3 4 5 6

What is the LAST column in an optimal edit sequence?

- Replacement: A[1..m-1] B[1..n-1] | A[m] B[n]
- Insertion: A[1..m] B[1..n-1] | B[n]
- Deletion: A[1..m-1] B[1..n] | A[m]

Edit(i, j) = edit distance between A[1..i] and B[1..j]

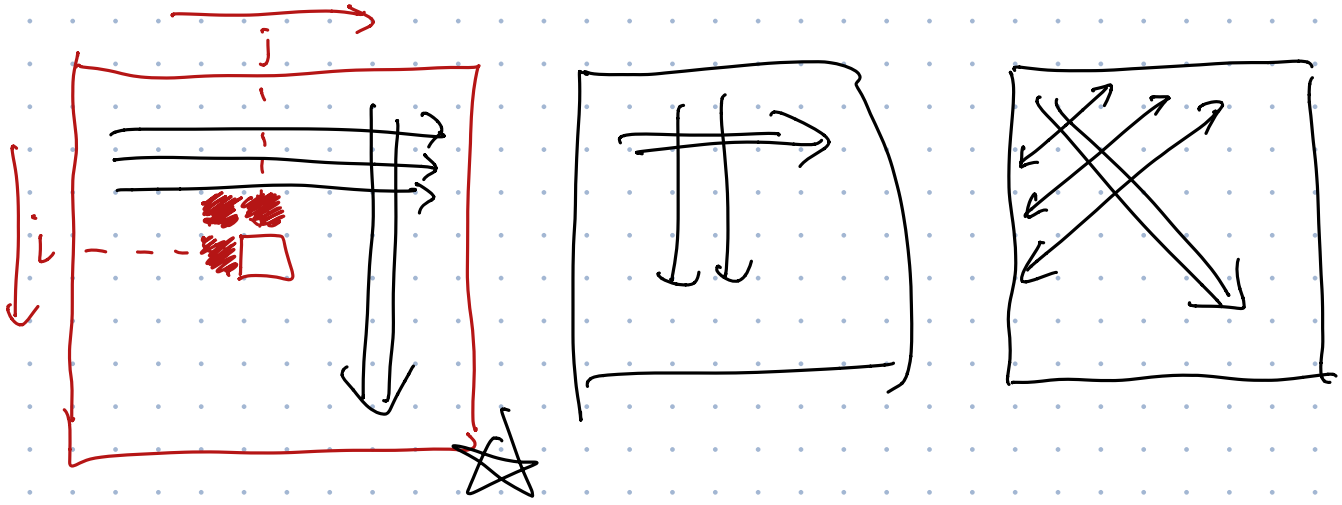
We need Edit(m, n).

$$Edit(i,j) = \begin{cases} j & \text{if } i = 0 \\ i & \text{if } j = 0 \\ \min \begin{cases} [A[i] \neq B[j]] + Edit(i-1, j-1) \\ 1 + Edit(i, j-1) \\ 1 + Edit(i-1, j) \end{cases} & \text{otherwise} \end{cases}$$

0..m 0..n

$$Edit(i,j) = \begin{cases} i & \text{if } j = 0 \\ j & \text{if } i = 0 \\ \min \left\{ \begin{array}{c} Edit(i,j-1)+1 \\ Edit(i-1,j)+1 \\ Edit(i-1,j-1)+[A[i] \neq B[j]] \end{array} \right\} & \text{otherwise} \end{cases}$$

Edit[0..m, 0..n]

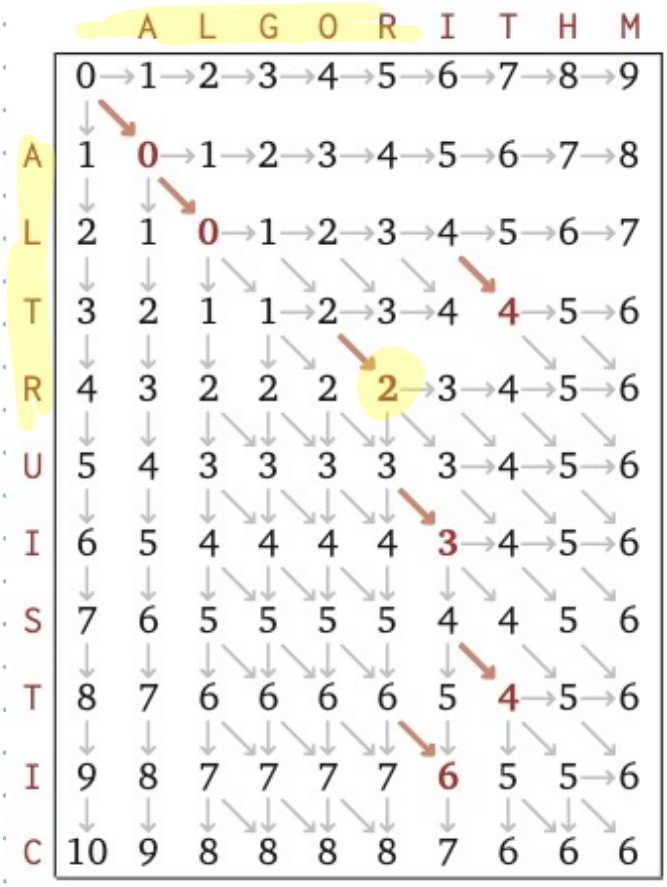



```
EditDistance(A[1..m], B[1..n]):
  for j ← 0 to n
      Edit[0, j] ← j

  for i ← 1 to m
      Edit[i, 0] ← i
      for j ← 1 to n
          ins ← Edit[i, j − 1] + 1
          del ← Edit[i − 1, j] + 1
          if A[i] = B[j]
              rep ← Edit[i − 1, j − 1]
          else
              rep ← Edit[i − 1, j − 1] + 1
          Edit[i, j] ← min {ins, del, rep}

  return Edit[m, n]
```

O(mn) time


A L G O R I T H M
0 1 2 3 4 5 6 7 8 9
A 1 0 1 2 3 4 5 6 7 8
L 2 1 0 1 2 3 4 5 6 7
T 3 2 1 1 2 3 4 4 5 6
R 4 3 2 2 2 2 3 4 5 6
U 5 4 3 3 3 3 3 4 5 6
I 6 5 4 4 4 4 3 4 5 6
S 7 6 5 5 5 5 4 4 5 6
T 8 7 6 6 6 6 5 4 5 6
I 9 8 7 7 7 7 6 5 5 6
C 10 9 8 8 8 8 7 6 6 6