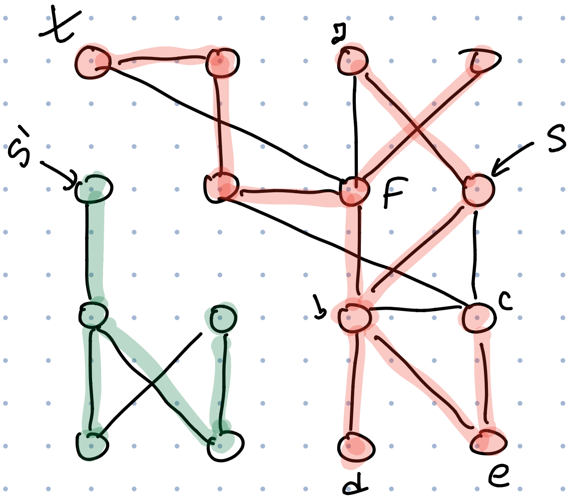
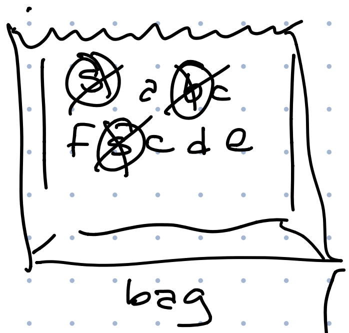


whatever-first search

traversal — connectivity / components / reachability

$uv = \{u,v\}$

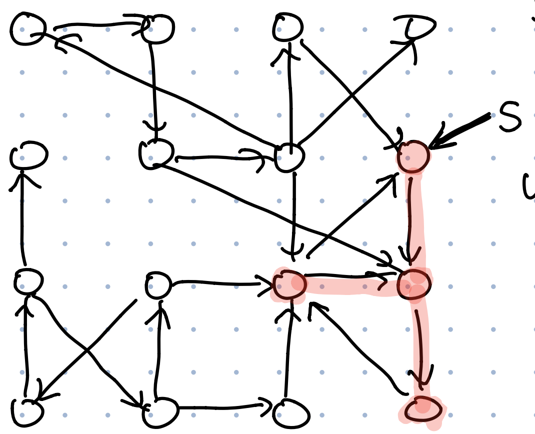
time $= O(V+E)$



connected $\Rightarrow E \geq V-1$

$V \leq E+1$

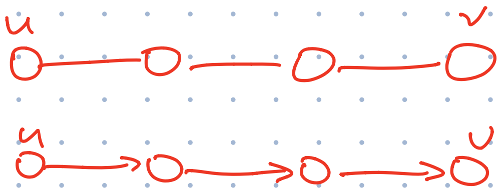
time $= O(E)$



$u \to v = (u,v)$

keep a bag of vertices, init s

```
while bag not empty
    remove v from bag
    if v unmarked
        mark v
        for all edges v→w
            put w in bag
```



u can reach v

DFS(v):

```
mark v
PREVISIT(v)
for every edge v→w
    if w is unmarked
        parent(w) ← v
        DFS(w)
POSTVISIT(v)
```

DFSALL(G):

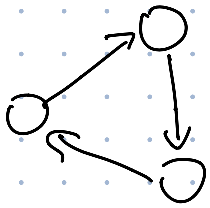
```
PREPROCESS(G)
for all vertices v
    unmark v

for all vertices v
    if v is unmarked
        DFS(v)
```

$O(V+E)$ time

not counting PREP + VISIT

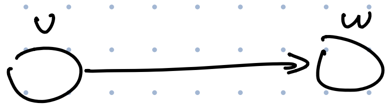
```
DFS(v):
  mark v
  v.pre ← clock++
  for every edge v→w
    if w is unmarked
      parent(w) ← v
      DFS(w)
  v.post ← clock++
```

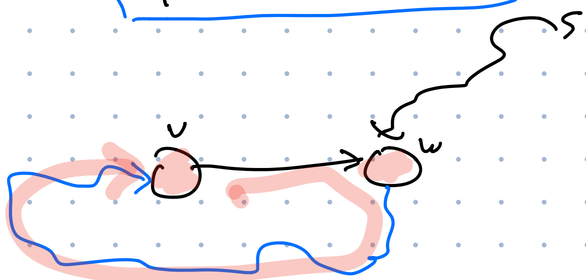
```
DFSAll(G):
  clock ← 0
  for all vertices v
    unmark v

  for all vertices v
    if v is unmarked
      DFS(v)
```



Lemma: After DFSAll(G)

If G has a dir. cycle, then for some edge $v \to w$ we have $\boxed{v.post < w.post}$





If DFS reaches v first:

$v.pre < w.pre < \boxed{w.post < v.post}$

If DFS reaches w first:

$w.pre < v.pre$

(1) w can reach v.

$w.pre < v.pre < \boxed{v.post < w.post}$

(2) w can't reach v.

$w.pre < \boxed{w.post < \boxed{v.pre} < v.post}$

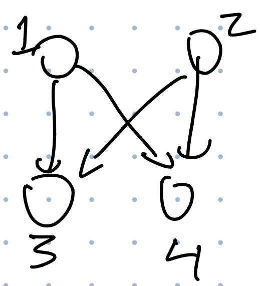
Suppose there is a dir cycle

Let w be first vertex reached by DFS in that cycle

Let $v \to w$ be edge in cycle

Iff $v.post > w.post$ for all $v \to w$ then $\boxed{\text{G has no directed cycles}}$

G is a dag.



Every dag has a topological ordering

$num(v) < num(w)$ for all $v \to w$

Proof: Let $num(v) = 2V - post(v)$ ∎

Topological sort = reverse postorder

Preprocess
clock ← V ← #vertices

Previsit(v):
yawn

Postvisit(v):
Top[clock--] ← v

$\boxed{O(V+E) \text{ time}}$

$\boxed{\text{ALL DP = DFS}}$