HW 10 only has 2 problems

Prove X is NP-hard

Reduce known NP-hard problem to X in poly time

Default → 3SAT

3! → 3SAT, 3Color

- CircuitSat
- 3SAT

] Binary choices

- Max Clique
- Max Ind Set
- Min Vertex Cover

] Largest/Smallest subset

- 3Color
- Min Colors

] ~~Many~~ more than 2 subsets/few subsets as possible

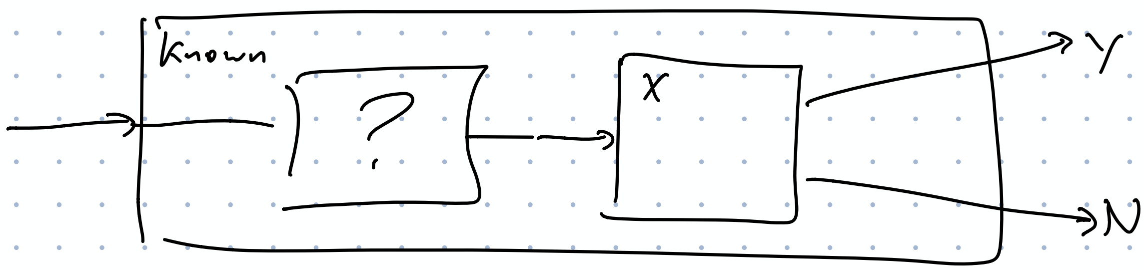
- Hamiltonian Cycle/Path

] ordering/path/cycle long sequence

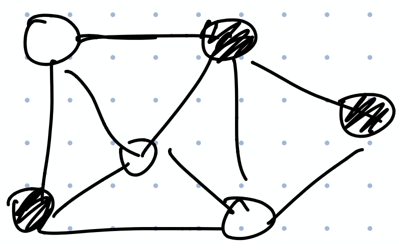
⋮

- Partition/SubsetSum
- 3Partition

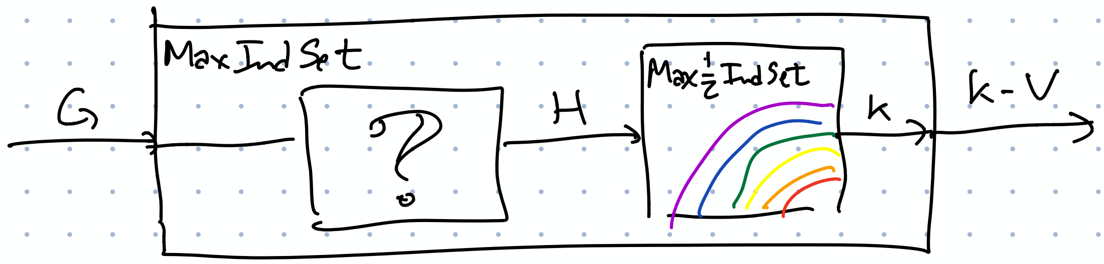
] balance ← ferry problem!

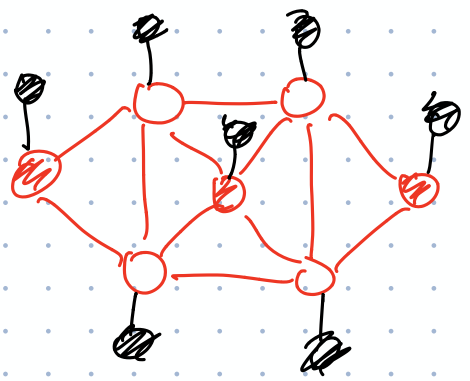


A subset S of vertices in an undirected graph G is *half-independent* if each vertex in S is adjacent to *at most one* other vertex in S. Prove that finding the size of the largest half-independent set of vertices in a given undirected graph is NP-hard.



Reduce from Max Independent Set





To build H

add a new leaf to every vertex of G

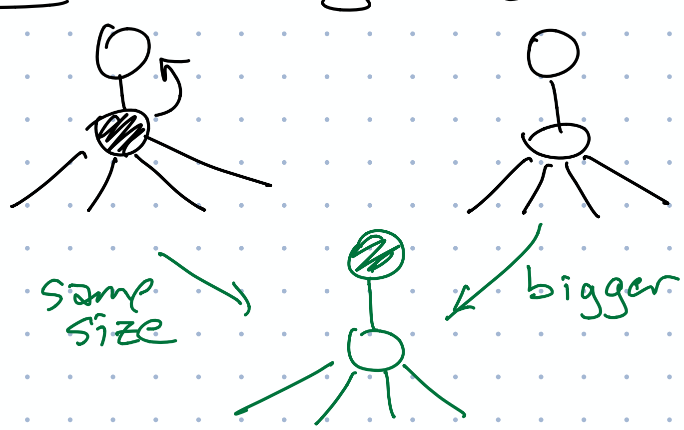
Call new leaves L

⟹ Let S be any independent set in G

Then $S \cup L$ is a half-ind set in H ✓

⟸ Let S be any largest half-ind set in H

Claim: Some largest ½-ind set in H contains L



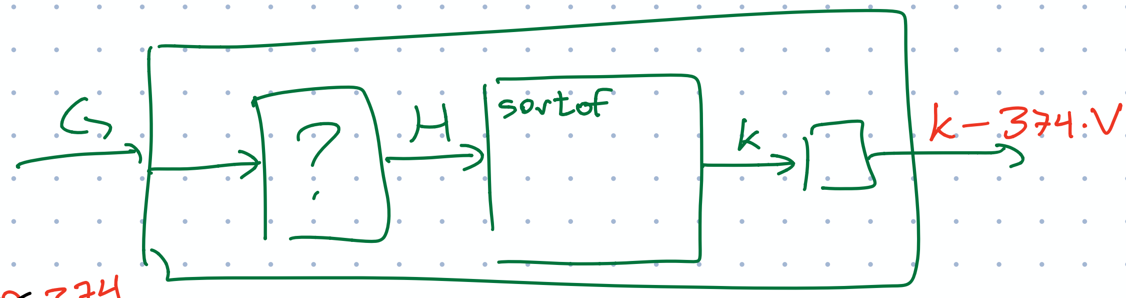
Move/add marks to L

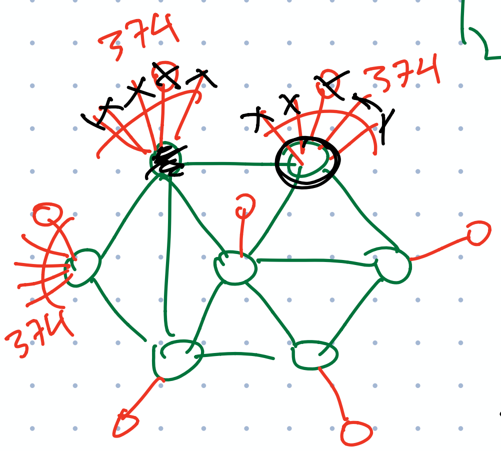
→ new ½ ind set S′

Now $S' \setminus L$ is indep set in G. ✓

A subset S of vertices in an undirected graph G is *sort-of-independent* if if each vertex in S is adjacent to *at most 374* other vertices in S. Prove that finding the size of the largest sort-of-independent set of vertices in a given undirected graph is NP-hard.

Reduce from Max Ind Set





⇒ If S is ind set in G

then S∪L is sort-of ind set in H

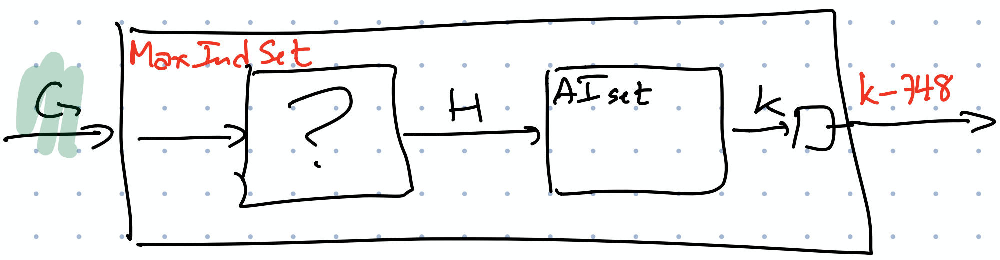
⇐ Largest sort-of ind set S in H

Move marks to leaves → S'

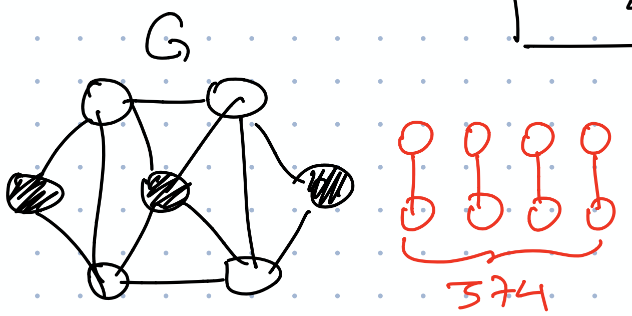
S' \ L is ind set in G.

A subset S of vertices in an undirected graph G is *almost independent* if at most 374 edges in G have both endpoints in S. Prove that finding the size of the largest almost-independent set of vertices in a given undirected graph is NP-hard.

Reduce from MaxIndepSet!



Add 374 indep edges – call vertices W

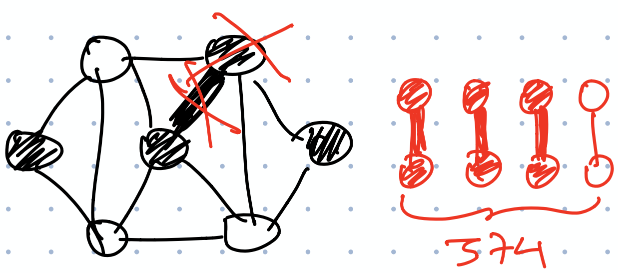


⇒ S is indep set in G ⟶ S∪W is almost-ind set in H. ✓

⇐ Let S be largest almost-ind set in H

spose some vertex in W is unmarked
spose some edge e in G is bad

↑ unmark one end of e
mark some vertex in W
→ new almost-ind set



⟶ S′ almost indep $|S'| = |S|$

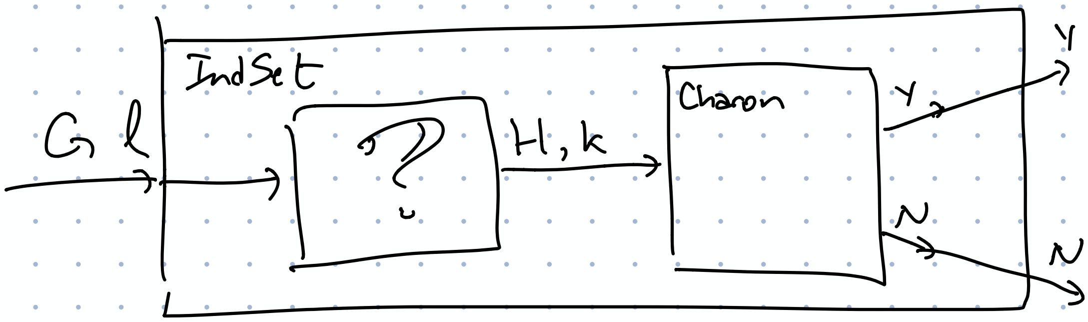
$S' \setminus W$ is indep in G ✓

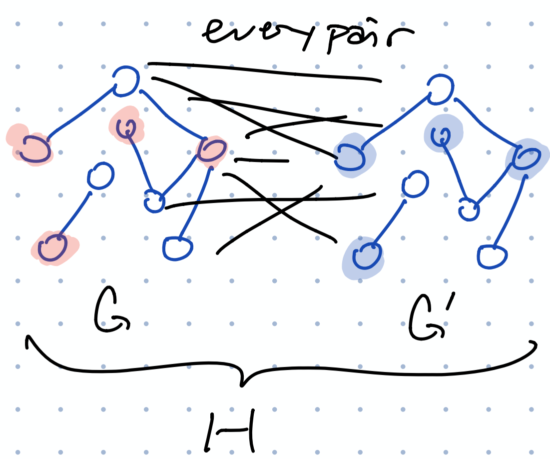
Charon needs to ferry n recently deceased people across the river Acheron into Hades. Certain pairs of these people are sworn enemies, who cannot be together on either side of the river unless Charon is also present. (If two enemies are left alone, one will steal the obol from the other's mouth, leaving them to wander the banks of the Acheron as a ghost for all eternity. Let's just say this is a Very Bad Thing.) The ferry can hold at most k passengers at a time, including Charon, and only Charon can pilot the ferry.

Prove that it is NP-hard to decide whether Charon can ferry all n people across the Acheron unharmed (aside from being, you know, dead). The input for Charon's problem consists of the integers k and n and an n-vertex graph G describing the pairs of enemies. The output is either TRUE or FALSE.

Reduce from Ind Set

Does G have an ind set of size k?





$k = 2V - \ell + 1$

⇒ Suppose S is ind set of size ℓ in G

① $S \xrightarrow{(V \cup V') \setminus S} \emptyset$

② $S \xleftarrow{(V \cup V') \setminus (S \cup S')} S'$

③ $\emptyset \xrightarrow{(V \cup V') \setminus S'} S'$